Identity Based Threshold Ring Signature

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Abstract. In threshold ring signature schemes, any group of t entities spontaneously conscript arbitrarily $n - t$ entities to generate a publicly verifiable t-out-of-n signature on behalf of the whole group, yet the actual signers remain anonymous. The spontaneity of these schemes is desirable for ad-hoc groups such as mobile ad-hoc networks. In this paper, we present an identity based (ID-based) threshold ring signature scheme. The scheme is provably secure in the random oracle model and provides trusted authority compatibility. To the best of authors' knowledge, our scheme is the first ID-based threshold ring signature scheme which is also the most efficient (in terms of number of pairing operations required) ID-based ring signature scheme (when $t = 1$) and threshold ring signature scheme from pairings.

Keywords: Threshold ring signature, identity-based signature, bilinear pairings, anonymity, spontaneity

1 Introduction

Anonymity is becoming a major concern in many multi-user electronic commerce applications such as e-lotteries [15], e-cash and online games [12]. Group-oriented signature schemes [9] enable an entity of a group to produce a signature on behalf of the group. There are two major paradigms in anonymous group-oriented signature schemes: group signature and ring signature. In a group signature scheme, the group is predefined and there is a group manager that can revoke this anonymity. Ring signature scheme provides a similar feature. It does not support anonymity revocation mechanism, but no setup stage is needed to produce and distribute a group secret explicitly. Hence it enables any individual to spontaneously conscript arbitrarily $n - 1$ entities and generate a publicly verifiable 1-out-of-n signature on behalf of the whole group, yet the actual signer remains unconditionally anonymous. Threshold ring signature is the t-out-of-n threshold version where t or more entities can jointly generate a valid signature but $t - 1$ or fewer entities cannot. These schemes are getting more and more popular due to the increasing prevalence of pervasive computing applications and mobile ad-hoc networks, where ad-hoc groups are very common [7].

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1.1 Motivation of ID-based Threshold Ring Signature

In traditional public key infrastructure (PKI), a user must pre-enroll the PKI or he/she cannot enjoy the cryptographic services provided by the PKI, e.g. no one can send them any encrypted message. Identity-based (ID-based) cryptography [5, 30] solves this problem: all users already have their corresponding public key before their enrollment since the public key can be derived via a public algorithm with input of a string that can uniquely identify each of them, such as an email address.

All previous threshold ring signature constructions are non ID-based, hence *real spontaneity* is not always possible: the public key of each member of the group is required to be published by the underlying PKI before it can be used to generate the signature. Removing this pre-requisite requirement motivates the construction of ID-based threshold ring signature scheme, which provide a better alternative than non-ID based solutions[1].

1.2 Related Work

Ring signature scheme was first formalized by Rivest *et al.* in [28]. After that, several other ring signature schemes (for examples [1, 20]) were proposed. Bresson *et al.* [7] extended the ring signature scheme into a threshold ring signature scheme using the concept of partitioning. Later, Wong *et al.* [34] proposed another threshold ring signature using tandem construction method. In [18], a constant-size ring signature was derived from the anonymous identification scheme proposed.

There are some threshold ring signature schemes with special properties recently. For examples, Liu *et al.* [24] introduced the concept of separability to threshold ring signature scheme, which enables the use of various flavours of public keys in a single threshold ring signature; Tsang *et al.* [32] introduced individual-linkability to threshold ring signature scheme, which enables anyone to determine if two ring signatures are signed with the help of the same signer; and Chan *et al.* [8] constructed CDS-type [17] t-out-of-n blind threshold ring signature, such that the signers do not know what exactly they are signers and cannot link which invocation of signing algorithm corresponding to which unblinded signature. In [24], a generic construction of threshold ring signature from any trapdoor-one-way type signature scheme and three-move type signature scheme is given. Yet, the authors have not illustrated the correctness and the security of this construction except the specific instantiations from RSA [27] and Schnorr signature [29].

Using bilinear pairing to construct ring signature is not a new idea. Inspired by the aggregate signature, a ring signature scheme was proposed in [6]. A technique similar to that of [6] was used to derive a new ring signature scheme

[1] Under the assumption that the trusted authority (the private key generator) will not reveal any information about who has requested for his/her private key and who has not.

in [35]. In [38], a ring signature was derived from the short signature proposed. A proxy ring signature was proposed in [39]. ID-based ring signature was introduced in [37] and subsequently a more efficient construction was proposed in [23]. Small inconsistencies in [37] and [23] were fixed by [2], together with a new proxy ring signature scheme from the delegation function due to [39]. Another ID-based ring signature with formally proven security was proposed in [22]. Threshold ring signature scheme from pairings was proposed in [33], but this scheme is not ID-based and has not addressed the requirement of TA (trusted authority) compatibility [36] in which not all the users join the same TA.

1.3 Our Contributions

In this paper, we present an ID-based threshold ring signature scheme. The scheme is provably secure in the random oracle model [4] and provides TA compatibility [36]. To the best of authors' knowledge, our scheme is the first ID-based threshold ring signature scheme. Our scheme is the most efficient (in terms of number of pairing operations required) ID-based ring signature scheme (when the threshold value $t = 1$) and also the most efficient threshold ring signature scheme from pairings.

1.4 Organization

The rest of the paper is organized as follows. The next section contains some preliminaries about the formal definitions of an ID-based threshold ring signature scheme, bilinear pairing as well as the Gap Diffie-Hellman group. Formal security definitions describing the adversary's capabilities and goals are presented in Section 3. Section 4 describes the proposed ID-based threshold ring signature scheme. The security and efficiency analysis of our scheme are given in Section 5. Finally, Section 6 concludes the paper.

2 Preliminaries

Before presenting our results, we give the framework of ID-based threshold ring signature schemes and review the definitions of bilinear pairing and Gap Diffie-Hellman groups.

2.1 Framework of ID-Based Threshold Ring Signature

An ID-based threshold ring signature scheme consists of four algorithms: `Setup`, `KeyGen`, `Sign`, and `Verify`.

- `Setup`: On an unary string input 1^k where k is a security parameter, it produces the master secret key s and the common public parameters $params$, which include a description of a finite signature space and a description of a finite message space.

- `KeyGen`: On an input of signer's identity $ID \in \{0,1\}^*$ and the master secret s, it outputs the signer's secret signing key S_{ID}.
- `Sign`: On input of a message m, a group of n users' identities $\{ID_i\}$, where $1 \leq i \leq n$, and the secret keys of t members $\{S_{ID_{i_j}}\}$, where $1 \leq i_j \leq n$, $1 \leq j \leq t$ and $t \leq n$; it outputs a (t, n) ID-based threshold ring signature σ on the message m.
- `Verify`: On a threshold ring signature σ, a message m, the threshold value t and the group of signers' identities $\{ID_i\}$ where $1 \leq i \leq n$ as the input, it outputs $\top$ for "true" or $\perp$ for "false", depending on whether σ is a valid signature signed by at least t members in the group $\{ID_i\}$ on a message m.

These algorithms must satisfy the standard consistency constraint of ID-based threshold ring signature schemes, i.e. if we have $\sigma = \mathtt{Sign}(m, \{ID_i\}, \{S_{ID_{i_j}}\})$ and $|\{S_{ID_{i_j}}\}| = t$ (where $|\{S_{ID_{i_j}}\}|$ denotes the number of elements in the set $\{S_{ID_{i_j}}\}$), we must have $\mathtt{Verify}(\sigma, \{ID_i\}, m, t) = \top$. Security requirements will be described in Section 3.

2.2 Bilinear Pairing and Gap Diffie-Hellman Groups

Bilinear pairing is an important primitive for many cryptographic schemes (for examples, [2, 5, 6, 11, 13, 14, 16, 21–23, 25, 33, 35–39]). Here, we describe some of its key properties.

Let $(\mathbb{G}_1, +)$ and $(\mathbb{G}_2, \cdot)$ be two cyclic groups of prime order q. The bilinear pairing is given as $\hat{e} : \mathbb{G}_1 \times \mathbb{G}_1 \to \mathbb{G}_2$, which satisfies the following properties:

1. *Bilinearity*: For all $P, Q, R \in \mathbb{G}_1$, $\hat{e}(P+Q, R) = \hat{e}(P, R)\hat{e}(Q, R)$, and $\hat{e}(P, Q+R) = \hat{e}(P, Q)\hat{e}(P, R)$.
2. *Non-degeneracy*: There exists $P, Q \in \mathbb{G}_1$ such that $\hat{e}(P, Q) \neq 1$.
3. *Computability*: There exists an efficient algorithm to compute $\hat{e}(P, Q)\ \forall P, Q \in \mathbb{G}_1$.

Definition 1. *Given a generator P of a group $\mathbb{G}$ and a 3-tuple (aP, bP, cP), the Decisional Diffie-Hellman problem (DDHP) is to decide whether $c = ab$.*

Definition 2. *Given a generator P of a group $\mathbb{G}$, (P, aP, bP, cP) is defined as a valid Diffie-Hellman tuple if $c = ab$.*

Definition 3. *Given a generator P of a group $\mathbb{G}$ and a 2-tuple (aP, bP), the Computational Diffie-Hellman problem (CDHP) is to compute abP.*

Definition 4. *If $\mathbb{G}$ is a group such that DDHP can be solved in polynomial time but no probabilistic algorithm can solve CDHP with non-negligible advantage within polynomial time, then we call $\mathbb{G}$ a Gap Diffie-Hellman (GDH) group.*

We assume the existence of a bilinear map $\hat{e} : \mathbb{G}_1 \times \mathbb{G}_1 \to \mathbb{G}_2$ that one can solve Decisional Diffie-Hellman Problem in polynomial time.

3 Formal Security Model

For an ID-based threshold ring signature scheme to be considered as secure, we need to consider its unforgeability and signer ambiguity.

3.1 Unforgeability of ID-Based Threshold Ring Signature

The following EUF-IDTR-CMIA2 game played between a challenger $\mathcal{C}$ and an adversary $\mathcal{A}$ formally defines the *existential unforgeability of ID-based threshold ring signature under adaptive chosen-message-and-identity attack*.

EUF-IDTR-CMIA2 Game:

Setup: The challenger $\mathcal{C}$ takes a security parameter k and runs the `Setup` to generate common public parameters $params$ and also the master secret key s. $\mathcal{C}$ sends $params$ to $\mathcal{A}$.

Attack: The adversary $\mathcal{A}$ can perform a polynomially bounded number of queries in an adaptive manner (that is, each query may depend on the responses to the previous queries). The types of queries allowed are described below.

- Hash functions queries: $\mathcal{A}$ can ask for the values of the hash functions (e.g. $H(\cdot)$ and $H_0(\cdot)$ in our proposed scheme) for any input.
- `KeyGen`: $\mathcal{A}$ chooses an identity ID. $\mathcal{C}$ computes $\texttt{KeyGen}(ID) = S_{ID}$ and sends the result to $\mathcal{A}$.
- `Sign`: $\mathcal{A}$ chooses a group of n users' identities $\{ID_i\}$ where $1 \leq i \leq n$, a threshold value t' where $t' \leq n$, and any message m. $\mathcal{C}$ outputs a (t', n) ID-based threshold ring signature σ.

Forgery: The adversary $\mathcal{A}$ outputs an ID-based threshold ring signature σ on message m "signed" by at least t' members ($t' \leq n$) of a group of n users $\{ID_i\}$ where $1 \leq i \leq n$. The only restriction is that $(m, \{ID_i\})$ does not appear in the set of previous `Sign` queries and less than t' private keys of $\{ID_i\}$ are returned by previous `KeyGen` queries. It wins the game if $\texttt{Verify}(\sigma, \{ID_i\}, m, t')$ is equal to $\top$. The advantage of $\mathcal{A}$ is defined as the probability that it wins.

Definition 5. *An ID-based threshold ring signature scheme is said to have the existential unforgeability against adaptive chosen-message-and-identity attacks property (EUF-IDTR-CMIA2 secure) if no adversary has a non-negligible advantage in the EUF-IDTR-CMIA2 game.*

3.2 Signer Ambiguity of ID-Based Threshold Ring Signature

Definition 6. *An ID-based threshold ring signature scheme is said to have the unconditional signer ambiguity if for any group of n users $\{ID_i\}$ where $1 \leq i \leq n$, any t' signers indexed by $\{i_j\}$, where $1 \leq i_j \leq n$, $1 \leq j \leq t'$ and $t' \leq n$, any message m and any signature σ, where $\sigma = \texttt{Sign}(m, \{ID_i\}, \{S_{ID_{i_j}}\})$, any* *verifier $\mathcal{A}$ (i.e. not a signer in the group $\{ID_{i_j}\}$), even with unbounded computing resources, cannot identify any of the signer with probability better than a random guess. That is, $\mathcal{A}$ can only output any member of $\{i_j\}$ with probability no better than $\frac{t'}{n}$.*

4 Our Proposed Scheme

In this section, we show how to adopt the techniques introduced in [24] with the elegancy of bilinear pairings to spawn an efficient ID-based threshold ring signature scheme with reasonable signature size.

4.1 Basic Construction

Define $\mathbb{G}_1, \mathbb{G}_2$, and $\hat{e}(\cdot,\cdot)$ as in the Section 2 where $\mathbb{G}_1$ is a GDH group. $H(\cdot)$ and $H_0(\cdot)$ are two cryptographic hash functions where $H : \{0,1\}^* \to \mathbb{G}_1$ and $H_0 : \{0,1\}^* \to \mathbb{Z}_q^*$.

`Setup`: TA randomly chooses $s \in_R \mathbb{Z}_q^*$, keeps it as the master secret key and computes the corresponding public key $P_{pub} = sP$. The system parameters are:

$$params = \{\mathbb{G}_1, \mathbb{G}_2, \hat{e}(\cdot,\cdot), q, P, P_{pub}, H(\cdot), H_0(\cdot)\}.$$

`KeyGen`: The signer with identity $ID \in \{0,1\}^*$ submits ID to TA. TA sets the signer's public key Q_{ID} to be $H(ID) \in \mathbb{G}_1$, computes the signer's private signing key S_{ID} by $S_{ID} = sQ_{ID}$. Then TA sends the private signing key to the signer.

`Sign`: Let L be the set of all identities of the n users. Without loss of generality, we assume user indexed by $\{1, 2, \cdots, t\}$ are the participating signers while user indexed by $\{t+1, t+2, \cdots, n\}$ are the non-participating signers. The participating signers carry out the following steps to give an ID-based threshold ring signature.

1. An arbitrary entity (which is trusted to keep the identities of the participating signers in confidential) "prepares the signature on behalf of" other entities in the group by performing the following computations: For $i \in \{t+1, \cdots, n\}$, chooses x_i and $h_i \in_R \mathbb{Z}_q^*$ and computes $U_i = x_iP - h_iP_{pub}$ and $V_i = x_iQ_{ID_i}$.
2. For $j \in \{1, \cdots, t\}$, each signer ID_j chooses $r_j \in_R \mathbb{Z}_q^*$ and computes $U_j = r_jP$.
3. Anyone in the group of t participating signers who got the knowledge of $\cup_{k=1}^n\{U_k\}$ computes $h_0 = H_0(L, t, m, \cup_{k=1}^n\{U_k\})$ and construct a polynomial f of degree $n-t$ over $\mathbb{Z}_q$ such that $f(0) = h_0$ and $f(i) = h_i$ for $t+1 \leq i \leq n$.
4. For $j \in \{1, \cdots, t\}$, each signer ID_j computes $h_j = f(j)$ and $V_j = r_jQ_{ID_j} + h_jS_{ID_j}$.
5. Anyone in the group of t participating signers who got the knowledge of $\cup_{k=1}^n\{V_k\}$ computes $V = \sum_{k=1}^n V_k$.
6. Output the signature for m and L as $\sigma = \{\cup_{k=1}^n\{U_k\}, V, f\}$.
   (The polynomial f only contains information for the hash values used and its inclusion will not compromise the unforgeability and the anonymity of the scheme.)

`Verify`: A verifier checks whether a signature $\sigma = \{\cup_{k=1}^{n}\{U_k\}, V, f\}$ for the message m is given by at least t signers from the set of users L as follows.

1. Check if the degree of polynomial f is $n-t$ and $H_0(L, t, m, \cup_{k=1}^{n}\{U_k\})$ is the constant term of f. Proceed if both conditions are true, reject otherwise.
2. For $k \in \{1, \cdots, n\}$, compute $h_k = f(k)$.
3. Check whether $\prod_{k=1}^{n} \hat{e}(Q_{ID_k}, U_k + h_k P_{pub}) = \hat{e}(P, V)$. If the equality holds, return $\top$. Otherwise, return $\perp$.

4.2 Trusted Authority Compatibility

It is quite often that different users joined different trusted authorities (TAs) in the reality. In [36], the notion of TA compatibility is introduced in the ID-based signcryption [16, 36] scenario. We extend their notion into TA compatibility in ID-based threshold ring signature. For ID-based threshold ring signature schemes, spontaneity will be affected if the intended group of signers joined different TAs. However, our scheme can be easily extended to handle this situation without compromising the spontaneity. We just need to change the equality to be checked in the verification algorithm to $\prod_{k=1}^{n} \hat{e}(Q_{ID_k}, U_k + h_k P_{pub_k}) = \hat{e}(P, V)$, where P_{pub_k} is the public key of the TA of the k-th user.

4.3 Robustness

Robustness is often desirable in group-oriented signature scheme. For a threshold ring signature scheme that does not support robustness, the misbehavior of any participating signer cannot be detected, and the final signature generated by the group of signers will be invalid even there is only one misbehaving signer. In our scheme, the partial signature $\sigma_j = \{h_j, U_j, V_j\}$ generated by the signer ID_j can be verified easily by checking whether $\hat{e}(Q_{ID_j}, U_j + h_j P_{pub}) = \hat{e}(P, V_j)$ holds.

5 Analysis of the Proposed Scheme

We analyze consistency, efficiency, existential unforgeability and signer ambiguity of our proposed scheme.

5.1 Consistency

The consistency of our basic construction can be easily verified by the following equations.

$$
\begin{aligned}
\hat{e}(P,V) &= \hat{e}(P,\sum_{k=1}^{n} V_k) \\
&= \prod_{i=1}^{t} \hat{e}(P,V_i) \prod_{j=t+1}^{n} \hat{e}(P,V_j) \\
&= \prod_{j=1}^{t} \hat{e}(P, r_j Q_{ID_j} + h_j S_{ID_j}) \prod_{i=t+1}^{n} \hat{e}(P, x_i Q_{ID_i}) \\
&= \prod_{j=1}^{t} \hat{e}(P, (r_j + h_j s) Q_{ID_j}) \prod_{i=t+1}^{n} \hat{e}(x_i P, Q_{ID_i}) \\
&= \prod_{j=1}^{t} \hat{e}(Q_{ID_j}, (r_j + h_j s)P) \prod_{i=t+1}^{n} \hat{e}(Q_{ID_i}, x_i P - h_i P_{pub} + h_i P_{pub}) \\
&= \prod_{j=1}^{t} \hat{e}(Q_{ID_j}, U_j + h_j P_{pub}) \prod_{i=t+1}^{n} \hat{e}(Q_{ID_i}, U_i + h_i P_{pub}) \\
&= \prod_{k=1}^{n} \hat{e}(Q_{ID_k}, U_k + h_k P_{pub})
\end{aligned}
$$

The consistency of the checking for the sake of robustness and that of our extended scheme with TA compatibility can be verified easily in a similar manner.

5.2 Efficiency

Although some research has been done in analyzing the complexity and speeding up the computation of pairing function (for examples, [3, 10, 19]), the pairing operations are still rather expensive. Our scheme is the most efficient (in terms of number of pairing operations required) ID-based ring signature scheme (when the threshold value $t = 1$). Taken into account the computational costs for signature generation and verification, [37] uses $4n - 1$ pairing operations while both of [2] and [23] use $2n + 1$ of them. While the most efficient 1-out-of-n ID-based ring signature scheme before the birth of our scheme is [22], which uses $n + 3$ pairings in total (i.e. signing and verification), our scheme only uses $n+1$ pairing operations. Although the difference is not great, our scheme can be further optimized since the multiplication of a series of pairings in `Verify` can be optimized by using the concept of "Miller lite" of Tate pairing presented in [31]. Moreover, the pairing operations in our scheme can be executed in parallel, which is not possible in schemes like [2, 23, 37].

The previous non ID-based threshold ring signature scheme from bilinear pairings in [33], requires $n + t$ pairing operations (or $(n + 1)t$ of them without optimization) for verification. Our scheme is more efficient since it only requires n pairing operations in verification and none of them in signing.

Considering the signature size, our scheme is also up to the state-of-the-art. Signature sizes in [7] and [34] are $O(n \lg n)$ and $O(n^t)$, respectively. We share the same order of space complexities as in [24] and [33]. However, due to the elegancy of elliptic curve, our scheme should achieve shorter signature size than [24].

5.3 Existential Unforgeability and Signer Ambiguity

The security of our proposed scheme is summarized in the following two theorems.

Theorem 1 *In the random oracle model (the hash functions are modeled as random oracles), if there is an algorithm $\mathcal{A}$ that can win the EUF-IDTR-CMIA2 game in polynomial time, then CDHP can be solved with non-negligible probability in polynomial time.*

Proof. Suppose the challenger $\mathcal{C}$ receives a random instance (P, aP, bP) of the CDHP and has to compute the value of abP. $\mathcal{C}$ will run $\mathcal{A}$ as a subroutine and act as $\mathcal{A}$'s challenger in the EUF-IDTR-CMIA2 game. During the game, $\mathcal{A}$ will consult $\mathcal{C}$ for answers to the random oracles H and H_0. Roughly speaking, these answers are randomly generated, but to maintain the consistency and to avoid collision, $\mathcal{C}$ keeps three lists to store the answers used. We assume $\mathcal{A}$ will ask for $H(ID)$ before ID is used in any other queries.

$\mathcal{C}$ gives $\mathcal{A}$ the system parameters with $P_{pub} = bP$. Note that b is unknown to $\mathcal{C}$. This value simulates the master key value for the TA in the game.

H requests and `KeyGen` requests: When $\mathcal{A}$ asks queries on the hash values of identities, $\mathcal{C}$ checks the list L_1, If an entry for the query is found, the same answer will be given to $\mathcal{A}$; otherwise, a value d_i from $\mathbb{Z}_q^*$ will be randomly generated and d_iP will be used as the answer, the query and the answer will then be stored in the list. Note that the associated private key is d_ibP which $\mathcal{C}$ knows how to compute.

The only exception is that $\mathcal{C}$ has to randomly choose one of the H queries from $\mathcal{A}$, say the k-th query, and answers $H(ID^*) = aP$ for this query. Since aP is a value in a random instance of the CDHP, it does not affect the randomness of the hash function H. Since both a and b are unknown to $\mathcal{C}$, a `KeyGen` request on this identity will make $\mathcal{C}$ fails.

H_0 requests: When $\mathcal{A}$ asks queries on the hash values, $\mathcal{C}$ checks the corresponding list L_2. If an entry for the query is found, the same answer will be given to $\mathcal{A}$; otherwise, a randomly generated value will be used as an answer to $\mathcal{A}$, the query and the answer will then be stored in the list.

`Sign` requests: $\mathcal{A}$ chooses a group of n users' identities $L = \{ID_j\}$, and a threshold value t' where $t' \leq n$ and any message m. On input of (m, L, t'), $\mathcal{C}$ outputs a (t', n) ID-based threshold ring signature σ as follows.

1. For $i \in \{0, t', t'+1, \cdots, n\}$, randomly choose $h_i \in_R \mathbb{Z}_q^*$.

2. Construct a polynomial f over $\mathbb{Z}_q$ such that the degree of f is $n - t'$ and $f(i) = h_i$ for $i = 0, t', t'+1, \cdots, n$.
3. For $j \in \{1, \cdots, t\}$, compute $h_j = f(j)$.
4. For $k \in \{1, \cdots, n\}$, randomly choose h_k and compute $U_k = x_k P - h_k P_{pub}$.
5. Compute $V = \sum_{k=1}^{n} x_k Q_{ID_k}$.
6. Assign h_0 as the value of $H_0(L, t, m, \cup_{k=1}^{n}\{U_k\})$; if collision occurs, generate another h_0 and repeat.
7. Output the signature as $\sigma = \{\cup_{k=1}^{n}\{U_k\}, V, f\}$.

Finally, $\mathcal{A}$ outputs a forged signature $\sigma = \{U, V, f\}$ that is "signed" by some t' members in the group $\{ID_i\}$, $ID^* \in \{ID_i\}$ and $\mathcal{A}$ only requested for the private key of some $t'-1$ members in the group. If $ID^* \notin \{ID_i\}$, $\mathcal{C}$ fails.

It follows from the forking lemma [26] that if $\mathcal{A}$ is a sufficiently efficient forger in the above interaction, then we can construct a Las Vegas machine $\mathcal{A}'$ that outputs two signed messages (U, V, f) and (U, V', f'). To do so we keep all the random tapes in two invocations of $\mathcal{A}$ the same except h_0 returned by H_0 query of the forged message.

Now we consider the probability that ID^* is the chosen target of forgery. Let π be the index of ID^* in L, we need $f(\pi) \neq f'(\pi)$ to solve for CDHP. From the signing algorithm, $f(i) = f'(i)$ if ID_i is a non-participating signer; together with the fact that $f(0) \neq f'(0)$, we know that $f(j) \neq f'(j)$ if ID_j is a participating signer. Since $\mathcal{A}'$ knows $t'-1$ private keys among the group $\{ID_i\}$, the probability that ID^* is a participating signer (and hence $(f(\pi) \neq f'(\pi))$ is $\frac{1}{n-t'+1}$.

Given the machine $\mathcal{A}'$ derived from $\mathcal{A}$, we can solve the CDHP by computing $abP = (h_\pi - h'_\pi)^{-1}(V - V')$. We calculate the probability of success of $\mathcal{C}$ as follows. For $\mathcal{C}$ to succeed, $\mathcal{A}$ did not ask a `KeyGen` query on ID^*. And the corresponding probability is at least $\frac{q_H - q_E}{q_H}$. Further, with a probability of $(n-t'+1)(\frac{q_H-q_E-1}{q_H-q_E})(\frac{q_H-q_E-2}{q_H-q_E-1})\cdots(\frac{q_H-q_E-(n-t')}{q_H-q_E-(n-t'-1)})(\frac{1}{q_H-q_E-(n-t')}) = \frac{n-t'+1}{q_H-q_E}$, $ID^* \in \{ID_i\}$, hence the probability for using $\mathcal{A}$ to solve the CDHP is $\frac{1}{q_H}$.

□

Theorem 2 *Our ID-based threshold ring signature scheme satisfies the property of unconditional signer ambiguity.*

Proof. The polynomial f with degree $n-t$ can be considered as a function chosen randomly from the collection of all polynomials over $\mathbb{Z}_q$ with degree $n-t$ since $h_{t+1}, \cdots, h_n$ are randomly generated and h_0 is the output of the random oracle H_0.

For $i \in \{t+1, \cdots, n\}$, and for $j \in \{1, \cdots, t\}$, $\{x_i\}$ and $\{r_j\}$ are chosen independently and distributed uniformly over $\mathbb{Z}_q^*$. So $\{U_i\} \cup \{U_j\}$ and hence $\cup_{k=1}^{n}\{U_k\}$ are also uniformly distributed.

The polynomial f is determined by $h_{t+1}, \cdots, h_n$ and h_0, then the distributions of $h_1, \cdots, h_t$ are also uniform over the underlying range, with the fact that $\{S_{ID_j}\}$ is independent of $\{r_j\}$ and $\{h_j\}$, we say that $\{V_i\} \cup \{V_j\}$ and hence V are also uniformly distributed.

To conclude, for any fixed message m and fixed set of identities L, the distribution of $\{\cup_{k=1}^{n}\{U_k\}, V, f\}$ are independent and uniformly distributed no matter which t participating signers are. So we conclude that even an adversary with all the private keys corresponding to the set of identities L and unbounded computing resources has no advantage in identifying any one of the participating signers over random guessing. □

6 Conclusion

In this paper, we present an ID-based threshold ring signature scheme. We prove the security of our scheme in the random oracle model [4]. Moreover, our scheme provides trusted authority compatibility [36]. To the best of authors' knowledge, our scheme is the first ID-based threshold ring signature scheme, which is also the most efficient ID-based ring signature scheme (when the threshold value $t = 1$) and threshold ring signature scheme from pairings in terms of the number of pairing operations. Due to the elegancy of bilinear pairing, signatures generated by our scheme are much shorter and simpler than signatures from other previous threshold ring signature schemes. Future research directions include devising an ID-based threshold ring signature scheme with constant signature size or making the threshold ring signature scheme works in a hierarchical setting [14].

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Appendix

The work described in this paper was first publicly available in [13]. In an independent and more or less concurrent work [21], an ID-based ring signature scheme for general access structure was proposed. However, their scheme is inefficient for t-out-of-n threshold access structure; the space complexity of the signature and the time complexities of signing and verification are all in $O(n^t)$. Subsequently, they proposed a different construction specific to threshold access structure, which requires $2n - t$ pairing operations in signing and $2n$ of them in verification.

At ICISC 2004, a new bilinear threshold ring signature was proposed [25]. Since the structure of the threshold scheme in [21] and that of the scheme in [25] are similar, they share the same number of pairing operations in signing and verification.

To conclude, our scheme is still the most efficient ID-based threshold ring signature and the most efficient threshold ring signature scheme from pairings.