Provable Efficient Certificateless Public Key Encryption

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Abstract. Certificateless public key cryptography was introduced to overcome the key escrow limitation of the identity-based cryptography. It combines the advantages of the identity-based cryptography and the traditional PKI. Recently, Dae Hyun Yum1 and Pil Joong Lee have proposed a generic series construction model of certificateless public key encryption (CL-PKE) which is built from generic primitives: identity-based encryption and public key encryption. However, this model pays much attention on the generic construction and neglects the nice properties of the bilinear pairings. In this paper, we propose an efficient CL-PKE scheme which is based on the nice algebraic properties of Weil pairing. The scheme works in a kind of parallel model and it is more efficient on computation or published public key information than the existing schemes.

1 Introduction

Traditionally, a Public Key infrastructure (PKI) is used to provide an assurance to the user about the relationship between a public key and the identity of the holder of the corresponding private key by certificates. However, a PKI faces may challenges in the practice, especially the scalability of the infrastructure and the management of the certificates. To simplify the management of certificates, Shamir [1] proposed identity-based public key cryptography (ID-PKC) in which the public key of each party is derived directly from certain aspects of its identity, for example, an IP address belonging to a network host, or an e-mail address associated with a user. Private keys are generated for entities by a trusted third party called Key Generation Center (KGC). For a long while it was an open problem to obtain a secure and efficient identity based encryption (IBE) scheme. Until 2001, Boneh and Franklin [2] presented an efficient and provably secure identity-based encryption scheme (BF-IBE) using the bilinear pairings on elliptic curves.

The direct derivation of public keys in ID-PKC eliminates the need for certificates and some of the problems associated with them. However, the dependence on a KGC who can generate private keys inevitably introduces key escrow to the identity-based cryptography. Then in [3] Al-Riyami and Patersion introduced

the notion of Certificateless Public Key Cryptography (CL-PKC). CL-PKC can overcome the key escrow limitation of ID-PKC without introducing certificates and the management overheads that this entails. It combines the advantages of the ID-PKC and the PKI.

In this paper, we concentrate on the certificateless public key encryption (CL-PKE) schemes. So far almost all the CL-PKE schemes [3,4,5,6] are based on the BF-IBE scheme. Recently, Dae Hyun Yum and Pil Joong Lee [9] have proposed a generic series construction of CL-PKE which is built from generic primitives: identity-based encryption and public key encryption. The CL-PKE scheme in [4] is an instance of such model. However, this model pays much attention on the generic construction and neglects the nice properties of the bilinear pairings. In this paper, we propose an efficient CL-PKE scheme which is based on the nice algebraic properties of Weil pairing. The scheme works in a kind of parallel model and it is more efficient on computation or published public key information than the existing schemes.

The paper is organized as follows: First we review the concepts of CL-PKE and its security model. In section 3, we introduce some mathematic basis of bilinear maps. Then we present our new efficient CL-PKE scheme in section 4 and prove its security. In section 5, we compare our scheme with the existing CL-PKE schemes on performance. Finally, section 6 gives conclusions.

2 Certificateless Public Key Encryption

In this section, we review the definition and security model for CL-PKE from [3].

Definition 1. [3] A CL-PKE scheme is specified by seven algorithm (Setup, Partial-Private-Key-Extract, Set-Secret-Value, Set-Private-Key, Set-Public-Key, Encrypt, Decrypt) such that:

- **Setup** is a probabilistic algorithm that takes security parameter κ as input and returns the system parameters $params$ and the $masterkey$. The system parameters include a description of the message space $\mathcal{M}$ and ciphertext space $\mathcal{C}$.
- **Partial-Private-Key-Extract** is a deterministic algorithm which takes $params, masterkey$ and an identifier for entity A, $ID_A \in \{0,1\}^n$, as inputs. It returns a partial private key D_A .
- **Set-Secret-Value** is a probabilistic algorithm that takes as input $params$ and outputs a secret value x_A.
- **Set-Private-Key** is a deterministic algorithm that takes $params$, D_A and x_A as inputs. The algorithm returns S_A, a (full) private key.
- **Set-Public-Key** is a deterministic algorithm that takes $params$ and x_A as inputs and outputs a public key P_A.
- **Encrypt** is a probabilistic algorithm that takes $params$, $M \in \mathcal{M}$, x_A and ID_A as inputs and returns either a ciphertext $C \in \mathcal{C}$ or the null symbol $\perp$ indicating an encryption failure.

– **Decrypt** is a deterministic algorithm that takes as inputs *params*, $C \in \mathcal{C}$ and S_A. It returns a message $M \in \mathcal{M}$ or a message $\perp$ indicating a decryption failure.

Algorithms **Set-Private-Key** and **Set-Public-Key** are normally run by an entity A for himself, after running **Set-Secret-Value**. Usually, A is the only entity in possession S_A and x_A. Algorithms **Setup** and **Partial-Private-Key-Extract** are usually run by a trusted third party, called Key Generation Center (KGC) [3].

2.1 Security Model for CL-PKE

Al-Riyami and Patersion presented the full IND-CCA security model for CL-PKE in [3]. The following is the actions that an general adversary $\mathcal{A}$ against a CL-PKE scheme may carry out and discuss how each action should be handled by the challenger C for that adversary.

1. **Extract partial private key of entity A:** Challenger C responds by running algorithm **Partial-Private-Key-Extract** to generate the partial private key D_A for entity A.

2. **Extract private key for entity A:** If A's public key has not been replaced then C can respond by running algorithm **Set-Private-Key** to generate the private key S_A for entity A. But it is unreasonable to expect C to be able to respond to such a query if $\mathcal{A}$ has already replaced A's public key.

3. **Request public key of entity A:** C responds by running algorithm **Set-Public-Key** to generate the public key P_A for entity A (first running **Set-Secret-Value** for A if necessary).

4. **Replace public key of entity A:** The adversary $\mathcal{A}$ can repeatedly replace the public key P_A for any entity A with any value P_0 of its choice. The current value of an entity's public key is used by C in any computations or responses to the adversary's requests.

5. **Decryption query for ciphertext C and entity A:** In the model of [3], adversary can issue a decryption query for any entity and any ciphertext. It is assumed in [3] that C should properly decrypt ciphertexts, even for those entities whose public keys have been replaced. This is a rather strong property for the security model (after all, the challenger may no longer know the correct private key). However, it ensures that the model captures the fact that changing an entity's public key to a value of the adversary's choosing may give that adversary an advantage in breaking the scheme. For further discussion of this feature, see [3].

The IND-CCA security model of [3] distinguishes two types of adversary. A type I adversary $\mathcal{A}_I$ is able to change public keys of entities at will, but does not have access to the *masterkey*. A Type II adversary $\mathcal{A}_{II}$ is equipped with the *masterkey* but is not allowed to replace public keys of entities. This adversary models security against an eavesdropping KGC.

CL-PKE Type I IND-CCA Adversary: Such an adversary $\mathcal{A}_I$ does not have access to the *masterkey*. However, $\mathcal{A}_I$ may request public keys and replace public keys with values of its choice, extract partial private and private keys and make decryption queries, all for identities of its choice. As discussed above, we make several natural restrictions on such a Type I adversary:

1. Adversary $\mathcal{A}_I$ cannot extract the private key for ID_{ch} at any point.

2. Adversary $\mathcal{A}_I$ cannot request the private key for any identifier if the corresponding public key has already been replaced.

3. Adversary $\mathcal{A}_I$ cannot both replace the public key for the challenge identifier ID_{ch} before the challenge phase and extract the partial private key for ID_{ch} in some phase.

4. In Phase 2, $\mathcal{A}_I$ cannot make a decryption query on the challenge ciphertext C^* for the combination (ID_{ch}, P_{ch}) that was used to encrypt M_b.

CL-PKE Type II IND-CCA Adversary: Such an adversary $\mathcal{A}_{II}$ does have access to the *masterkey*, but may not replace public keys of entities. $\mathcal{A}_{II}$ can compute partial private keys for himself, given the *masterkey*. It can also request public keys, make private key extraction queries and decryption queries, both for identities of its choice. The restrictions on this type of adversary are:

1. Adversary $\mathcal{A}_{II}$ cannot replace public keys at any point.

2. Adversary $\mathcal{A}_{II}$ cannot extract the private key for ID_{ch} at any point.

3. In Phase 2, $\mathcal{A}_{II}$ cannot make a decryption query on the challenge ciphertext C^* for the combination (ID_{ch}, P_{ch}) that was used to encrypt M_b.

Definition 2. A CL-PKE scheme is said to be IND-CCA secure if no polynomially bounded adversary $\mathcal{A}$ of Type I or Type II has a non-negligible advantage in the following game:

Setup: The challenger C takes a security parameter κ as input and runs the Setup algorithm. It gives $\mathcal{A}$ the resulting system parameters *params*. If $\mathcal{A}$ is of Type I, then C keeps the *masterkey* to himself, otherwise, he gives the *masterkey* to $\mathcal{A}$.

Phase 1: $\mathcal{A}$ issues a sequence of requests described above. These queries may be asked adaptively, but are subject to the rules on adversary behavior defined above.

Challenge Phase: Once $\mathcal{A}$ decides that Phase 1 is over it outputs the challenge identifier ID_{ch} and two equal length plaintexts $M_0, M_1 \in \mathcal{M}$. Again, the adversarial constraints given above apply. C now picks a random bit $b \in \{0, 1\}$ and computes C^*, the encryption of M_b under the current public key P_{ch} for ID_{ch}. Then C^* is delivered to $\mathcal{A}$.

Phase 2: Now $\mathcal{A}$ issues a second sequence of requests as in Phase 1, again subject to the rules on adversary behavior above.

Guess: Finally, $\mathcal{A}$ outputs a guess $b' \in \{0, 1\}$. The adversary wins the game if $b = b'$. We define $\mathcal{A}$'s advantage in this game to be $\text{Adv}(\mathcal{A}) := 2|Pr[b = b'] - 1/2|$.

IND-CPA security is similar to IND-CCA security except that the adversary is more limited. That is the adversary cannot issue decryption queries while attacking the scheme.

3 Mathematic Basic

Before presenting the new CL-PKE scheme, we first review a few concepts related to bilinear maps. Let E/F_q be an elliptic curve and $m = \#E(F_q)$ be the group order of the curve. Let n be a prime such that $n \mid m$ and $n \nmid q$. Then the group of n-torsion points has the structure $E[n] \cong Z_n \oplus Z_n$ and is thus generated by two elements, say P_1 and P_2 ($< P_1 > \neq < P_2 >$) . We can denote the elements in the set of $E[n]$ using the formaP_1+bP_2, $a, b \in Z_n^*$. Denote the group generated by P_1 by G_1 and the group generated by P_2 by G_2, i.e. $G_1 =< P_1 >$ and $G_2 =< P_2 >$. ψ is an isomorphism from G_2 to G_1 with $\psi(P_2) = P_1$. The Weil pairing is a function [13]:

$e_n : E[n] \times E[n] \rightarrow \mu_n$.

e_n maps to the group μ_n of nth roots of unity, which is a cyclic group of order n as well. Denote this group by G_T. The following are some useful properties of the Weil Pairing:

1. Identity: For all $P \in E[n]$, $e_n(P, P) = 1$.

2. Alternation: For all $P, Q \in E[n]$, $e_n(P, Q) = e_n(Q, P)^{-1}$.

3. Bilinearity: For all $P, Q, R \in E[n]$, $e_n(P, +Q, R) = e_n(P, R) + e_n(Q, R)$, and $e_n(P, Q + R) = e_n(P, Q) + e_n(P, R)$.

4. Non-degeneracy: For all $P \in G_1$ and $Q \in G_2$, $e_n(P, Q) \neq 1$.

5. Computable: For all $P, Q \in E[n]$, $e_n(P, Q)$ is computable in polynomial time.

Note that from [11], we can either assume that ψ is efficiently computable or make our security proof relative to some oracle which computes ψ.

In the following, we consider some problems.

co-BIDH Assumption: For $a, b, c \in_R Z_q^*, P_2 \in G_2^*, P_1 = \psi(P_2) \in G_1^*, e_n$, given (P_1, P_2, aP_2, bP_2), to compute $e_n(P_1, P_2)^{a^{-1}b}$ is hard.

k-BCAA1 Assumption: [8] For an integer k, and $x \in_R Z_n^*, P_2 \in G_2^*, P_1 = \psi(P_2) \in G_1^*, e_n$, given $(P_1, P_2, xP_2, h_0, (h_1, \frac{1}{h_1+x}P_2), ..., (h_k, \frac{1}{h_k+x}P_2))$ where $h_i \in_R Z_q^*$ and different from each other for $0 \leq i \leq k$, to compute $e_n(P_1, P_2)^{1/(x+h_0)}$ is hard.

k-BDHI Assumption: [8,12] For an integer k, and $x \in_R Z_n^*, P_2 \in G_2^*, P_1 = \psi(P_2) \in G_1^*, e_n$, given $(P_1, P_2, xP_2, x^2P_2, ...x^kP_2)$, to compute $e_n(P_1, P_2)^{1/x}$ is hard.

In [8] Chen and Cheng have proved that the following relationship between the k-BCAA1 problem and the k-BDHI problem.

Theorem 1. *[8] If there exists a polynomial time algorithm to solve (k-1)-BDHI, then there exists a polynomial time algorithm for k-BCAA1. If there*

exists a polynomial time algorithm to solve (k-1)-BCAA1, then there exists a polynomial time algorithm for k-BDHI.

4 A New CL-PKE Scheme

Inspired by the provable secure SK-IBE scheme [7, 8], we propose a new CL-PKE scheme. We describe our new scheme in a similar method of [2]. First, we give a basic CL-PKE scheme which is only IND-CPA secure. Then in the next section, we will extend the basic scheme to the full scheme which is secure against an IND-CCA attack using a technique due to Fujisaki-Okamoto [10].

4.1 BasicCL-PKE

Our basic scheme is consisted of the following algorithms.

Setup: Given a security parameter κ, the generator takes the following steps.

1. Generate a Weil pairing $e : E[q] \times E[q] \rightarrow G_T$ with $E[q] = G_1 \oplus G_2$ and an isomorphism ψ from G_2 to G_1. Pick a random generator $P_2 \in G_2^*$ and set $P_1 = \psi(P_2)$.
2. Pick a random $s \in Z_q^*$ and compute $P_{pub} = sP_1$.
3. Compute $g = e(P_1, P_2)$.
4. Pick five cryptographic hash functions $H_1 : \{0,1\}^* \rightarrow Z_q^*$ and $H_2 : G_T \rightarrow \{0,1\}^n$,for some n.

The message space is $\mathcal{M} = \{0,1\}^n$. The ciphertext space is $\mathcal{C} = E_n \times \{0,1\}^n$. The system parameters are $params =< q, G_1, G_2, G_T, e, n, P_1, P_2, g, P_{pub}, H_1, H_2 >$. The $masterkey$ is s.

Partial-Private-Key-Extract: The algorithm takes as input an identifier $ID \in \{0,1\}^*$, $params$ and the $masterkey$ s and returns the partial private key $D_{ID} = \frac{1}{H_1(ID)+s} P_2$.

Set-Secret-Value: The algorithm takes as inputs $params$ and identifier ID, selects a random $x_{ID} \in Z_q^*$ and outputs x_{ID} as the entity's secret value.

Set-Private-Key: The algorithm takes an inputs $params$, entity ID's partial private key D_{ID} and secret value x_{ID}. The output of the algorithm is the pair $S_{ID} =< D_{ID}, x_{ID} >$.

Set-Public-Key: The algorithm takes $params$ and entity ID's secret value x_{ID} as inputs and constructs ID's public key as $P_{ID} = x_{ID}P_2$.

Encrypt: To encrypt $M \in \mathcal{M}$ for entity ID with the public key P_{ID}, perform the following steps:

1. Check that P_{ID} is in G_2^*, if not output $\perp$. This checks the validity of the public key.
2. Compute $Q_{ID} = H_1(ID)P_1 + P_{pub}$.
3. Choose random values r_1 and r_2 and compute the ciphertext:

$$C =< r_1 Q_{ID} + r_2 P_{ID}, M \oplus H_2(g^{(r_1+r_2)}) >$$

Decrypt: Suppose $C =< U, V >$. To decrypt this ciphertext using the private key $S_{ID} =< D_{ID}, x_{ID} >$ compute:

$$M = V \oplus H_2(e(U, D_{ID} - \frac{1}{x_{ID}} P_1)).$$

According to the Weil Pairing's properties, we know $e(P_1, P_1) = 1$, $e(P_2, P_2) = 1$, and $e(P_2, -P_1) = e(P_1, P_2)$. Hence the consistency of the scheme can be verified by

$$\begin{aligned} e(U, D_{ID} - \tfrac{1}{x_{ID}} P_1) &= e(r_1 Q_{ID} + r_2 P_{ID}, D_{ID} - \tfrac{1}{x_{ID}} P_1) \\ &= e(r_1(H_1(ID) + s)P_1 + r_2 x_{ID} P_2, \tfrac{1}{H_1(ID)+s} P_2 - \tfrac{1}{x_{ID}} P_1) \\ &= e(r_1(H_1(ID) + s)P_1, \tfrac{1}{H_1(ID)+s} P_2) e(r_2 x_{ID} P_2, -\tfrac{1}{x_{ID}} P_1) \\ &= e(P_1, P_2)^{r_1} e(P_1, P_2)^{r_2} \\ &= g^{(r_1 + r_2)} \end{aligned}$$

4.2 Security of BasicCL-PKE

In this section, we study the security of the BasicCL-PKE scheme. The following theorems show that Basic-PKE scheme is a CPA secure certificateless encryption scheme.

Theorem 2. *If there exists a Type-I IND-CPA adversary $\mathcal{A}_{\mathcal{I}}$ against BasicCL-PKE with advantage, then there exists an adversary $\mathcal{B}$ which can solve the k-BCAA1 problem with non-negligible advantage in the random oracle model.*

Theorem 3. *If there exists a Type-II IND-CPA adversary $\mathcal{A}_{\mathcal{II}}$ against BasicCL-PKE with advantage, then there exists an adversary $\mathcal{B}$ which can solve the co-BIDH problem with non-negligible advantage in the random oracle model.*

To prove the theorem 2, we define the following public key encryption scheme called BasicPub-I.

BasicPub-I The scheme includes the following algorithms:

Key-generation: Given a security parameter κ, the generator takes the following steps.

1. Generate the parameters $< q, G_1, G_2, G_T, e, P_1, P_2, g >$ which are identical to the ones of the BasicCL-PKE.

2. Pick a random $s \in Z_q^*$ and compute $P_{pub} = sP_1$. Randomly choose different elements $h_i \in Z_q^*$ and compute $\frac{1}{h_i+s} P_2$ for for $0 \leq i < q_1$.

3. Pick a random $x \in Z_q^*$ and compute $P_{ID} = xP_2$.

4. Pick a hash function $H_2 : G_T \rightarrow \{0,1\}^n$ for some n.

The public parameters are $K_{pub-I} =< q, G_1, G_2, G_T, e, n, P_1, P_2, g, P_{pub}, x, P_{ID}, h_0, (h_1, \frac{1}{h_1+s} P_2), (h_2, \frac{1}{h_2+s} P_2), ..., (h_{q_1-1}, \frac{1}{h_{q_1-1}+s} P_2), H_2 >$ and the private key is $K_{pri-I} = \frac{1}{h_0+s} P_2$.

Encrypt: To encrypt $M \in \mathcal{M}$, perform the following steps:

1. Check that P_{ID} is in G_1^*, if not output $\perp$. This checks the validity of the public key.

2. Choose two random $r_1, r_2 \in Z_q^*$ and compute the ciphertext:

$$C =< r_1(h_0 P_1 + P_{pub}) + r_2 P_{ID}, M \oplus H_2(g^{(r_1+r_2)}) >$$

Decrypt: Suppose $C =< U, V >$. To decrypt this ciphertext using the private key K_{pri-I} compute:

$$M = V \oplus H_2(e(U, K_{pri-I} - x^{-1}P_1)).$$

Proof of Theorem 2 This theorem straightforwardly follows from the following lemma 1 and lemma 2.

Lemma 1: *Let H_1 is a random oracle. Suppose $\mathcal{A}_\mathcal{I}$ is an IND-CPA adversary that has advantage ε against BasicCL-PKE. Suppose $\mathcal{A}_\mathcal{I}$ makes at most q_1 queries to H_1. Then there is an IND-CPA adversary $\mathcal{B}$ that at lest ε/q_1 against BasicPub-I.*

Proof: Let $\mathcal{A}_\mathcal{I}$ be a Type I advesary against the new BasicCL-PKE. Suppose $\mathcal{A}_\mathcal{I}$ has advantage ε and makes q_1 queries to random oracle H_1. We show how to construct from $\mathcal{A}_\mathcal{I}$ an IND-CPA adversary $\mathcal{B}$ against the BasicPub-I. Let $\mathcal{C}$ denote the challenger against $\mathcal{B}$ for BasicPub-I. The challenger $\mathcal{C}$ begins by supplying $\mathcal{B}$ with a public key: $K_{pub-I} =< q, G_1, G_2, G_T, e, n, P_1, P_2, g, P_{pub}, x, P_{ID}, h_0, (h_1, \frac{1}{h_1+s}P_2), (h_2, \frac{1}{h_2+s}P_2), ..., (h_{q_1-1}, \frac{1}{h_{q_1-1}+s}P_2), H_2 >$

Algorithm $\mathcal{B}$ works by interacting with $\mathcal{A}_I$ in an IND-CPA game as follows:

Setup: $\mathcal{B}$ chooses an index I with $1 \leq I \leq q_1$. Then $\mathcal{B}$ gives $\mathcal{A}_\mathcal{I}$ the BasicCL-PKE public parameters $params =< q, G_1, G_2, G_T, e, n, P_1, P_2, g, P_{pub}, H_1, H_2 >$. Here $< q, G_1, G_2, G_T, e, n, P_1, P_2, g, P_{pub}, H_2 >$ are taken from K_{pub-I} and H_1 is a random oracle controlled by $\mathcal{B}$ as described below.

H_1-queries: At any time algorithm $\mathcal{A}_\mathcal{I}$ can query the random oracle H_1. To response to these queries $\mathcal{B}$ maintains a list of tuples $< ID_i, h_i, x_i >$. We refer to this list as the H_1^{list}. When $\mathcal{A}$ queries the oracle H_1 at a point ID_i algorithm $\mathcal{B}$ responds as follows:

1. If the query ID_i already appears on the H_1^{list} in a tuple $< ID_i, h_i, x_i >$, then algorithm $\mathcal{B}$ responds with $H_1(ID_i) = h_i$.

2. Otherwise, if the query is on the Ith distinct ID, then $\mathcal{B}$ return $H_1(ID_I) = h_0$ and add the tuple $< ID_I, h_0, \perp >$ into the H_1^{list}.

3. Otherwise, $\mathcal{B}$ selects a random integer $h_i (i > 0)$ from K_{pub-I} which has not been chosen by $\mathcal{B}$ and returns $H_1(ID_i) = h_i$. Then $\mathcal{B}$ chooses a random $x_i \in Z_q^*$ and adds the tuple $< ID_i, h_i, x_i >$ to the H_1^{list}.

Phase 1: Now $\mathcal{A}_\mathcal{I}$ launches Phase 1 of its attack by making a series of queries. B replies to these queries as follows:

Partial Private Key Extraction: Suppose the query is on ID_i. There are two cases:

If $i = I$, $\mathcal{B}$ aborts **(Event 1)**.

If $i \neq I$, then $\mathcal{B}$ replies with $\frac{1}{h_i+s}P_2$.

Private Key Extraction: Suppose the query is on ID_i. There are two cases:

If $i = I$, $\mathcal{B}$ aborts **(Event 2)**.

If $i \neq I$, then $\mathcal{B}$ replies with $< \frac{1}{h_i+s}P_2, x_i >$.

Request for Public Key: If the query is on ID_I then $\mathcal{B}$ returns P_{ID}. Otherwise, if the request is on ID_i with $i \neq I$, then $\mathcal{B}$ outputs x_iP_2.

Replace Public Key: Suppose the query is to replace the public key for ID_i with value P_i'. $\mathcal{B}$ responds as follows.

1. If no tuple corresponding to ID_i exists on the H_1^{list}, $\mathcal{B}$ follows the $H_1 - queries$ algorithm to create the tuple with $x_i = \perp$ and sets $P_{ID_i} = P_i'$.

2. Otherwise, $\mathcal{B}$ updates P_{ID_i} with P_i' and sets $x_i = \perp$.

Challenge Phase: At some point, $\mathcal{A}_{\mathcal{I}}$ decides to end Phase 1 and picks ID_{ch} and two messages M_0, M_1 on which it wants to be challenged. $\mathcal{B}$ responds as follows.

1. If $ID_{ch} \neq ID_I$ then $\mathcal{B}$ aborts **(Event 3)**.

2. Otherwise, $ID_{ch} = ID_I$ and $\mathcal{B}$ gives $\mathcal{C}$ the pair M_0, M_1 as the messages on which it wishes to be challenged. $\mathcal{C}$ responds with the challenge ciphertext $C =< U, V, W >$ which is the BasicPub-I encryption of M_b for a random $b \in \{0, 1\}$. Then $\mathcal{B}$ delivers C to $\mathcal{A}_{\mathcal{II}}$. It is easy to see that C is the CL-PKE encrypton of M_b for identifier ID_I with public key P_{ID}.

Phase 2: $\mathcal{B}$ responds to queries as in Phase 1.

Guess: Eventually, $\mathcal{A}_{\mathcal{II}}$ outputs b' for b. Algorithm $\mathcal{B}$ outputs b' as its guess for b.

If the algorithm $\mathcal{B}$ does not abort during the simulation then algorithm $\mathcal{A}_{\mathcal{I}}$'s view is identical to its view in the real attack. To complete the proof it remains to calculate the probability that $\mathcal{B}$ does not abort during the simulation. $\mathcal{B}$ could abort when one of the following events happens:(1)Event 1: $\mathcal{A}_{\mathcal{I}}$ queries the Partial Private key Extraction for ID_I at some point; (2)Event 2: $\mathcal{A}_{\mathcal{I}}$ queries the Private key Extraction for ID_I at some point; (3) Event 3: $\mathcal{A}$ does not choose ID_I as ID_{ch}. Notice that the $\neg event3$ implies that $\neg Event1$ and $\neg Event2$ (if $\mathcal{A}_{\mathcal{I}}$ choose ID_{ch} equal to ID_I, then no partial private key extraction and private key extraction are allowed). Hence, the probability that $\mathcal{B}$ does not abort during the simulation is: $Pr[\neg Event1]Pr[\neg Event2]Pr[\neg Event3] = Pr[\neg Event3] = 1/q_1$. This shows that $\mathcal{B}$'s advantage is at least ε/q_1 as required.

Lemma 2. *Let H_2 is a random oracle and there exists an IND-CPA adversary $\mathcal{A}$ against BasicPub-I which has non-negligible advantage ε. Suppose $\mathcal{A}$ makes a total of q_2 queries to H_2. Then there exists an algorithm $\mathcal{B}$ to solve the $(q_1 - 1)$-BCAA1 problem with non-negligible advantage $2\varepsilon/q_2$.*

Proof: Algorithm $\mathcal{B}$ is given as input a random $(q_1 - 1)$-BCAA1 instance $< q, G_1, G_2, G_T, e, \psi, P_1, P_2, xP_2, h_0, (h_1, \frac{1}{h_1+x}P_2), ..., (h_{q_1-1}, \frac{1}{h_{q_1-1}+x}P_2) >$ where $x \in Z_q^*$ is a random element. Algorithm $\mathcal{B}$ finds $D = e(P_1, P_2)^{1/(x+h_0)}$ by interacting with $\mathcal{A}$ as follows:

Setup: Algorithm $\mathcal{B}$ first simulates algorithm Key-generation of BasicPub-I to create the public parameters as belew.

1.Computes $P_{pub} = \psi(xP_2) \in G_1$.

2. Pick a random $r \in Z_q^*$ and set $P_{ID} = rP_2$.

2. Now $\mathcal{B}$ pasees $\mathcal{A}$ the public parameters $K_{pub-I} =< q, G_1, G_2, G_T, e, \psi, P_1, P_2, P_{pub}, r, P_{ID}, h_0, (h_1, \frac{1}{h_1+x}P_2), ..., (h_{q_1-1}, \frac{1}{h_{q_1-1}+x}P_2) >$. The private key is $K_{pri-I} = \frac{1}{h_0+x}P_2$.

H_2-queries: At any time algorithm $\mathcal{A}$ can query the random oracle H_2. To response to these queries $\mathcal{B}$ maintains a list of tuples $< X_i, H_i >$. We refer to this list as the H_2^{list}. When $\mathcal{A}$ queries the oracle H_2 at a point X_i algorithm $\mathcal{B}$ responds as follows:

1. If the query X_i already appears on the H_2^{list} in a tuple $< X_i, H_i >$, then algorithm $\mathcal{B}$ responds with $H_2(X_i) = H_i$.

2. Otherwise, $\mathcal{B}$ chooses a random $H_i \in \{0,1\}^n$, return $H_2(X_i) = H_i$, and adds the tuple $< X_i, H_i >$ to the H_2^{list}.

Challenge: Algorithm $\mathcal{A}$ outputs two message M_0 and M_1 on which it wants to be challenged. $\mathcal{B}$ chooses a random string $R \in \{0,1\}^n$ and two random integers $r_1, r_2 \in Z_q^*$, and then defines the challenged ciphertext to be $C =< U, V >=< r_1P_1 + r_2P_{ID}, R >$. Observe that the decryption of C is $R \oplus H_2(r_1P_1 + r_2rP_2, \frac{1}{h_0+x}P_2 - r^{-1}P_1) = R \oplus H_2(D^{r_1} * e(P_1, P_2)^c)$.

Guess: Algorithm $\mathcal{A}$ outputs it guess $b \in \{0,1\}$. At this point $\mathcal{B}$ pick a random tuple $< X_i, H_i >$ from the H_2^{list} and outputs $(X_i/e(P_1, P_2)^c)^{-r_1}$ as the solution to the given instance of $(q_1 - 1)$-BCAA1 problem.

Let $\mathcal{H}$ be the event that algorithm $\mathcal{A}$ issues a query for $H_2(D^{r_1} * e(P_1, P_2)^c)$ at some point during the simulation. From Claim 1 of [2], we know that $Pr[\mathcal{H}] \geq 2\varepsilon$. Hence, at the end of the simulation, $D^{r_1} * e(P_1, P_2)^c$ appear in some tuple on the H_2^{list} with probability at least 2ε. Assume that $\mathcal{A}$ has queries q_2 distinct value to H_2. Hence, we know that $\mathcal{B}$ produces the correct answer with probability at least $2\varepsilon/q_2$.

To prove the theorem 3, we define the following public key encryption scheme called BasicPub-II.

BasicPub-II This scheme includes the following algorithms:

Key-generation: Given a security parameter κ, the generator takes the following steps.

1. Generate the parameters $< q, G_1, G_2, G_T, e, P_1, P_2, g >$ which are identical to the ones of the BasicCL-PKE.

2. Pick a random $s \in Z_q^*$ and compute $P_{pub} = sP_1$. Randomly choose element $h_0 \in Z_q^*$.

3. Pick a random $x \in Z_q^*$ and compute $P_{ID} = xP_2$.

4. Pick a hash function $H_2 : G_T \rightarrow \{0,1\}^n$ for some n.

The public $K_{pub-II} =< q, G_1, G_2, G_T, e, n, P_1, P_2, g, s, P_{pub}, P_{ID}, h_0, H_2 >$ and the private key is $K_{pri-II} = x$.

Encrypt: To encrypt $M \in \mathcal{M}$, perform the following steps:

1. Check that P_{ID} is in G_1^*, if not output $\perp$. This checks the validity of the public key.

2. Choose two random $r_1, r_2 \in Z_q^*$ and compute the ciphertext:

$$C =< r_1(h_0 P_1 + P_{pub}) + r_2 P_{ID}, M \oplus H_2(g^{(r_1+r_2)})$$

Decrypt: Suppose $C =< U, V >$. To decrypt this ciphertext using the private key K_{pri-II} compute:

$$M = V \oplus H_2(e(U, \tfrac{1}{h_0+s} P_2 - \tfrac{1}{K_{pri-II}} P_1)).$$

Proof of Theorem 3 This theorem straightforwardly follows from the following lemma 3 and lemma 4.

Lemma 3. *Let H_1 is a random oracle. Suppose $\mathcal{A}_{\mathcal{II}}$ is an IND-CPA adversary that has advantage ε against BasicCL-PKE. Suppose $\mathcal{A}_{\mathcal{II}}$ makes at most q_1 queries to H_1 and q_E Private Key Extraction queries. Then there is an IND-CPA adversary $\mathcal{B}$ that at lest $\frac{\varepsilon}{q_1}(1 - \frac{1}{q_1})^{q_E}$ against BasicPub-II.*

Proof: Let $\mathcal{A}_{\mathcal{II}}$ be a Type II advesary against the new CL-PKE. Suppose $\mathcal{A}_{\mathcal{II}}$ has advantage ε and makes q_1 queries to random oracle H_1. We show how to construct from $\mathcal{A}_{\mathcal{II}}$ an IND-CPA adversary $\mathcal{B}$ against the BasicPub-II. Let $\mathcal{C}$ denote the challenger against $\mathcal{B}$ for BasicPub-II. The challenger $\mathcal{C}$ begins by supplying $\mathcal{B}$ with a public key:

$$K_{pub-II} =< q, G_1, G_2, G_T, e, n, P_1, P_2, g, s, P_{pub}, P_{ID}, h_0, H_2 >.$$

Algorithm $\mathcal{B}$ works by interacting with $\mathcal{A}$ in an IND-CPA game as follows:

Setup: $\mathcal{B}$ chooses an index I with $1 \leq I \leq q_{H_1}$. Then $\mathcal{B}$ gives $\mathcal{A}_{\mathcal{II}}$ the CL-PKE public parameters $params =< q, G_1, G_2, G_T, e, n, P_1, P_2, g, P_{pub}, H_1, H_2 >$ and the value of s. Here $< q, G_1, G_2, G_T, e, n, P_1, P_2, P_{pub}, g, H_2 >$ and s are taken from K_{pub-II}. H_1 is a random oracle controlled by $\mathcal{B}$ as described below.

H_1-queries: At any time algorithm $\mathcal{A}_{\mathcal{II}}$ can query the random oracle H_1. To response to these queries $\mathcal{B}$ maintains a list of tuples $< ID_i, h_i, x_i >$. We refer to this list as the H_1^{list}. When $\mathcal{A}_{\mathcal{II}}$ queries the oracle H_1 at a point ID_i algorithm $\mathcal{B}$ responds as follows:

1. If the query ID_i already appears on the H_1^{list} in a tuple $< ID_i, h_i, x_i >$, then algorithm $\mathcal{B}$ responds with $H_1(ID_i) = h_i$.

2. Otherwise, if the query is on the Ith distinct ID, then $\mathcal{B}$ return $H_1(ID_I) = h_0$ and add $< ID_{ID}, h_0, \perp >$ into the H_1^{list}.

3. Otherwise, $\mathcal{B}$ chooses a random $h_i \in Z_q^*$ and return $H_1(ID_i) = h_i$. Then $\mathcal{B}$ chooses a random $x_i \in Z_q^*$ and adds the tuple $< ID_i, h_i, x_i >$ to the H_1^{list}.

Phase 1: Now $\mathcal{A}_{\mathcal{II}}$ launches Phase 1 of its attack by making a series of queries. B replies to these queries as follows:

Private Key Extraction: If the query is on ID_I then $\mathcal{B}$ aborts **(Event 1)**. Otherwise, if the request in on ID_i with $i \neq I$, then $\mathcal{B}$ outputs $< \frac{1}{h_i+s} P_2, x_i >$.

Request for Public Key: If the query is on ID_I then $\mathcal{B}$ returns P_{ID}. Otherwise, if the request is on ID_i with $i \neq I$, then $\mathcal{B}$ outputs $x_i P_2$.

Challenge Phase: At some point, $\mathcal{A}_{\mathcal{II}}$ decides to end Phase 1 and picks ID_{ch} and two messages M_0, M_1 on which it wants to be challenged. $\mathcal{B}$ responds as follows.

1. If $ID_{ch} \neq ID_I$ then $\mathcal{B}$ aborts **(Event 2)**.

2. Otherwise, $ID_{ch} = ID_I$ and $\mathcal{B}$ gives $\mathcal{C}$ the pair M_0, M_1 as the messages on which it wishes to be challenged. $\mathcal{C}$ responds with the challenge ciphertext $C =< U, V, W >$ which is the BasicPub-II encryption of M_b for a random $b \in \{0, 1\}$. Then $\mathcal{B}$ delivers C to $\mathcal{A}_{\mathcal{II}}$. It is easy to see that C is the CL-PKE encrypton of M_b for identifier ID_I with public key P_{ID}.

Phase 2: $\mathcal{B}$ responds to queries as in Phase 1.

Guess: Eventually, $\mathcal{A}_{\mathcal{II}}$ outputs b' for b. Algorithm $\mathcal{B}$ outputs b' as its guess for b.

If the algorithm $\mathcal{B}$ does not abort during the simulation then algorithm $\mathcal{A}_{\mathcal{II}}$'s view is identical to its view in the real attack. To complete the proof it remains to calculate the probability that $\mathcal{B}$ does not abort during the simulation. $\mathcal{B}$ could abort when one of the following events happens:(1)Event 1:$\mathcal{A}$ queries the Private key Extraction for ID_I at some point; (2) Event 2: $\mathcal{A}$ does not choose ID_I as ID_{ch}. Hence, the probability that $\mathcal{B}$ does not abort during the simulation is: $Pr[\neg Event1]Pr[\neg Event2] = (1 - \frac{1}{q_1})^{q_E}(\frac{1}{q_1})$. This shows that $\mathcal{B}$'s advantage is at least $\frac{\varepsilon}{q_1}(1 - \frac{1}{q_1})^{q_E}$ as required.

Lemma 4. *Let H_2 is a random oracle and there exists an IND-CPA adversary $\mathcal{A}$ against BasicPub-II which has non-negligible advantage ε. Suppose $\mathcal{A}$ makes a total of q_2 queries to H_2. Then there exists an algorithm $\mathcal{B}$ to solve the co-BIDH problem with non-negligible advantage $2\varepsilon/q_2$.*

Proof: Algorithm $\mathcal{B}$ is given as input a random co-BIDH problem instance $< P_1, P_2, aP_2, bP_2 >$. Let $D = e(P_1, P_2)^{a^{-1}b}$ be the solution to the co-BIDH problem. Algorithm $\mathcal{B}$ finds D by interacting with $\mathcal{A}$ as follows:

Setup: Algorithm $\mathcal{B}$ simulate algorithm Key-generation of the BasicPub-II to create the public $K_{pub-II} =< q, G_1, G_2, G_T, e, n, P_1, P_2, g, s, P_{pub}, P_{ID}, h_0, H_2 >$ by randomly selecting $s, h_0 \in Z_q^*$ and setting $P_{pub} = sP, P_{ID} = aP_2$. H_2 is a random oracle controlled by $\mathcal{B}$. The private key K_{pri-II} equals to a which $\mathcal{B}$ does not know. Then algorithm $\mathcal{B}$ passes the public key K_{pub-II} to $\mathcal{A}$ and responds queries as follows.

H_2-queries: At any time algorithm $\mathcal{A}$ can query the random oracle H_2. To response to these queries $\mathcal{B}$ maintains a list of tuples $< X_i, H_i >$. We refer to this list as the H_2^{list}. When $\mathcal{A}$ queries the oracle H_2 at a point X_i algorithm $\mathcal{B}$ responds as follows:

1. If the query X_i already appears on the H_2^{list} in a tuple $< X_i, H_i >$, then algorithm $\mathcal{B}$ responds with $H_2(X_i) = H_i$.

2. Otherwise, $\mathcal{B}$ chooses a random $H_i \in \{0, 1\}^n$, return $H_2(X_i) = H_i$, and adds the tuple $< X_i, H_i >$ to the H_2^{list}.

Challenge: Algorithm $\mathcal{A}$ outputs two message M_0 and M_1 on which it wants to be challenged. $\mathcal{B}$ chooses a random string $R \in \{0,1\}^n$ and a random integer $c \in Z_q^*$, and then defines the challenged ciphertext to be $C =< U, V >=< (h_0 + s)cP_1 + bP_2, R >$. Observe that the decryption of C is $R \oplus H_2(e((h_0 + s)cP_1 + bP_2, \frac{1}{h_0+s}P_2 - a^{-1}P_1) = R \oplus H_2(D * e(P_1, P_2)^c)$.

Guess: Algorithm $\mathcal{A}$ outputs it guess $b \in \{0,1\}$. At this point $\mathcal{B}$ pick a random tuple $< X_i, H_i >$ from the H_2^{list} and outputs $X_i/e(P_1, P_2)^c$ as the solution to the given instance of co-BIDH problem.

Let $\mathcal{H}$ be the event that algorithm $\mathcal{A}$ issues a query for $H_2(D * e(P_1, P_2)^c)$ at some point during the simulation. From Claim 1 of [2], we know that $Pr[\mathcal{H}] \geq 2\varepsilon$. Hence, at the end of the simulation, $D * e(P_1, P_2)^c$ appear in some tuple on the H_2^{list} with probability at least 2ε. Assume that $\mathcal{A}$ has queries q_2 distinct value to H_2. Hence, we know that $\mathcal{B}$ produces the correct answer with probability at least $2\varepsilon/q_2$.

4.3 FullCL-PKE

In this section, we use a technique due to Fujisaki-Okamoto [10] to convert the IND-CPA secure BasicCL-PKE scheme into an IND-CCA secure certificateless encryption scheme in the random oracle. We obtain the following IND-CCA certificateless encryption called FullCL-PKE by applying the Fujisaki-Okamoto transformation.

Setup: As in the BasicCL-PKE scheme. In addition, we select two hash functions $H_3 : \{0,1\}^n \times \{0,1\}^n \rightarrow Z_q^*, H_5 : \{0,1\}^n \times \{0,1\}^n \rightarrow Z_q^*, H_4 : \{0,1\}^n \rightarrow \{0,1\}^n$.

Partial-Private-Key-Extract: As in the BasicCL-PKE scheme.

Set-Secret-Value: As in the BasicCL-PKE scheme.

Set-Private-Key: As in the BasicCL-PKE scheme.

Set-Public-Key: As in the BasicCL-PKE scheme.

Encrypt: To encrypt $M \in \mathcal{M}$ for entity ID with the public key P_{ID}, perform the following steps:

1. Check that P_{ID} is in G_2^*, if not output $\perp$. This checks the validity of the public key.
2. Compute $Q_{ID} = H_1(ID)P_1 + P_{pub}$.
3. Choose a random $\sigma \in \{0,1\}$ and set $r_1 = H_3(\sigma, M), r_2 = H_5(\sigma, M)$.
4. Choose random values r_1 and r_2 and compute the ciphertext:

$$C =< r_1 Q_{ID} + r_2 P_{ID}, \sigma \oplus H_2(g^{(r_1+r_2)}), M \oplus H_4(\sigma) >$$

Decrypt: Suppose $C =< U, V, W >$. To decrypt this ciphertext using the private key $S_{ID} =< D_{ID}, x_{ID} >$ compute:

1. Compute $V \oplus H_2(e(U, D_{ID} - \frac{1}{x_{ID}}P_1)) = \sigma$.
2. Compute $W \oplus H_4(\sigma) = M$.
3. Set $r_1 = H_3(\sigma, M), r_2 = H_5(\sigma, M)$. Test that $U = r_1 Q_{ID} + r_2 P_{ID}$. If not, reject the ciphtext.

4. Output M as the decryption of C.

The security proof of the FullCL-PKE scheme is based on the results of Fujisaki and Okamoto (Theorem 14 in [10]). We can prove the FullCL-PKE to be IND-CCA secure formally in the similar method in [2]. The detail proof will be written in the full paper version.

5 Performance Analysis

In this section, we will show that our proposed FullCL-PKE scheme has the best performance, comparing with other existing CL-PKE schemes [3,4,5,6]. All the schemes have three major operations, i.e., Pairing (p), Scalar(s) and Exponentiation (e). Without considering the pre-computation, the properties and performance of the CL-PKE schemes are listed in Table 1, where we compare the schemes on the computation complexity, public key length (Pubkey Len) and the hardness assumption.

We know that pairing computation is more time-consuming than scalar and exponentiation computation [14]. From Table 1 we can see that our new scheme requires no pairing computation in Encrypt and the public key consists of only one element of G_2 rather than two required in AP's scheme I and CC's scheme I. Hence, our scheme is more efficient than the existing CL-PKE schemes.

Table 1. Comparison of the CL-PKE Schemes

Schemes	Encrypt	Decrypt	Pubkey Len
AP's scheme I [3]	3p+1s+1e	1p+1s	2
CC's scheme I [5]	3p+1s+1e	1p+1s	2
AP's scheme II [4]	1p+2s+1e	1p+2s	1
CC's scheme II [6]	1p+2s+1e	1p+2s	1
New scheme	3s+1e	1p+2s	1

6 Conclusions

In this paper, we present an efficient CL-PKE scheme which is constructed by a kind of parallel model rather than a serial model. Based on the security of two public key encryption schemes we prove the security of our scheme formally in the similar method in [2]. Furthermore, our scheme is more efficient than the existing CL-PKE schemes on computation or published public key information.

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