Efficient Hybrid Encryption from ID-Based Encryption

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Abstract

This paper deals with generic transformations from ID-based key encapsulation mechanisms (IBKEM) to hybrid public-key encryption (PKE). The best generic transformation known until now is by Boneh and Katz and requires roughly 704-bit overhead in the ciphertext. We present new generic transformations that are applicable to *partitioned* IBKEMs. A partitioned IBKEM is an IBKEM that provides some extra structure. Such IBKEMs are quite natural and in fact nearly all known IBKEMs have this additional property. Our first transformation yields chosen-ciphertext secure PKE schemes from selective-ID secure partitioned IBKEMs with a 256-bit overhead in ciphertext size plus one extra exponentiation in encryption/decryption. As the central tool a Chameleon Hash function is used to map the identities. We also propose other methods to remove the use of Chameleon Hash, which may be of independent technical interest.

Applying our transformations to existing IBKEMs we propose a number of novel PKE schemes with different trade-offs. In some concrete instantiations the Chameleon Hash can be made "implicit" which results in improved efficiency by eliminating the additional exponentiation.

Since our transformations preserve the public verifiability property of the IBE schemes it is possible to extend our results to build threshold hybrid PKE schemes. We show an analogue generic transformation in the threshold setting and present a concrete scheme which results in the most efficient threshold PKE scheme in the standard model.

Keywords: Hybrid Encryption, Selective-ID, KEM, Threshold PKE, Chameleon Hash

1 Introduction

1.1 Background

Research on efficient and secure public-key encryption (PKE) has been central in cryptography. A vast number of papers are devoted to construct cryptosystems that achieves security against adaptive chosen ciphertext attacks, aka CCA-security [38, 42]. Many of the schemes are only secure in the random oracle model [4]. The random oracle model is a *heuristic*, and a proof of security in the random oracle model does not directly imply anything about the security of a system in the real world. In fact, it has been demonstrated that there exist cryptographic

schemes which are secure in the random oracle model but which are inherently insecure when the random oracle is instantiated with any real hash function (see, e.g., [15]).

In this paper we focus on CCA-secure cryptosystems under standard cryptographic assumptions, without using random oracles. Early such constructions were given in [38, 42, 43, 24], which are considered as theoretical due to the use of a general technique for non-interactive zero-knowledge proofs. The first practical breakthrough was introduced by Cramer and Shoup [21], followed by its generalization [22].

Identity-based encryption (IBE) [45] extends the framework of public-key encryption such that one's public-key consists of an arbitrarily chosen string, possibly a string associated to one's real identity, on top of a set of system parameters shared by all users. The first instantiation given by Sakai et al. [44] and Boneh and Franklin [11] opened vistas for further research on IBE itself and its numerous applications. While most early constructions of CCA-secure PKE followed the Naor-Yung paradigm [38], a completely different approach based on IBE schemes was presented by Canetti et al. [18]. The technique, aka CHK transform, is a *generic transformation* that converts any selective-ID CPA-secure IBE scheme [17] into a CCA-secure PKE scheme by employing a one-time signature. Due to the inefficiency of one-time signatures, the transformation adds about 65k bits[1] of overhead to the original ciphertext in a typical setting. Later, Boneh and Katz gave an alternative transformation (BK transformation) with improved efficiency by essentially using a MAC [13] (Recently [18, 13] have been merged into one journal paper [10]). However, the BK transformation still requires 704 bits of overhead. Since typical IBE schemes have ciphertext sizes of about 512 bits this transformation still doubles the ciphertext overhead.

Meanwhile more efficient but dedicated constructions of CCA-secure key-encapsulation mechanisms (KEMs) were developed in [14, 30, 31, 33]. Whereas a PKE scheme encrypts messages, in a KEM random session keys are encapsulated. According to the KEM/DEM composition theorem [22], these KEMs in combination with CCA-secure symmetric encryption schemes eventually yield CCA-secure hybrid PKE schemes. These dedicated KEM constructions are based on "identity-based techniques" and exploit certain algebraic structures of known IBE schemes, mostly the one from Boneh and Boyen [7]. Compared to the PKE schemes obtained by the generic transformations these dedicated PKE schemes are more efficient in ciphertext length.

Constructing hybrid PKEs in the threshold setting is yet another interesting issue. There are some dedicated constructions in the standard model based on the Cramer-Shoup encryption, which need interaction between the decryption servers [16, 1]. The paper [9] extends the CHK transformation to the threshold setting and presents a *non-interactive* threshold CCA-secure PKE based on a threshold variant of the Boneh-Boyen IBE. Unfortunately the more efficient BK transformation cannot be applied here since due to the MAC consistency of the resulting scheme can only be privately verified. Accordingly, the threshold scheme in [9] suffers from a large overhead of about 65k caused by the CHK transform.

Some efficient CCA-secure threshold KEMs are presented in [14] and [26]. Unfortunately, the KEM/DEM composition theorem *does not* work in the threshold setting [3] and it is not rigorously clarified how to generically convert a threshold KEM into a threshold PKE.

[1] There are various ways to build one-time signatures with different tradeoffs in size and computational cost. Our estimated 65k overhead comes from the a hash-tree based one-time signature scheme [25]. Using a one-time signature based on the discrete-log problem one can reach to a ciphertext overhead of roughly 1.2k-6k with two extra multi-base exponentiations. See Section 7.5 and Appendix A for details.

1.2 Our Contributions

New generic transformations. We address the issue of reducing the ciphertext overhead introduced in generic transformations from IBE to PKE. As our main contribution we present new constructions that transform an identity-based key encapsulation mechanism (IBKEM) into a hybrid public-key encryption scheme. Our constructions are applicable to so called *partitioned* IBKEMs which will be explained shortly.

Our new transformations have the following properties.

- The first transformation in Section 4 transforms selective-ID secure partitioned IBKEMs into chosen-ciphertext secure PKE schemes. The additional overhead in the transformation is one application of a Chameleon hash [34] (also called trapdoor hash function) in encryption and decryption, respectively. For a typical implementation of a chameleon hash this results in a ciphertext expansion of 128-bits plus one extra multi-exponentiation in encryption/decryption.[2]

- To explain the essential key idea, we also develop methods to remove the use of Chameleon hash in Section 7. They may be of independent theoretical interest, as well as of practical advantage. For example, one of transformations can achieve chosen-ciphertext secure PKE schemes with no additional overhead, in the case of (a strong variant of) adaptive-ID secure partitioned IBKEMs.

We remark that in contrast to the CHK and BK transformations, our construction adds another computational assumption to the security of the resulting PKE scheme. However, efficient Chameleon hashes can be built a relatively weak assumptions such as discrete logarithms or factoring [34] which are implied by the security assumptions on which all known IBE schemes are based on.

Our transformations yield tag-KEMs [3], not PKE schemes. This is more general since every tag-KEM can again be transformed into a hybrid PKE scheme by pairing it with a passively secure symmetric encryption scheme. The advantage of a tag-KEM (rather than PKE) is its great flexibility, i.e. it completely decouples the key encapsulation from the asymmetric part. Furthermore, in contrast to a standard KEM [22], a tag-KEM only requires a passively secure symmetric encryption scheme for the hybrid tag-KEM/DEM construction. We refer to [3] for more details.

Partitioned IBKEMs. A small price we pay in our transformations is a restriction on the ingredient, i.e. that the underlying IBKEM is required to be *partitioned*. Roughly, an IBKEM is *partitioned*, if (i) the encapsulated session key does not depend on the identity, and (ii) the ciphertext can be split into two parts such that one part depends on the identity but the other one does not. We will argue that partitioned IBKEMs are a natural extension of IBKEMs — all known IBKEMs, except the one in [28], are in fact partitioned. There are only few known IBKEMs that are not partitioned [44, 12] in the random oracle model.

We think that our notion of partitioned IBKEMs may be also of independent theoretical interest. As we will discuss later, it provides more insight in the problem of construction efficient PKE schemes in the standard model.

New PKE schemes. We demonstrate the usefulness of our new transformations by providing example instantiations of PKE schemes based on known IBE schemes from Boneh-Boyen [7], Waters [47].

[2] One important detail is that, in contrast to the CHK and BK transformations, we do not have to include the public parameters of the Chameleon hash in the PKE ciphertext.

- For the selective-ID secure Boneh-Boyen IBE scheme we use our first transformation to obtain an efficient PKE scheme. We further demonstrate how to improve efficiency of our transformation by using an *implicit Chameleon Hash*. The implied PKE scheme saves one multi-exponentiation in encryption/decryption.

- For Waters' adaptive-ID secure IBE scheme we use our second (loss-free) transformation. The resulting PKE scheme resembles exactly the PKE scheme presented in [14]. Using our generic transformation we are able to explain its security in terms of the original IBE scheme.

New Generic Transformations in the Threshold Setting. Since our transformations preserve the public verifiability property of the IBE schemes it is possible to extend our results to the threshold setting where some servers shares the private key and collaborate to decrypt a ciphertext. We first rigorously prove a threshold Tag-KEM/DEM composition theorem, which was first presented in [3] but without a proof. Next, we provide the required transformations and give a concrete example instantiation of a threshold Tag-KEM scheme, which is more efficient than some recently proposed solutions in the standard model [9].

1.3 Related Work

A similar but weaker result is independently introduced in [48] for constructing PKE schemes while ours also concerns Tag-KEM schemes that provides more flexibility. [48] heavily relies on the use of the trapdoor hashing (Chameleon hash), and its direct use of Chameleon hash (by Pedersen commitment and a collision resistant hash, in [48, Section 2.5]) as a one-time signature results in a scheme only provable in a (stronger) generic model. As we discuss later in Appendix A, though a more complex three-base Pedersen Chameleon hash could be proven as chosen message attack secure one-time signature, it is not known how to prove a two-base one ([48] used) as a secure one-time signature in the standard model. Since our work focus on the standard model and could avoid the use of Chameleon hash, it is clear to see our results are more general. Besides, our proposals are in particular advantageous to build threshold hybrid PKE scheme, because it is able to be built smoothly from Tag-KEM/DEM framework. On the contrary, it is not known how to generically convert threshold KEM(PKE) to threshold hybrid PKE.

2 Preliminaries

2.1 Notations

If x is a string, then $|x|$ denotes its length, while if S is a set then $|S|$ denotes its size. If $k \in \mathbb{N}$ then 1^k denotes the string of k ones. If S is a set then $s \xleftarrow{\$} S$ denotes the operation of picking an element s of S uniformly at random. We write $\mathcal{A}(x, y, \ldots)$ to indicate that $\mathcal{A}$ is an algorithm with inputs $x, y, \ldots$ and by $z \xleftarrow{\$} \mathcal{A}(x, y, \ldots)$ we denote the operation of running $\mathcal{A}$ with inputs $(x, y, \ldots)$ and letting z be the output. We write $\mathcal{A}^{\mathcal{O}_1, \mathcal{O}_2, \cdots}(x, y, \ldots)$ to indicate that $\mathcal{A}$ is an algorithm with inputs $x, y, \ldots$ and access to oracles $\mathcal{O}_1, \mathcal{O}_2, \ldots$ and by $z \xleftarrow{\$} \mathcal{A}^{\mathcal{O}_1, \mathcal{O}_2, \cdots}(x, y, \ldots)$ we denote the operation of running $\mathcal{A}$ with inputs $(x, y, \ldots)$ and access to oracles $\mathcal{O}_1, \mathcal{O}_2, \ldots$, and letting z be the output.

2.2 Public-Key Encryption

A public-key encryption (PKE) scheme consists of three algorithms $\mathcal{PKE} = (\mathsf{PKE.kg}, \mathsf{PKE.Enc}, \mathsf{PKE.Dec})$.

$(pk, sk) \xleftarrow{\$} \mathsf{PKE.kg}(1^k)$; A probabilistic key-generation algorithm that produces a key pair for given security parameter $k \in \mathbb{N}$. The public-key pk defines a message space $MsgSp$.

$C \xleftarrow{\$} \mathsf{PKE.Enc}(pk, M)$; A probabilistic algorithm that outputs a ciphertext C for a message $M \in MsgSp$.

$M \xleftarrow{\$} \mathsf{PKE.Dec}(sk, C)$; A probabilistic decryption algorithm that decrypts ciphertext C to recover a message M. It may also output a special symbol $\perp$ to represent rejection.

For consistency, we require that for all $k \in \mathbb{N}$, all (pk, sk), all messages $M \in MsgSp$, it must hold that $\Pr[\mathsf{PKE.Dec}(sk, \mathsf{PKE.Enc}(pk, M)) = M] = 1$, where the probability is taken over the above randomized algorithms.

The security we require for PKE is IND-CCA security [42]. It is captured by defining the following experiment.

$$
\begin{array}{l}
\textbf{Experiment } \mathbf{Exp}^{pke\text{-}\mathrm{cca}}_{\mathcal{PKE},\mathcal{A}}(k) \\
\quad (pk, sk) \xleftarrow{\$} \mathsf{PKE.kg}(1^k) \\
\quad (M_0, M_1, St_1) \xleftarrow{\$} \mathcal{A}_1^{\mathsf{PKE.Dec}(sk,\cdot)}(pk) \\
\quad b \xleftarrow{\$} \{0,1\}\ ;\ C^* \xleftarrow{\$} \mathsf{PKE.Enc}(pk, M_b) \\
\quad b' \xleftarrow{\$} \mathcal{A}_2^{\mathsf{PKE.Dec}(sk,\cdot)}(C^*, St_1) \\
\quad \text{If } b \neq b' \text{ then return 0 else return 1.}
\end{array}
$$

The adversary $\mathcal{A}_2$ is restricted not to ask C^* to the decryption oracle $\mathsf{PKE.Dec}$ and the two messages M_0 and M_1 have to be of equal length.

We define the advantage of $\mathcal{A}$ in the chosen-ciphertext experiment as

$$\mathbf{Adv}^{pke\text{-}\mathrm{cca}}_{\mathcal{PKE},\mathcal{A}}(k) \;=\; |\Pr[\mathbf{Exp}^{pke\text{-}\mathrm{cca}}_{\mathcal{PKE},\mathcal{A}}(k) = 1] - 1/2| \;.$$

A PKE scheme $\mathcal{PKE}$ is said to be indistinguishable against chosen-ciphertext attacks (IND-CCA secure in short) if the advantage function $\mathbf{Adv}^{pke\text{-}\mathrm{cca}}_{\mathcal{PKE},\mathcal{A}}(k)$ is a negligible function in k for all polynomial-time adversaries $\mathcal{A}$.

2.3 Identity-based Key-Encapsulation Mechanism

An identity-based key-encapsulation mechanism (IBKEM) scheme $\mathcal{IBKEM} = (\mathsf{IBKEM.Kg}, \mathsf{IBKEM.Extract}, \mathsf{IBKEM.Enc}, \mathsf{IBKEM.Dec})$ [45, 12] consists of four polynomial-time algorithms.

$(pk, sk) \xleftarrow{\$} \mathsf{IBKEM.Kg}(1^k)$; A probabilistic key-generation algorithm that produces a master key pair for given security parameter $k \in \mathbb{N}$. The public-key pk defines a key space $KeySp$ and an identity space $IDSp$.

$sk[id] \xleftarrow{\$} \mathsf{IBKEM.Extract}(sk, id)$; A probabilistic algorithm that outputs a user secret key $sk[id]$ for identity id.

$(C, K) \xleftarrow{\$} \mathsf{IBKEM.Enc}(pk, id)$; A probabilistic algorithm that creates a key K and a ciphertext C for identity id.

$K \leftarrow \mathsf{IBKEM.Dec}(sk[id], id, C)$**;** A deterministic decryption algorithm that decapsulates ciphertext C with identity id to recover a session key K. It may also outputs a special symbol $\perp$ to represent rejection.

For consistency, we require that for all $k \in \mathbb{N}$, all identities id, and all $(C, K) \xleftarrow{\$} \mathsf{IBKEM.Enc}(pk, id)$, we have $\Pr[\mathsf{IBKEM.Dec}(\mathsf{IBKEM.Extract}(sk, id), id, C) = K] = 1$, where the probability is taken over the choice of $(pk, sk) \xleftarrow{\$} \mathsf{IBKEM.Kg}(1^k)$, and the coins of all the algorithms in the expression above.

For fixed public key pk and identity id we define the set of valid ciphertexts $\mathit{CipherSp} = \mathit{CipherSp}(pk, id)$ as all possible C that can be output from the probabilistic $(C, K) \xleftarrow{\$} \mathsf{IBKEM.Enc}(pk, id)$.

2.4 Tag-KEM

A *key-encapsulation mechanism with tags* (tag-KEM) [3] $\mathcal{TKEM}$ consists of four polynomial-time algorithms.

$(pk, sk) \xleftarrow{\$} \mathsf{TKEM.Kg}(1^k)$**;** A probabilistic key-generation algorithm that produces a master key pair for given security parameter $k \in \mathbb{N}$. The public-key pk defines a key space KeySp and a tag space TagSp.

$(K, \omega) \xleftarrow{\$} \mathsf{TKEM.SKey}(pk)$**;** A probabilistic algorithm that outputs a key K and some state information ω.

$C \xleftarrow{\$} \mathsf{TKEM.Enc}(t, \omega)$**;** A probabilistic algorithm that creates a ciphertext C for tag t and key K which is implicitly transmitted by state information ω.

$K \xleftarrow{\$} \mathsf{TKEM.Dec}(sk, t, C)$**;** A probabilistic decryption algorithm that decapsulates ciphertext C with tag t to recover a session key K. It may also output a special symbol $\perp$ to represent rejection.

Note that in contrast to the original definition [3] we allow decapsulation also to be a probabilistic algorithm.

For consistency, we require that for all $k \in \mathbb{N}$, all (pk, sk), all tags $t \in \mathit{TagSp}$, and all $(K, \omega) \xleftarrow{\$} \mathsf{TKEM.SKey}(pk)$ and $C \xleftarrow{\$} \mathsf{TKEM.Enc}(t, \omega)$, it must hold that $\Pr[\mathsf{TKEM.Dec}(sk, t, C) = K] = 1$, where the probability is taken over the above randomized algorithms.

The security we require for tag-KEMs is IND-CCA security [3]. It is captured by defining the following experiment.

Experiment $\mathbf{Exp}^{\mathit{tkem}\text{-cca}}_{\mathcal{TKEM},\mathcal{A}}(k)$
$(pk, sk) \xleftarrow{\$} \mathsf{TKEM.Kg}(1^k)$; $(K_1^*, \omega) \xleftarrow{\$} \mathsf{TKEM.SKey}(pk)$; $K_0^* \xleftarrow{\$} \mathit{KeySp}$; $b \xleftarrow{\$} \{0,1\}$
$(t^*, St_1) \xleftarrow{\$} \mathcal{A}_1^{\mathsf{TKEM.Dec}(sk,\cdot,\cdot)}(pk, K_b^*)$
$C^* \xleftarrow{\$} \mathsf{TKEM.Enc}(t^*, \omega)$
$b' \xleftarrow{\$} \mathcal{A}_2^{\mathsf{TKEM.Dec}(sk,\cdot,\cdot)}(C^*, St_1)$
If $b \neq b'$ then return 0 else return 1.

The adversary is restricted not to ask (t^*, C^*) to the decryption oracle $\mathsf{TKEM.Dec}$.

We define the advantage of $\mathcal{A}$ in the chosen-ciphertext experiment as

$$\mathbf{Adv}^{\mathit{tkem}\text{-cca}}_{\mathcal{TKEM},\mathcal{A}}(k) \;=\; |\Pr[\mathbf{Exp}^{\mathit{tkem}\text{-cca}}_{\mathcal{TKEM},\mathcal{A}}(k) = 1] - 1/2| \;.$$

A tag-KEM $\mathcal{TKEM}$ is said to be indistinguishable against chosen-ciphertext attacks (IND-CCA secure in short) if the advantage function $\mathbf{Adv}^{tkem\text{-}\mathrm{cca}}_{\mathcal{TKEM},\mathcal{A}}(k)$ is a negligible function in k for all polynomial-time adversaries $\mathcal{A}$.

2.5 Data Encapsulation Mechanism

A data encapsulation mechanism (DEM, in short) $\mathcal{DEM} = (\mathsf{DEM.Enc}, \mathsf{DEM.Dec})$ with key-space $\{0,1\}^k$ is specified by its encryption algorithm $\mathsf{DEM.Enc}$ and decryption algorithm $\mathsf{DEM.Dec}$. The DEM needs to be indistinguishable against one-time attacks captured by defining the ind-ot-advantage of an adversary $\mathcal{B}$ as

$$\mathbf{Adv}^{dem\text{-}\mathrm{ind\text{-}ot}}_{\mathcal{DEM},\mathcal{B}}(k) = |\Pr[b = b' \ : \ K_S \xleftarrow{\$} \{0,1\}^k \ ; \ b \xleftarrow{\$} \{0,1\} \ ; \ b' \xleftarrow{\$} \mathcal{B}^{\mathsf{LOR}(K_s,\cdot,\cdot,b)}(1^k)] - 1/2|$$

Above, $\mathsf{LOR}(K_S, M_0, M_1, b)$ returns $\psi \xleftarrow{\$} \mathsf{DEM.Enc}(K_S, M_b)$. $\mathcal{B}$ is allowed only one query to this left-or-right encryption oracle, consisting of a pair of equal-length messages.

2.6 Tag-KEM/DEM: From Tag-KEM to Hybrid PKE

We transform a tag-KEM $\mathcal{TKEM} = (\mathsf{TKEM.Kg}, \mathsf{TKEM.SKey}, \mathsf{TKEM.Enc}, \mathsf{TKEM.Dec})$ and a DEM $\mathcal{DEM} = (\mathsf{DEM.Enc}, \mathsf{DEM.Dec})$ into a public-key encryption scheme $\mathcal{PKE} = (\mathsf{PKE.kg} = \mathsf{TKEM.Kg}, \mathsf{PKE.Enc}, \mathsf{PKE.Dec})$ as follows. For completeness the formal definition of a PKE scheme is given in Section 2.2.

$\mathsf{PKE.Enc}(pk, M)$	$\mathsf{PKE.Dec}(sk, C\|\|\psi)$
$(K, \omega) \xleftarrow{\$} \mathsf{TKEM.SKey}(pk)$	$K \xleftarrow{\$} \mathsf{TKEM.Dec}(sk, C)$
$\psi \leftarrow \mathsf{DEM.Enc}(K, M)$	$M \leftarrow \mathsf{DEM.Dec}(K, \psi)$
$C \xleftarrow{\$} \mathsf{TKEM.Enc}(\psi, \omega)$	Return M
Return $C\|\|\psi$	

The following was proved in [3].

Theorem 1. *Suppose $\mathcal{TKEM}$ is IND-CCA secure and $\mathcal{DEM}$ is IND-OT secure. Then the PKE scheme is IND-CCA secure.*

2.7 Chameleon Hash

A Chameleon Hash $\mathcal{CMH} = (\mathsf{CMH.Kg}, \mathsf{CMH.H}, \mathsf{CMH.Trap})$ is specified by the following algorithms [34].

$(pk_{ch}, sk_{ch}) \xleftarrow{\$} \mathsf{CMH.Kg}(1^k)$**;** A key-generation algorithm $\mathsf{CMH.Kg}$ that outputs a pair of hash and trapdoor keys pk_{ch}, sk_{ch}, respectively. The hash key defines the message space $\mathsf{Msg_{CMH}}$ and randomness space $\mathsf{Rand_{CMH}}$.

$h \leftarrow \mathsf{CMH.H}(pk_{ch}, m, s)$**;** A hash algorithm that takes the hash key pk_{ch}, message $m \in \mathsf{Msg_{CMH}}$, and randomness $s \in \mathsf{Rand_{CMH}}$ and outputs a hash value h.

$s \leftarrow \mathsf{CMH.Trap}(sk_{ch}, m', s', m)$**;** An algorithm that outputs randomness s uniformly such that $\mathsf{CMH.H}(pk_{ch}, m', s') = \mathsf{CMH.H}(pk_{ch}, m, s)$ for given $m', m \in \mathsf{Msg_{CMH}}$ and $s' \in \mathsf{Rand_{CMH}}$ by using trapdoor-key sk_{ch}.

In the original security definition of a Chameleon hash, full collision resistance [23] was considered. However, in our case something weaker is sufficient, i.e., random prefix collision resistance [3], which seems close to the target collision resistance [37, 5] rather than full collision resistance. It is captured by defining the following experiment.

Experiment $\mathbf{Exp}^{cmh\text{-rpc}}_{\mathcal{CMH},\mathcal{A}}(k)$
$(pk_{ch}, sk_{ch}) \xleftarrow{\$} \mathsf{CMH.Kg}(1^k)$
$(m, St) \xleftarrow{\$} \mathcal{A}_1(pk_{ch})$
$s \xleftarrow{\$} \mathsf{Rand_{CMH}}$
$(m', s') \xleftarrow{\$} \mathcal{A}_1(s, St)$
If $\mathsf{CMH.H}(pk_{ch}, m', s') = \mathsf{CMH.H}(pk_{ch}, m, s) \bigwedge (m', s') \neq (m, s)$ then return 1 else return 0.

We define the success probability of $\mathcal{A}$ in the above experiment as

$$\mathbf{Adv}^{cmh\text{-rpc}}_{\mathcal{CMH},\mathcal{A}}(k) \;=\; \Pr[\mathbf{Exp}^{cmh\text{-rpc}}_{\mathcal{CMH},\mathcal{A}}(k) = 1] \,.$$

A chameleon hash $\mathcal{CMH}$ is said to be random prefix collision resistant (RPC secure, in short) if $\mathbf{Adv}^{cmh\text{-rpc}}_{\mathcal{CMH},\mathcal{A}}(k)$ is a negligible function in k for all polynomial-time adversaries $\mathcal{A}$.

Chameleon Hash based on DLOG. To apply our generic transformation in Section 5, we recall a simple construction from [34] of a discrete-log based Chameleon hash. The key generator creates a random group element $h = g^z$, from a multiplicative cyclic group $\mathbb{G}$ of prime order p. The public hash key pk_{ch} is h, the trapdoor key sk_{ch} is z. Message space and randomness are defined as $\mathbb{Z}_p$. To hash an element $m \in \mathbb{Z}_p$ using randomness $s \in \mathbb{Z}_p$, compute $g^m h^s \leftarrow \mathsf{CMH.H}(pk_{ch}, m, s)$. To compute a collision for (m', s', m) the trapdoor algorithm $\mathsf{CMH.Trap}(sk_{ch} = z, m', s', m)$ computes $s = (m - m')/z + s' \bmod p$. If $m \neq m'$ so is $s \neq s'$. We verify the correctness of the trapdoor algorithm by $\mathsf{CMH.H}(pk_{ch}, m', s') = g^{m'} h^{s'} = g^{m'+zs'} = g^{m+zs} = \mathsf{CMH.H}(pk_{ch}, m, s)$. This Chameleon hash is prefix collision resistant under the assumption that computing discrete logs in $\mathbb{G}$ is computational infeasible (DLOG assumption).

3 Partitioned IBKEM

3.1 Definition

Intuitively, an IBKEM is said to be *partitioned* if the encapsulated key K does not depend on id, and the ciphertext C can be split into two parts $C = (c_1, c_2)$. We requre the first part c_1 not to depend on id, and (c_1, id) uniquely determines c_2. Furthermore, it is needed that $\mathsf{IBKEM.Dec}$ rejects all *invalid* ciphertexts by returning a random key $K \in KeySp$. Note that, since $\mathsf{IBKEM.Dec}$ is deterministic, the randomness for K would come from $\mathsf{IBKEM.Extract}(sk, id)$. Though such strong properties are indeed provided by most of existing schemes in the standard model, we will show that some requirements can be relaxed or generalized in the sequel. Partitioned IBKEM is formally defined as follows.

Definition 2. *An IBKEM scheme is* partitioned *if there exist three functions* $\mathsf{IBKEM.SKey}$, $\mathsf{IBKEM.EncKey}$, *and* $\mathsf{IBKEM.EncID}$ *such that:*

$(K, \varphi) \xleftarrow{\$} \mathsf{IBKEM.SKey}(pk)$**;** *A probabilistic algorithm that takes a master public-key, and outputs a session key K and state information φ.*

$c_1 \leftarrow \mathsf{IBKEM.EncKey}(\varphi)$; *A deterministic algorithm that takes a state information* φ *generated by* IBKEM.SKey*, and outputs the first part of ciphertext* c_1*.*

$c_2 \leftarrow \mathsf{IBKEM.EncID}(\varphi, id)$; *A deterministic algorithm that takes a state information* φ *and an identity* id*, and computes the second part of ciphertext* c_2*.*

And they fulfill the following properties.

Consistency: *Let* $\mathsf{IBKEM.EncID} \circ \mathsf{EncKey} \circ \mathsf{SKey}(pk, id)$ *be a composed function that computes* $(K, \varphi) \xleftarrow{\$} \mathsf{IBKEM.SKey}(pk)$*,* $c_1 \leftarrow \mathsf{IBKEM.EncKey}(\varphi)$*,* $c_2 \leftarrow \mathsf{IBKEM.EncID}(\varphi, id)$ *and outputs* $((c_1, c_2), K)$*. Then for any* pk *and* id*, we requre that* $\mathsf{IBKEM.EncID} \circ \mathsf{EncKey} \circ \mathsf{SKey}(pk, id) = \mathsf{IBKEM.Enc}(pk, id)$*.*

Unique Split Property: *For any* c_1, c_2 *and* id *such that* $(c_1, c_2) \in \mathit{CipherSp}(pk, id)$ *for some* pk*, there exists no* (c_1', id') *such that* $(c_1', id') \neq (c_1, id)$ *and* $(c_1', c_2) \in \mathit{CipherSp}(pk, id')$*.*

$-Rejection Property: *If* $(\tilde{c}_1, \tilde{c}_2) \notin \mathit{CipherSp}(pk, id)$*, then* $\mathsf{IBKEM.Dec}(\mathsf{IBKEM.Extract}(sk, id), id, (\tilde{c}_1, \tilde{c}_2))$ *distributes statistically close to uniform over* KeySp*.*

Note that it is important to require c_2 to be uniquely defined on c_1 and id, since otherwise a trivial splitting of the ciphertext as $(c_1, c_2) = (\varepsilon, c_1||c_2)$ is always possible, where ε denotes the empty string.

At this point it is worth noting that, to our best knowledge, most of known IBKEMs [7, 47] are in fact partitioned. We will present concrete instantiations of the resulting tag-KEMs in Section 5. Most of known IBKEMs, except the Sakai et al. scheme [44] and the Boneh-Franklin scheme [12], are partitioned. However, both schemes are out of our main purpose in this paper, since they are only secure in the random-oracle model.

3.2 Security Model

The notion of indistinguishability against selective-identity and chosen-plaintext attacks, sID IND-CPA in short, is defined in the same way as given in [17, 12] regardless of the partitioned structure. Formally, it is defined through the following game between a challenger and an adversary $\mathcal{A} = (\mathcal{A}_0, \mathcal{A}_1)$. Let scheme $\mathit{IBKEM} = (\mathsf{IBKEM.Kg}, \mathsf{IBKEM.Extract}, \mathsf{IBKEM.SKey}, \mathsf{IBKEM.EncKey}, \mathsf{IBKEM.EncID}, \mathsf{IBKEM.Dec})$ be a partitioned IBKEM.

To an adversary $\mathcal{A}$ we associate the following experiment:

Experiment $\mathbf{Exp}^{\textit{ibkem}\text{-sid-cpa}}_{\mathit{IBKEM},\mathcal{A}}(k)$

$(id^*, St_0) \xleftarrow{\$} \mathcal{A}_0(1^k)$

$(pk, sk) \xleftarrow{\$} \mathsf{IBKEM.Kg}(1^k)$; $(K_1^*, \varphi) \xleftarrow{\$} \mathsf{IBKEM.SKey}(pk)$; $K_0^* \xleftarrow{\$} \mathit{KeySp}$; $b \xleftarrow{\$} \{0, 1\}$

$c_1^* \leftarrow \mathsf{IBKEM.EncKey}(\varphi)$; $c_2^* \leftarrow \mathsf{IBKEM.EncID}(\varphi, id^*)$

$b' \xleftarrow{\$} \mathcal{A}_1^{\mathsf{IBKEM.Extract}(sk, \cdot)}(pk, K_b^*, c_1^*, c_2^*, St_0)$

If $b \neq b'$ then return 0 else return 1

Adversary $\mathcal{A}$ is not allowed to query oracle $\mathsf{IBKEM.Extract}(sk, \cdot)$ for the target identity id^*.

It is stressed that though the above experiment is described by using the separated encryption functions of an 'partitioned' IBKEM, from the viewpoint of the adversary it is exactly the same experiment as in the standard selective-ID IBKEM security definition presented in [12]. Hence an adversary having some advantage in attacking the standard IBKEM security has exactly the same advantage in attacking the partitioned IBKEM security described in the above experiment.

We define the advantage of $\mathcal{A}$ in the experiment as

$$\mathbf{Adv}^{\mathit{ibkem}\text{-sid-cpa}}_{\mathit{IBKEM},\mathcal{A}}(k) \;=\; |\Pr[\mathbf{Exp}^{\mathit{ibkem}\text{-sid-cpa}}_{\mathit{IBKEM},\mathcal{A}}(k)=1] - 1/2| \;.$$

A partitioned IBKEM IBKEM is said to be selective-identity IND-CPA if the advantage functions $\mathbf{Adv}^{\mathit{ibkem}\text{-sid-cpa}}_{\mathit{IBKEM},\mathcal{A}}(k)$ is a negligible function in k for all polynomial-time adversaries $\mathcal{A}$.

4 Transformation using Chameleon Hash

We construct a tag-KEM $\mathit{TKEM} = (\mathsf{TKEM.Kg}, \mathsf{TKEM.SKey}, \mathsf{TKEM.Enc}, \mathsf{TKEM.Dec})$ from a partitioned selective-ID IBKEM scheme IBKEM and a chameleon hash scheme $\mathcal{CMH}$. For ease of notation, $\mathcal{CMH}$ is considered as global so that $\mathsf{CMH.H}$ is accessible from a function without explicitly taking its key from outside.

First of all, we set the key generation algorithm $\mathsf{TKEM.Kg} = \mathsf{IBKEM.Kg}$.[3] The rest of the algorithms are constructed as follows.

$\mathsf{TKEM.SKey}(pk)$
  $(K, \varphi) \xleftarrow{\$} \mathsf{IBKEM.SKey}(pk)$
  Return (K, φ)

$\mathsf{TKEM.Enc}(t, \varphi)$
  $c_1 \leftarrow \mathsf{IBKEM.EncKey}(\varphi)$
  $s \xleftarrow{\$} \mathsf{Rand}_{\mathsf{CMH}}$; $id \leftarrow \mathsf{CMH.H}(t||c_1, s)$
  $c_2 \leftarrow \mathsf{IBKEM.EncID}(\varphi, id)$
  Return $c \leftarrow c_1||c_2||s$

$\mathsf{TKEM.Dec}(sk, t, c)$
  $c_1||c_2||s \leftarrow c$
  $id \leftarrow \mathsf{CMH.H}(t||c_1, s)$
  $sk[id] \xleftarrow{\$} \mathsf{IBKEM.Extract}(sk, id)$
  $K \leftarrow \mathsf{IBKEM.Dec}(sk[id], id, (c_1, c_2))$
  Return K

In this transformation we implicitly require that the Chameleon hash maps elements from correct domains. In a more general setting we can apply (target) collision resistant hash functions, see Section 5 for more details.

Theorem 3. *If the partitioned IBKEM is selective-ID IND-CPA secure and the chameleon hash is random prefix collision resistant, then the above TKEM is IND-CCA secure. In particular,*

$$\mathbf{Adv}^{\mathit{tkem}\text{-cca}}_{\mathit{TKEM},\mathcal{A}}(k) \le \mathbf{Adv}^{\mathit{ibkem}\text{-sid-cpa}}_{\mathit{IBKEM},\mathcal{B}}(k) + \mathbf{Adv}^{\mathit{cmh}\text{-rpc}}_{\mathcal{CMH},\mathcal{C}}(k).$$

Proof. (Sketch.) Assume there exists an adversary $\mathcal{A}$ against the IND-CCA security of the tag-KEM TKEM. We show that then there exists either an adversary $\mathcal{B}$ against the selective-ID IND-CPA security of IBKEM or an adversary $\mathcal{C}$ against the random prefix collision resistance of $\mathcal{CMH}$.

We first describe adversary $\mathcal{B}$.

Setup. Given security parameter k, adversary $\mathcal{B}$ sets up a random instance (pk_{ch}, sk_{ch}) of the chameleon hash via $\mathsf{CMH.Kg}(1^k)$. It then generates the target identity id^* by $id^* \leftarrow \mathsf{CMH.H}(t', s')$ where t' and s' are chosen randomly from appropriate domains. It sends id^* to the challenger and receives a public-key pk and a challenge (K_b^*, c_1^*, c_2^*). Adversary $\mathcal{B}$ now runs $\mathcal{A}$ by giving pk and pk_{ch}.

Challenge Simulation. At some point, $\mathcal{A}$ outputs a target tag t^*. $\mathcal{B}$ then computes $s^* \leftarrow \mathsf{CMH.Trap}(sk_{ch}, t', s', t^*||c_1^*)$ and sends K_b^* and (c_1^*, c_2^*, s^*) to $\mathcal{A}$.

[3] When $\mathcal{CMH}$ is local, $\mathsf{TKEM.Kg}$ also invokes $\mathsf{CMH.Kg}$ and include the hash keys to its public-key.

Decryption Oracle Simulation. If $\mathcal{A}$ makes a decryption query with a ciphertext (c_1, c_2, s) and a tag t, adversary $\mathcal{B}$ simulates decryption oracle $\mathsf{TKEM.Dec}(sk, t, (c_1, c_2, s))$ in the following way. Let $id = \mathsf{CMH.H}(t||c_1, s)$.

Case 0: If $(c_1, c_2, s, t) = (c_1^*, c_2^*, s^*, t^*)$, return nothing.

Case 1: If $(c_1, s, t) \neq (c_1^*, s^*, t^*)$ and $id = id^*$, abort. (Denote this event by COL.)

Case 2: If $(c_1, s, t) = (c_1^*, s^*, t^*)$ and $c_2 \neq c_2^*$, return a random K.

Case 3: If none of the above happens, send id to oracle $\mathsf{IBKEM.Extract}$ and receive $sk[id]$. Then compute $K \leftarrow \mathsf{IBKEM.Dec}(sk[id], id, (c_1, c_2))$ and return K to $\mathcal{A}$.

Output. Finally, $\mathcal{A}$ returns a guess bit b'. Adversary $\mathcal{B}$ returns the same bit b' and terminates the game.

Challenge simulation is perfect since $id^* = \mathsf{CMH.H}(t', s') = \mathsf{CMH.H}(t^*, s^*)$. In the simulation of the decryption oracle, Case 0 and 3 are just as defined. In Case 2, the ciphertext (c_1^*, c_2) is clearly incorrect since $c_2^* \neq c_2$ is the only correct second part for (c_1^*, s^*, t^*). Consequently, due to the perfect \$-rejection property, the decryption oracle in the original experiment outputs a random K, just as done by $\mathcal{B}$. Accordingly, $\mathcal{B}$ perfectly simulates $\mathcal{A}$'s view in the IND-CCA experiment unless event COL happens.

If COL does not happen then $\mathcal{B}$ has the same advantage in winning the experiment as $\mathcal{A}$. On the other hand, we can build an adversary $\mathcal{C}$ against the random prefix collision resistance of the chameleon hash that wins with probability one if COL happens during $\mathcal{A}$'s simulation. This concludes the proof. □

Remark 4. Following Thoerem 1, one can combine the above tag-KEM and DEM to obtain an IND-CCA secure hybrid PKE.

Remark 5. There may exist some particular IBKEMs where the ciphertext C can not even be split into the two parts c_1 and c_2. One possible reason is that all elements in C depend on id. This is in particular the case when C only consists of one element. In that case one can add one "dummy element" c_1 to the ciphertext such that: (i) the security of the IBKEM is not influenced; (ii) the new split (c_1, c_2) now fulfills the "unique split property" as defined in Definition 2. The IBKEM from Sakai et al. [44] is such an example where this dummy element can easily be added without harming the security of the scheme, but only secure in the random oracle model.

From a theoretical point of view we find the need for element c_1 in the transformation quite interesting. Since a fixed id uniquely links c_1 with c_2 we observe that c_1 it can be viewed as a "witness" to prove correctness of the resulting tag-KEM ciphertext. Without witness c_1 such correctness cannot be verified. This may enables an adversary to modify c_2 in a certain way to mount a CCA attack.

If the element c_2 does not uniquely depend on c_1 and id, we instead require a limited sense of non-malleability. Namely, given correct (c_1, c_2, id), it is possible to create another correct (c_1, c_2', id) only with negligible probability even with access to the oracle $\mathsf{IBKEM.Extract}$. Such a property, however, would result in needing a dedicated proof, which is not preferable for generic construction we concern.

Remark 6. In the proof of Theorem 3, adversary $\mathcal{B}$ must simulate the behavior of the decryption oracle $\mathsf{IBKEM.Dec}(\mathsf{IBKEM.Extract}(sk, id^*), id^*, (c_1^*, c_2))$ without knowing sk. The unique split and perfect \$-rejection properties from Definition 2 allow to do so.

Such a simulation is also possible if $\mathsf{IBKEM.Dec}(\mathsf{IBKEM.Extract}(sk, id^*), id^*, (c_1^*, c_2))$ returns a special symbol, say $\perp$, for every incorrect c_2. In such a case, $\mathcal{B}$ simply outputs $\perp$ instead of random K. Hence such a property, which we call $\perp$-rejection property, is also acceptable instead of \$-rejection. Indeed, any kind of rejection is acceptable as long as the output distribution of $\mathsf{IBKEM.Dec}(\mathsf{IBKEM.Extract}(sk, id^*), id^*, (c_1^*, c_2))$ is efficiently simulatable without knowing sk. One can also relax the "perfect" part by introducing a small error probability that additively affects to the reduction cost shown in Theorem 3 and 10.

5 Concrete Tag-KEMs

In this section we demonstrate the usefulness of our transformation by giving possible instantiations of tag-KEMs based on known IBE schemes. Note that all our tag-KEMs imply IND-CCA secure PKE schemes using the generic transformation from Section 2.6.

Bilinear Groups. All pairing based schemes will be parameterized by a *pairing parameter generator*. This is a polynomial-time algorithm $\mathcal{G}$ that on input 1^k returns the description of an multiplicative cyclic group $\mathbb{G}$ of prime order p, where $2^k < p < 2^{k+1}$, the description of a multiplicative cyclic group $\mathbb{G}_T$ of the same order, and a non-degenerate bilinear pairing $\hat{e}$: $\mathbb{G} \times \mathbb{G} \to \mathbb{G}_T$. See [12] for a description of the properties of such pairings. We use $\mathbb{G}^*$ to denote $\mathbb{G} \setminus \{1\}$, i.e. the set of all group elements except the neutral element. Throughout the paper we use $\mathbb{PG} = (\mathbb{G}, \mathbb{G}_T, p, \hat{e}, g)$ as shorthand for the description of bilinear groups, where g is a generator of $\mathbb{G}$.

The BDDH assumption in $\mathbb{PG}$ is captured by defining the bddh-advantage of an adversary $\mathcal{B}$ as

$$\mathbf{Adv}^{\mathrm{bddh}}_{\mathbb{PG},\mathcal{B}}(k) = |\Pr[\mathcal{B}(g^x, g^y, g^z, \hat{e}(g,g)^{xyz}) = 1] - \Pr[\mathcal{B}(g^x, g^y, g^z, g^r) = 1]|$$

where $x, y, z, r \xleftarrow{\$} \mathbb{Z}_p^*$.

In the following concrete transformation, we make use of a Chameleon hash from [34] based on DLOG assumption recalled in Section 2.7. Note that the BDDH assumption in $\mathbb{PG}$ implies the DLOG assumption in $\mathbb{G}$. The Chameleon hash maps elements from $\mathbb{Z}_p$ to $\mathbb{G}$. Unfortunately, in our application we additionally need to modify it to map elements from $\mathit{TagSp} \times \mathbb{G}$ to $\mathbb{Z}_p$. To this end we employ two (target collision-resistant) hash functions $\mathsf{CR} : \{0,1\}^* \times \mathbb{G} \to \mathbb{G}$ and $\mathsf{TCR} : \mathbb{G} \to \mathbb{Z}_p$. The modified Chameleon Hash function is defined as $\mathsf{CMH.H}_{\mathrm{dl}}(h, t||c_1, s) = \mathsf{TCR}(g^{\mathsf{CR}(t||c_1)} h^s)$. If CR is a collision resistant hash function and TCR is a target collision resistant hash function then the modified Chameleon Hash is prefix collision resistant.

5.1 Based on Boneh-Boyen's IBE

We first recall the (first) IBE scheme from Boneh and Boyen [7] which we already present in its partitioned IBKEM form $\mathcal{BB} = (\mathsf{BB.Kg}, \mathsf{BB.SKey}, \mathsf{BB.EncKey}, \mathsf{BB.EncID}, \mathsf{BB.Extract}, \mathsf{BB.Dec})$.

$\mathsf{BB.Kg}(1^k)$
$x_0, x_1, y \xleftarrow{\$} \mathbb{Z}_p^*$
$u_0 \leftarrow g^{x_0}$; $u_1 \leftarrow g^{x_1}$
$v \leftarrow \hat{e}(g,g)^y$
$pk \leftarrow (u_0, u_1, v)$
$sk \leftarrow (x_0, x_1, y)$
Return (pk, sk)

$\mathsf{BB.SKey}(pk)$
$r \xleftarrow{\$} \mathbb{Z}_p^*$; $K \leftarrow v^r \in \mathbb{G}_T$
Return $(K, \varphi = (pk, r))$

$\mathsf{BB.EncKey}(\varphi = (pk, r))$
Return $c_1 \leftarrow g^r$

$\mathsf{BB.EncID}(id, \varphi = (pk, r))$
Return $c_2 \leftarrow (u_0 u_1^{id})^r$

$\mathsf{BB.Extract}(sk, id)$
$s \xleftarrow{\$} \mathbb{Z}_p^*$
$d_1 \leftarrow g^s$; $d_2 \leftarrow g^y \cdot (u_0 u_1^{id})^s$
Return $sk[id] = (d_1||d_2)$

$\mathsf{BB.Dec}(sk[id], id, c_1||c_2)$
Parse $sk[id] = (d_1, d_2)$
Return $K \leftarrow \hat{e}(c_1, d_2)/\hat{e}(c_2, d_1)$

The above IBKEM scheme is known to be selective-ID IND-CPA secure [7] under the BDDH assumption. We quickly verify it has the perfect \$-rejection property. Fix pk and id and let $\tilde{C} = (\tilde{c_1}, \tilde{c_2}) \notin \mathit{CipherSp}$ be a ciphertext that was not generated by the encapsulation algorithms. That means that $\tilde{c_1} = g^{r_1}$ and $\tilde{c_2} = (u_0 u_1^{id})^{r_2}$ for some $r_1 \neq r_2$. Then, probabilistic $\mathsf{IBKEM.Dec}(\mathsf{IBKEM.Extract}(sk, id]), \tilde{c})$ picks a random $s \in \mathbb{Z}_p$ and returns

$$K = \hat{e}(\tilde{c_1}, g^y \cdot (u_0 u_1^{id})^s) / \hat{e}(\tilde{c_2}, g^s) = \hat{e}(g^{r_1}, g^y) \cdot \hat{e}(g^{r_1}, (u_0 u_1^{id})^s) / \hat{e}((u_0 u_1^{id})^{r_2}, g^s) = v^{r_1} \cdot \hat{e}(g, u_0 u_1^{id})^{s(r_1 - r_2)}.$$

Since for every decapsulation a fresh random value $s \in \mathbb{Z}_p$ is chosen, K is indeed a random key, uniformly distributed, and independent of anything else. On the other hand, a consistent ciphertext $\tilde{C} \in \mathit{CipherSp}$ has $r := r_1 = r_2$ and yield the correct key $K = v^r$.

Using the transformation from Section 4 and the modified DLOG-based Chameleon Hash $\mathsf{CMH.H}_{\mathrm{dl}}(h, \cdot, \cdot)$ we get the following tag-KEM. As it was already done in [17, 14, 30, 31], decapsulation can be further simplified by using sk to reject all inconsistent ciphertexts that are not contained in $\mathit{CipherSp}$.

$\mathsf{TKEM.Kg}(1^k)$	$\mathsf{TKEM.SKey}(pk')$	$\mathsf{TKEM.Dec}(sk, t, c_1 \Vert c_2 \Vert s)$
$(pk, sk) \xleftarrow{\$} \mathsf{BB.Kg}(1^k)$	$r \xleftarrow{\$} \mathbb{Z}_p^*$; $K \leftarrow v^r \in \mathbb{G}_T$	$id \leftarrow \mathsf{CMH.H}_{\mathrm{dl}}(h, t \Vert c_1, s)$
$z \xleftarrow{\$} \mathbb{Z}_p^*$; $h \leftarrow g^z$	Return $(K, \varphi = (pk', r))$	If $c_1^{x_0 + id \cdot x_1} \neq c_2$
$pk' \leftarrow (pk, h)$		then return $\perp$
Return (pk', sk)	$\mathsf{TKEM.Enc}(t, \varphi = (pk', r))$	Else Return $K \leftarrow \hat{e}(c_1, g^y)$
	$c_1 \leftarrow g^r$	
	$s \xleftarrow{\$} \mathbb{Z}_p$; $id \leftarrow \mathsf{CMH.H}_{\mathrm{dl}}(h, t \Vert c_1, s)$	
	$c_2 \leftarrow (u_0 u_1^{id})^r$	
	Return $C = c_1 \Vert c_2 \Vert s$	

Security of the above tag-KEM can be reduced by Theorem 3 to the security of the underlying IBKEM and the Chameleon hash. Hence the tag-KEM is IND-CCA under the BDDH assumption.

There is also a second IBE scheme in the paper by Boneh and Boyen [7], i.e. a (more efficient) scheme based on the stronger q-BDDHI assumption. We remark that applying our transformation to this scheme leads to a tag-KEM that is almost identical to the above tag-KEM. We therefore forbear from giving more details about the resulting here. This is reminiscent to what what happens to the two Boneh-Boyen IBE schemes applied to the CHK transformation from [18], see [31] for more details.

The above scheme uses a Chameleon Hash which additionally employs two hash functions to map elements to the proper domains. We now show a non-generic improvement of the above scheme that avoids these problems. The idea is to use an *implicit Chameleon Hash* in the exponent defined as $id = \mathsf{ICMH.H}(t', s) = t' + x_2 s$. In the public key of the Chameleon hash we only include $u_1^{x_2}$, so $u_1^{\mathsf{ICMH.H}(t', s)}$ can be publicly evaluated by computing $u_1^{t'} \cdot u_2^s$. Hence the element c_2 of the IBKEM ciphertext can be computed as $c_2 = (u_0 u_1^{id})^r = (u_0 u_1^{t'} \cdot u_2^s)^r$. Breaking prefix collision resistance of this implicit Chameleon hash is as hard as breaking the DLOG assumption in $\mathbb{G}$. A similar technique was already used in [8] to obtain short signatures without random oracles. Again, we need a collision-resistant hash function $\mathsf{CR} : \mathit{TagSp} \times \mathbb{G}_1 \to \mathbb{Z}_p$.

$\mathsf{TKEM.Kg}(1^k)$
$(pk, sk) \xleftarrow{\$} \mathsf{BB.Kg}(1^k)$
$x_2 \xleftarrow{\$} \mathbb{Z}_p^*$; $u_2 \leftarrow u_1^{x_2}$
$pk' \leftarrow (pk, u_2)$
$sk' \leftarrow (sk, x_2)$
Return (pk', sk')

$\mathsf{TKEM.SKey}(pk')$
$r \xleftarrow{\$} \mathbb{Z}_p^*$; $K \leftarrow v^r \in \mathbb{G}_T$
Return $(K, \varphi = (pk', r))$

$\mathsf{TKEM.Enc}(t, \varphi = (pk', r))$
$s \xleftarrow{\$} \mathbb{Z}_p^*$; $c_1 \leftarrow g^r$
$t' \leftarrow \mathsf{CR}(c_1||t)$; $c_2 \leftarrow \left(u_0 u_1^{t'} u_2^s\right)^r$
Return $C = c_1||c_2||s$

$\mathsf{TKEM.Dec}(sk', t, c_1||c_2||s)$
$t' \leftarrow \mathsf{CR}(c_1||t)$
If $c_1^{x_0 + t' \cdot x_1 + s \cdot x_2} \neq c_2$
then return $\perp$
Else Return $K \leftarrow \hat{e}(c_1, g^y)$

We apply Theorem 3 (with obvious modifications to take into account the implicit Chameleon hash) to prove that the above tag-KEM is IND-CCA secure if the hash function CR is collision resistant and the BDDH assumption holds in $\mathbb{PG}$.

5.2 Based on Waters' IBE

Viewing Waters' (adaptive-identity secure) IBE scheme [47] as an IBKEM we note that in fact it is already a partitioned IBKEM.

Let $\mathsf{CR} : \mathit{TagSp} \times \mathbb{G} \to \{0,1\}^n$ be a collision resistant hash function, where $n \approx k$ is a security parameter of the scheme. The transformation from Section 7.3 gives the following tag-KEM.

$\mathsf{TKEM.Kg}(1^k)$
$x_0, x_1, \ldots, x_n, y \xleftarrow{\$} \mathbb{Z}_p^*$
$u_0 \leftarrow g^{x_0}$; ... ; $u_n \leftarrow g^{x_n}$
$z \leftarrow \hat{e}(g,g)^y$
$pk \leftarrow (u_0, \ldots, u_n, z)$
$sk \leftarrow (x_0, \ldots, x_n, g^y)$
Return (pk, sk)

$\mathsf{TKEM.SKey}(pk)$
$r \xleftarrow{\$} \mathbb{Z}_p^*$; $K \leftarrow z^r$
Return $(K, \varphi = (pk, r))$

$\mathsf{TKEM.Enc}(t, \varphi = (pk, r))$
$c_1 \leftarrow g^r$
$id \leftarrow \mathsf{CR}(t||c_1)$; $c_2 \leftarrow (u_0 \prod_{i=1}^n u_i^{id_i})^r$
Return $C = c_1||c_2$

$\mathsf{TKEM.Dec}(sk, t, c_1||c_2)$
$id \leftarrow \mathsf{CR}(t||c_1)$
If $c_1^{x_0 + \sum_{i=1}^n id_i x_i} \neq c_2$
then return $\perp$
Else Return $K \leftarrow \hat{e}(c_1, g^y)$

Here id_i means the ith bit of $id \in \{0,1\}^n$. In fact, one can readily verify that the proof in [47] already shows that, under the BDDH assumption, Waters' IBKEM is strongly adaptive-ID IND-CPA. Using the tag-KEM/DEM framework from Section 2.6 the obtained PKE scheme is the same as the "direct PKE scheme based on IBE techniques" from [14]. Security of the PKE scheme can now be directly understood in terms of Theorem 10, i.e. by reducing it to the security of the hash function plus the strong security properties of Water's original IBE scheme. In contrast, [14] had to rely on a dedicated proof.

Kiltz [32] recently showed that the technique from Waters [47] can also be applied to the "inversion-based" IBE scheme from Boneh and Boyen [7] to obtain an another adaptive-identity secure IBE scheme based on the q-BDHI assumption. Viewing this scheme as an IBKEM we note that in fact it is already a partitioned IBKEM and hence it leads to another PKE scheme, similar to the one described above.

Gentry proposed yet another adaptive-identity secure IBE scheme in the standard model [28]. Unfortunately, we are not able to prove the implied IBKEM secure in the sense of Section 7.3.

5.3 Comparison

A quick comparison of all tag KEMs discussed in this section is given in Table 1. As security parameter we fixed $k = 128$ bits. For pairing groups that means that elements in $\mathbb{G}$ can be represented in ≈ 256 bits. We remark that using the concept of sequential multiplications all

Tag-KEM	Security Assumption	Ciphertext Overhead	Encryption #pairings +	Decryption #[multi,regular]-exp	Keysize (pk/sk)
§5.1	BDDH	768	$0 + [1, 2]$	$1 + [0, 1]$	4/3
§5.1 (implicit CMH)	BDDH	768	$0 + [1, 1]$	$1 + [0, 1]$	4/4
§5.2	BDDH	512	$0 + [3, 0]$	$1 + [0, 1]$	$\approx 130/130$

Table 1: Efficiency comparison for our chosen-ciphertext secure tag KEMs. For efficiency we count the number of pairings + [multi exponentiations, regular exponentiations] used for encryption, decryption, and key generation. All "symmetric" operations (such as the hash function) are ignored. Ciphertext overhead represents the expected difference (in bits) between ciphertext and plaintext length for $k = 128$ bit security. For comparison we mention that relative timings for the various operations are as follows: bilinear pairing $\approx 3 - 5$ [39], multi(=sequential)-exponentiation ≈ 1.2 [6], and regular exponentiation $= 1$.

decapsulation algorithms can be optimized to use only one pairing plus one sequential exponentiation. The security reduction of Waters' IBE to the BDDH assumption is not tight, i.e. it introduces a $\log q$ bit of security, where q is the maximal number of decryption queries made by an adversary. Also note that the key-size of the Waters' IBE based tag-KEM from can be reduced by the factor of l by loosing any other l bits of security in the reduction [20].

6 Transformation in the Threshold Setting

Observe that the generic transformation proposed in Section 4 does not require any private-key operation. The chameleon hashing in the decryption process comes *before* the decryption of the underlying IBKEM. Accordingly, if the underlying partitioned IBKEM is publicly verifiable, so is the resulting tag-KEM. This feature is quite useful in the threshold setting where every decryption server needs to be convinced of the correctness of the ciphertext, preferable without interacting with other servers.

Based on the above observation, we extend our framework from Section 4 to construct efficient threshold PKE schemes from selective-ID partitioned threshold IBKEMs. We quickly sketch the ingredients of this transformation.

- A threshold version of our generic transformation that transforms threshold selective-ID partitioned IBKEMs into threshold tag-KEMs, and further to a threshold hybrid encryption according to the following generic composition.
- An extension of the tag-KEM/DEM framework to the threshold setting. Every threshold tag-KEM can be combined with a passively secure DEM yielding efficient threshold PKE. This construction was briefly mentioned in the original work of [3], without a rigorous proof.
- A concrete example of a threshold selective-ID partitioned IBKEM that is secure on the BDDH assumption.

Putting all three ingredients together we get an efficient threshold PKE scheme which is IND-CCA secure under the BDDH assumption. Similar to the standard (non-threshold) case our proposal introduces a ciphertext overhead of 128 bits. To show the full details of the construction, we need to translate all the building blocks and the tag-KEM/DEM composition theorem into the threshold setting. Except for certain technical subtleties, all the results in the

previous sections hold in the threshold setting. Section 6.1 introduces detailed definitions in the threshold setting. Section 6.2 and Section 6.3 contain a generic transformation and a concrete example, respectively. For building threshold hybrid PKE from threshold Tag-KEM in a generic way, Appendix B.2 provides the threshold version of the tag-KEM/DEM composition theorem.

An existing proposal of a threshold PKE scheme was giving in [9]. This construction makes use of the CHK-transformation [18] based on a one-time signature scheme and therefore has a ciphertext overhead of roughly $65k$ bits. We note that the more efficient BK-transformation [13] is not applicable in this setting since it does not provide public verifiability of the threshold ciphertexts.

6.1 Definition in the Threshold Setting

For conventional definitions of threshold encryption, please refer to Appendix B.

6.1.1 Threshold Tag-KEM

We define threshold tag-KEM by following the definitions in [3]. As well as the threshold PKE in the previous section, we restrict ourselves to a simple case where *non-interactive* shared decryption is possible. A threshold tag-KEM $\mathcal{TTKEM}$ is a set of algorithms ($\mathsf{TTKEM.Kg}$, $\mathsf{TTKEM.SKey}$, $\mathsf{TTKEM.Enc}$, $\mathsf{TTKEM.Dec}$, $\mathsf{TTKEM.Com}$) with the following properties.

$(pk, vk, sk) \xleftarrow{\$} \mathsf{TTKEM.Kg}(n, \nu, k)$**:** an algorithm that generates a public-key pk, a public verification key vk and a private-key $sk = \{sk_1, ..., sk_n\}$, given the number of decryption players n ($n > 1$), a threshold ν ($1 \leq \nu \leq n$) and a security parameter k. Note that the space for tags $TagSp$ should be inherently defined by pk.

$(K, \omega) \xleftarrow{\$} \mathsf{TTKEM.SKey}(pk)$**:** an algorithm that generates a session key $K \in \mathcal{K}_D$ and some state information ω.

$C \xleftarrow{\$} \mathsf{TTKEM.Enc}(pk, \omega, t)$**:** given a tag t, an algorithm that encrypts K (embedded in ω) into C.

$\sigma_i \leftarrow \mathsf{TTKEM.Dec}(pk, sk_i, C, t)$**:** an algorithm that, given C and t, outputs $\perp$ or decryption share σ_i.

$K/\perp \leftarrow \mathsf{TTKEM.Com}(pk, vk, C, S, t)$**:** a combining algorithm that takes as input a public key pk, a verification key vk, a ciphertext C, and a set of decryption shares $S \subseteq \{\sigma_1, \ldots, \sigma_n\}$ and a tag t. It outputs K or $\perp$.

Consistency and Robustness. They are defined in the same way as those for $\mathcal{TPKE}$ and hence the details are omitted.

Real or Random Security against Chosen Ciphertext Attacks. Let $\mathcal{A}$ be a polynomial-time oracle machine. We consider the following experiment.

Experiment $\mathbf{Exp}^{\text{ttkem-cca}}_{\mathcal{TTKEM},\mathcal{A}}(k)$

$(I_c, St_0) \xleftarrow{\$} \mathcal{A}_0(1^k)$

$(pk, vk, sk) \xleftarrow{\$} \mathsf{TTKEM.Kg}(n, \nu, k)$

$(K_1, \omega) \xleftarrow{\$} \mathsf{TTKEM.SKey}(pk)\ ;\ K_0 \xleftarrow{\$} KeySp\ ;\ b \xleftarrow{\$} \{0,1\}$

$(t^*, St_1) \xleftarrow{\$} \mathcal{A}_1^{\mathsf{TTKEM.Dec}(sk_i,\cdot,\cdot)_{i\notin I_c}}(pk, vk, K_b, \{sk_i\}_{i\in I_c}, St_0)$

$C^* \xleftarrow{\$} \mathsf{TTKEM.Enc}(pk, \omega, t^*)$

$b' \xleftarrow{\$} \mathcal{A}_2^{\mathsf{TTKEM.Dec}(sk_i,\cdot,\cdot)_{i\notin I_c}}(C^*, St_1)$

If $b \neq b'$ then return 0 else return 1

Here $I_c \subset \{1, \ldots, n\}$ is the set of corrupted players, where $|I_c| < \nu$. $\mathcal{A}$ makes queries $\mathsf{Crrpt}(i)$ to get the secret key sk_i and internal state for the corrupted players $i \in I_c$. Also, $\mathcal{A}$ has access to a decryption oracle $\mathsf{TTKEM.Dec}(sk_i, \cdot, \cdot)$ for $i \notin I_c$, and gets the corresponding shares $\sigma_i \leftarrow \mathsf{TTKEM.Dec}(pk, sk_i, C, t)$, where $\mathcal{A}_2$ is not allowed to query $(C, t) = (C^*, t^*)$. We define the advantage of adversary $\mathcal{A}$ as

$$\mathbf{Adv}^{\text{ttkem-cca}}_{\mathcal{TTKEM},\mathcal{A}}(k) \;=\; |\Pr[\mathbf{Exp}^{\text{ttkem-cca}}_{\mathcal{TTKEM},\mathcal{A}}(k) = 1] - 1/2| \ .$$

We say that threshold tag-KEM $\mathcal{TTKEM}$ is CCA secure if for any polynomial-time $\mathcal{A}$, the advantage $\mathbf{Adv}^{\text{ttkem-cca}}_{\mathcal{TTKEM},\mathcal{A}}(k)$ is a negligible function in k.

6.1.2 Partitioned Threshold IBKEM

The partitioned threshold IBKEM is defined as follows. $\mathcal{TIBKEM}$ = (TIBKEM.Kg, TIBKEM.Extract, TIBKEM.SKey, TIBKEM.Enc, TIBKEM.Dec, TIBKEM.Com) with participating players $\{1, \ldots, n\}$ consists of above polynomial-time algorithms. Since $\mathcal{TIBKEM}$ could be partitioned, it means that TIBKEM.Enc is possibly divided into algorithms TIBKEM.SKey, TIBKEM.EncKey and TIBKEM.EncID, which will be explained in the following.

$(pk, vk, sk) \xleftarrow{\$} \mathsf{TIBKEM.Kg}(n, \nu, k)$**;** Via the randomized key-generation algorithm, it produces a public key pk, a public verification key vk, and the n master-key shares $sk = \{sk_i\}$ $(i \in \{1, \ldots, n\})$ for security parameter $k \in \mathbb{N}$ and threshold parameter $1 \leq \nu \leq n$.

$(sk[id]_i/\perp) \xleftarrow{\$} \mathsf{TIBKEM.Extract}(pk, id, sk_i)$**;** Via the extraction algorithm, for a certain id, the ith share $sk[id]_i$ of the user secret key $sk[id]$ is generated. $\mathsf{TIBKEM.Extract}(vk, id, sk[id]_i)$ also checks the validity of the created key shares $sk[id]_i$, outputs $sk[id]_i$ if shares are valid, otherwise $\perp$ for invalid shares. We require all correctly generated $sk[id]_i$ must be valid.

$(C, K) \xleftarrow{\$} \mathsf{TIBKEM.Enc}(pk, id)$**;** It creates an encapsulation C of the random session key K, with respect to identity id. Note that the underlying threshold IBKEM can be partitioned. Thus, we have the following, s.t.

- $(K, \varphi) \xleftarrow{\$} \mathsf{TIBKEM.SKey}(pk)$**;** A random session key K is generated without depending on the id, but on pk. Meanwhile, a state information φ of random session key K is also delivered.
- $c_1 \leftarrow \mathsf{TIBKEM.EncKey}(\varphi)$**;** A deterministic algorithm takes state information φ as input, then outputs the first part of ciphertext.
- $c_2 \leftarrow \mathsf{TIBKEM.EncID}(\varphi, id)$**;** A deterministic algorithm takes as input φ and id, to generate the second part of the ciphertext.

where $C = (c_1, c_2)$.

$(K_i/\perp) \leftarrow \mathsf{TIBKEM.Dec}(pk, id, sk[id]_i, C)$**;** The possessor of the ith user secret key share $sk[id]_i$ partially decapsulates the ciphertext C encrypted with respect to id to get back the ith decapsulation share K_i. The validity of decapsulation shares can be checked with vk, and the algorithm outputs $\perp$ when decapsulation shares are not valid.

$(K/\perp) \leftarrow \mathsf{TIBKEM.Com}(pk, vk, id, S, C)$**;** If sufficiently many valid shares of K_i are collected (denote the set as S), TIBKEM.Com can recover the session key K. Otherwise just output $\perp$ for failure.

Roughly speaking, for correctness it is required that all correctly generated shares pass their respective verification tests. And for completeness, any set of at least ν valid shares of a common identity id should be able to reconstruct the user secret key $sk[id]$, or alternatively to decapsulate any correctly generated encapsulation sent to id. We say that a user secret key or decapsulation share is correctly generated if it has been obtained by following the protocol specification. Moreover, a user secret key or decapsulation share is said to be valid if it passes the corresponding verification test.

Threshold Selective-ID Security Requirement. Next, we associate to a partitioned threshold IBKEM $\mathcal{TIBKEM}$ and an adversary $\mathcal{A}$ the experiment $\mathbf{Exp}^{tibkem\text{-sid-cpa}}_{\mathcal{TIBKEM},\mathcal{A}}$ as follows:

$$
\begin{array}{l}
\textbf{Experiment } \mathbf{Exp}^{tibkem\text{-sid-cpa}}_{\mathcal{TIBKEM},\mathcal{A}}(k) \\
(I_c, id^*, St_0) \xleftarrow{\$} \mathcal{A}(1^k) \\
(pk, sk, vk) \xleftarrow{\$} \mathsf{TIBKEM.Kg}(n, \nu, k) \\
(K_1^*, \varphi) \xleftarrow{\$} \mathsf{TIBKEM.SKey}(pk)\ ;\ c_1^* \leftarrow \mathsf{TIBKEM.EncKey}(\varphi) \\
c_2^* \leftarrow \mathsf{TIBKEM.EncID}(\varphi, id^*)\ ;\ K_0^* \xleftarrow{\$} KeySp\ ;\ b \xleftarrow{\$} \{0,1\} \\
b' \xleftarrow{\$} \mathcal{A}^{\mathsf{TIBKEM.Extract}(sk,\cdot)}(pk, vk, \{sk_i\}_{i\in I_c}, K_b^*, c_1^*, c_2^*, St_0) \\
\text{If } b = b' \text{ then return 1 else return 0}
\end{array}
$$

The set $I_c \subset \{1, \ldots, n\}$ is called the set of corrupted players and $|I_c|$ must be upper bounded by $\nu - 1$. The oracle $\mathsf{TIBKEM.Extract}(sk, \cdot)$ returns $sk[id] \xleftarrow{\$} \mathsf{TIBKEM.Extract}(pk, sk, id)$ with the restriction that $\mathcal{A}$ is not allowed to query for $id = id^*$. We define the advantage of $\mathcal{A}$ in the experiment as

$$
\mathbf{Adv}^{tibkem\text{-sid-cpa}}_{\mathcal{TIBKEM},\mathcal{A}}(k) \;=\; \left|\Pr\left[\,\mathbf{Exp}^{tibkem\text{-sid-cpa}}_{\mathcal{TIBKEM},\mathcal{A}}(k) = 1\,\right] - 1/2\right| \;.
$$

A partitioned threshold selective-ID IBKEM $\mathcal{TIBKEM}$ is said to be secure against chosen-plaintext attacks if for any ν, n with $0 \le \nu \le n$, the advantage function $\mathbf{Adv}^{tibkem\text{-sid-cpa}}_{\mathcal{TIBKEM},\mathcal{A}}(k)$ is a negligible function in k for all polynomial-time adversaries $\mathcal{A}$.

6.2 Generic Transformation

We construct a threshold tag-KEM $\mathcal{TTKEM}$ = (TTKEM.Kg,TTKEM.SKey, TTKEM.Enc, TTKEM.Dec, TTKEM.Com) from a partitioned threshold selective-ID IBKEM scheme $\mathcal{TIBKEM}$ = (TIBKEM.Kg, TIBKEM.Extract, TIBKEM.SKey, TIBKEM.EncKey TIBKEM.EncID, TIBKEM.Dec, TIBKEM.Com) and a chameleon hash scheme. As well as the description in Section 4, the chameleon hash is considered as global for simplicity.

The key generation function TTKEM.Kg is set to TTKEM.Kg = TIBKEM.Kg. Since the threshold setting does not change anything in encryption, TTKEM.SKey and TTKEM.Enc are

exactly the same as TKEM.SKey and TKEM.Enc given in Section 4. (Note that IBKEM.SKey, IBKEM.EncKey and IBKEM.EncID used in the construction are the same as TIBKEM.SKey, TIBKEM.EncKey and TIBKEM.EncID, respectively.) The decryption functions, TTKEM.Dec and TTKEM.Com, are as follows.

$\mathsf{TTKEM.Dec}(pk, sk_i, C, t)$	$\mathsf{TTKEM.Com}(pk, vk, C, S, t)$
$c_1 \| c_2 \| s \leftarrow C$	$c_1 \| c_2 \| s \leftarrow C$
$id \leftarrow \mathsf{CMH.H}(t \| c_1, s)$	$id \leftarrow \mathsf{CMH.H}(t \| c_1, s)$
$sk[id]_i \xleftarrow{\$} \mathsf{TIBKEM.Extract}(pk, id, sk_i)$	$K \leftarrow \mathsf{TIBKEM.Com}(pk, vk, id, C, S)$
$K_i \leftarrow \mathsf{TIBKEM.Dec}(pk, id, sk[id]_i, C)$	Return K
Return K_i	

Theorem 7. *If the partitioned IBKEM $\mathcal{TIBKEM}$ is selective-ID threshold IND-CPA secure and the chameleon hash is random prefix collision resistant, then the above threshold tag-KEM $\mathcal{TTKEM}$ is IND-CCA secure. In particular,*

$$\mathbf{Adv}^{\text{ttkem-cca}}_{\mathcal{TTKEM},\mathcal{A}}(k) \leq \mathbf{Adv}^{\textit{tibkem}\text{-sid-cpa}}_{\mathcal{TIBKEM},\mathcal{B}}(k) + \mathbf{Adv}^{\textit{cmh}\text{-rpc}}_{\mathcal{CMH},\mathcal{C}}(k).$$

Proof.[Sketch] The theorem is proved in almost the same way as done for the proof of Theorem 3. The only difference is in the decryption oracle simulation where the oracle is made by a set of servers and the adversary may corrupt some of the servers. The simulation is done as follows.

Decryption Oracle Simulation. If $\mathcal{A}$ makes a decryption query with a ciphertext (c_1, c_2, s) and a tag t, adversary $\mathcal{B}$ simulates $\mathsf{TTKEM.Dec}(pk, sk_i, C, t)$ and $\mathsf{TTKEM.Com}(pk, vk, C, S, t)$ executed by the decryption servers in the following way. Let $id = \mathsf{CMH.H}(t \| c_1, s)$.

- Case 0: If $(c_1, c_2, s, t) = (c_1^*, c_2^*, s^*, t^*)$, show the input to the corrupted servers and return $\perp$ as an output from each uncorrupted server.
- Case 1: If $(c_1, s, t) \neq (c_1^*, s^*, t^*)$ and $id = id^*$, abort.
- Case 2: If $(c_1, s, t) = (c_1^*, s^*, t^*)$ and $c_2 \neq c_2^*$, select a random K_i to simulate the output of $\mathsf{TIBKEM.Dec}$ for every uncorrupted server i. Next, set $S = \{K_{i_j}, \dots\}$ and return $K \leftarrow \mathsf{TIBKEM.Com}(pk, vk, id, C, S)$.
- Case 3: If none of the above happens, send id to oracle $\mathsf{TIBKEM.Dec}$ and receive K_i for every uncorrupted players. It also sends the input to the corrupted servers and obtain K_i (of any value). Then set $S = \{K_{i_j}, \dots\}$ and compute $K \leftarrow \mathsf{TIBKEM.Com}(pk, vk, id, C, S)$ and return K to $\mathcal{A}$.

In case 0, the corrupted servers see that honest players reject the input as prescribed. In case 3, the corrupted servers see correct outputs K_i from each uncorrupted server and the final output K as prescribed. Finally, in case 2, incorrect results in random K_i, as expected. This results in random K by the property of partitioned scheme. Thus, the decryption oracle simulation is perfect. □

6.3 A Concrete Threshold Tag-KEM

Next, we show a concrete scheme which originates from a partitioned threshold IBKEM. By our new transformation, it can be efficiently lifted to a CCA-secure threshold hybrid PKE. More precisely, it will be first converted into a CCA-secure threshold tag-KEM after applying the transformation due to Theorem 7, and further extended to a threshold hybrid PKE due to Theorem 12 (the threshold tag-KEM/DEM composition theorem).

6.3.1 The Scheme

We first investigate that a threshold KEM based on Boneh-Boyen selective-ID IBE [7] is actually a partitioned threshold selective-ID IBKEM. For simplicity, the description assumes a trusted party that distributes the shared keys to the parties. Later, we discuss how to do this key generation efficiently in a distributed way, without the trusted party.

$\mathsf{TIBKEM.Kg}(n, \nu, k)$**;** For an ν out of n threshold cryptosystem which security parameter is k, given (n, ν, k) as input, it works as follows. It first generates a bilinear group $\mathbb{G}$ with prime order p ($p > n$), and group $\mathbb{G}_T$ where bilinear mapping $\hat{e} : \mathbb{G} \times \mathbb{G} \to \mathbb{G}_T$ exists. It further chooses a random $\nu - 1$ degree polynomial $f \in \mathbb{Z}_p[X]$, such that

$$f(X) = \alpha_0 + \alpha_1 X + ... + \alpha_{\nu-1} X^{\nu-1}$$

keeping $\alpha_0 \in \mathbb{Z}_p$ secret. Then it chooses generators $g, g_1, u_0 \in \mathbb{G}$ at random and computes $u_1 = g^{\alpha_0}$. The public key is set to

$$pk = \{g, g_1, u_0, u_1\} \in \mathbb{G}^4 \times \mathbb{G}_T \ .$$

A hash function H is also chosen from a collision-free hash family, which is considered embedded in the public key. Correspondingly, for $i \in \{1, \ldots, n\}$, the master secret key of server i is $sk_i = g_1^{f(i)}$, where

$$sk = \{sk_1, \ldots, sk_n\} = \{g_1^{f(1)}, \ldots, g_1^{f(n)}\} \in \mathbb{G}^n \ .$$

Finally, the verification key is set to

$$vk = \{vk_1, \ldots, vk_n\} = \{g^{f(1)}, \ldots, g^{f(n)}\} \in \mathbb{G}^n \ .$$

For verifying whether the shared private key sk_i is legitimate, it can be checked if $\hat{e}(g, sk_i) \stackrel{?}{=} \hat{e}(vk_i, g_1)$.

$\mathsf{TIBKEM.Extract}(pk, id, sk_i)$**;** Given pk and id, the key extraction algorithm for server i outputs the private key share $sk[id]_i = (i, d_{i,0}, d_{i,1})$ as

$$d_{i,0} = sk_i \cdot (u_0 u_1^{id})^r, \qquad d_{i,1} = g^r$$

where r is randomly chosen from $\mathbb{Z}_p$. If the private key share was correctly generated the following must be satisfied:

$$\hat{e}(vk_i, g_1) \cdot \hat{e}(u_0 u_1^{id}, d_{i,1}) = \hat{e}(g, g_1^{f(i)}) \cdot \hat{e}(g, (u_0 u_1^{id})^r) = \hat{e}(g, d_{i,0}) \ .$$

$\mathsf{TIBKEM.Enc}(pk, id)$**;** The encapsulation algorithm can be divided into the following:

TIBKEM.SKey(pk); Given pk, randomly choose $\gamma \xleftarrow{\$} \mathbb{Z}_p$ and compute $K = Z^\gamma \in \mathbb{G}_T$. Output session-key K and state information γ.

TIBKEM.EncKey(pk, γ); Given pk, γ, output $c_1 = g^\gamma$.

TIBKEM.EncID(pk, id, γ); On input of pk, id, γ, compute $c_2 = (u_0 u_1^{id})^\gamma$. Output $C = (c_1, c_2) \in \mathbb{G}^2$.

TIBKEM.Dec($pk, id, sk[id]_i, C$); Parse ciphertext C to (c_1, c_2), check whether $\hat{e}(c_1, u_0 u_1^{id}) = \hat{e}(c_2, g)$. If not, output $\perp$. Otherwise, make use of $sk[id]_i = (i, d_{i,0}, d_{i,1})$ which has been checked by $\hat{e}(vk_i, g_1) \cdot \hat{e}(u_0 u_1^{id}, d_{i,1}) = \hat{e}(g, d_{i,0})$ and compute decryption share $K_i = (K_{i,0}, K_{i,1})$,

$$K_{i,0} = d_{i,0} \cdot (u_0 u_1^{id})^{s_i}, \qquad K_{i,1} = d_{i,1} \cdot g^{s_i}$$

where $s_i \xleftarrow{\$} \mathbb{Z}_p$, is randomly chosen from $\mathbb{Z}_p$.

TIBKEM.Com(pk, vk, id, S, C); Parse C to (c_1, c_2). Check the consistency of the decryption shares from the servers, in the following way. Given pk, vk, a set of decryption shares S, output $\perp$ if $|S| < \nu$. For K_i output by TIBKEM.Dec, use vk to verify whether there is

$$\hat{e}(g, K_{i,0}) \stackrel{?}{=} \hat{e}(g, g_1^{f(i)}) \cdot \hat{e}(g, (u_0 u_1^{id})^{r+s_i}) = \hat{e}(vk_i, g_1) \cdot \hat{e}(u_0 u_1^{id}, K_{i,1})$$

and output $\perp$ if it does not hold. Let S be the sufficient set of valid K_i, since the the ciphertext consistency and key consistency have been checked. Wlog, we assume $S = (K_1, \ldots, K_\nu)$. Let $\pi_i \in \mathbb{Z}_p$ $(1 \le i \le \nu)$ is the *Lagrange coefficients*, s.t. $\alpha_0 = f(0) = \sum_{i=1}^{\nu} \pi_i f(i)$.

Since there is an intermediate key (K_0, K_1),

$$K_0 = \prod_{i=1}^{\nu} K_{i,0}^{\pi_i} = g_1^{\alpha_0} \cdot (u_0 u_1^{id})^{\bar{r}}, \qquad K_1 = \prod_{i=1}^{\nu} K_{i,1}^{\pi_i} = g^{\bar{r}}$$

where $\bar{r}$ is some number in $\mathbb{Z}_p$. Then session-key K can be computed as follows,

$$K = \frac{\hat{e}(c_1, K_0)}{\hat{e}(c_2, K_1)} = \frac{\hat{e}(g^\gamma, g_1^{\alpha_0} \cdot (u_0 u_1^{id})^{\bar{r}})}{\hat{e}((u_0 u_1^{id})^\gamma, g^{\bar{r}})} = \hat{e}(g^\gamma, g_1^{\alpha_0}) = \hat{e}(u_1, g_1)^\gamma$$

The underlying scheme is partitioned and selective-ID threshold IND-CPA secure if the BDDH assumption holds, since it is based on the Boneh-Boyen selective-ID secure IBE. By applying the transformation from Section 6.2, it is straightforward to get a threshold tag-KEM from the underlying partitioned threshold selective-ID IBKEM. The technique we use is a modified discrete-log based Chameleon Hash $\mathcal{CMH}$. It is noteworthy that though using $\mathcal{CMH}$, we do not necessarily generate the trapdoor CMH.Trap, because it will be used only in the security proof. However, it is important to generate the parameters for the chameleon hash without leaking the trapdoor especially when the public-key is generated in a threshold manner. It may not be generally possible, but for the following instantiation, one can use the distributed discrete-log key-generation techniques from [40, 27, 2].

We finally remark that the key generation part can also be distributed by using known techniques such as [40, 27, 2]. Note that, while [40] and [27] works only on the discrete logarithm

Threshold Hybrid Encryption	Ciphertext Overhead	Encryption #pairings +	Decryption #[multi,regular]-exp	Public Parameters (vk+pk+hash)
Ours(§6.2)	768	$0 + [1, 2]$	$2 + [0, 2\nu]$	n+4+1
BBH+CHK [9, 18]	1792-6.5k	$0 + [1, 2]$+Sig	$2 + [0, 2\nu]$+Vfy	n+4

Table 2: Efficiency comparison for our transformed chosen-ciphertext secure threshold hybrid encryption. The table compares the similar parameters as Table 1, while all are based on the BDDH assumption. On the encryption and decryption (encapsulation and decapsulation), the generation and verification of the one-time signature is an obvious cost in CHK transformation [18]. The public parameter compares the number of group elements needed for public key and verification key, while secret key sk are required to be n due to the threshold setting. Note that we merely need a hash function, instead of a strong one-time signature. Usually, a one-time signature introduces at least 1280-6k bits overhead (see the footnote in the introduction). In addition to the two group elements, it appears to generate 1792-6.5k bits overhead totally.

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assumption in the secure-channels setting, the adaptively secure key generation by [2] over public-channel setting requires DDH assumption for non-committing encryption. However, the non-committing encryption can be instantiated independently from the bilinear groups used in the above construction, hence available for our purpose.

6.3.2 Security

Since the above construction is the result of applying Theorem 7 to the partitioned IBKEM of threshold Boneh-Boyen scheme in [9], the security can be shown only by inspecting that the scheme in [9] is partitioned. Since we only need selective-ID security, partitioning is only a syntactical issue which can be easily verified. By combining Theorem 7 and the original proof of threshold Boneh-Boyen selective-ID IBE in [9], we have the following theorem.

Theorem 8. *Under the BDDH assumption, the threshold tag-KEM is secure in the sense of threshold CCA security. In particular,*

$$\mathbf{Adv}^{\text{ttkem-cca}}_{\mathcal{TTKEM},\mathcal{A}}(k) \leq \mathbf{Adv}^{\text{bddh}}_{\mathbb{G},\mathcal{B}}(k) + \mathbf{Adv}^{cmh\text{-rpc}}_{\mathcal{CMH},\mathcal{A}}(k) + q_s/2p$$

if the BDDH assumption holds in $(\mathbb{G}, \mathbb{G}_T)$, where q_s is the upper bound of the decapsulation queries.

Following Theorem 12, this threshold tag-KEM and a one-time secure DEM yield a CCA-secure hybrid PKE.

6.3.3 Efficiency

A threshold PKE scheme was recently proposed in [9]. This construction makes use of the CHK-transform [18] based on a one-time signature scheme and therefore has a ciphertext overhead of roughly $6k$ bits. We note that the more efficient BK-transformation [13] is not applicable in this setting since it does not provide public verifiability of the threshold ciphertexts.

In Table 2 we make a brief comparison of efficient threshold hybrid encryption schemes in the standard model. We compare our threshold tag-KEM(§6.3.1) with the solution from BBH [9]. Obviously our schemes has an advantage in communication and computation cost. The data is taken when the security parameter k is set to 128-bit, similar to Table 1. The overhead of the

CHK transformation includes the computation of one-time signature and verification, and the typical overhead in ciphertext expansion.

7 Removing the Chameleon Hash

The heart of the transformation in Section 4 is the use of a chameleon hashing. Somewhat contradictory, this section shows several ways to eliminate the use of the chameleon hashing. The resulting transformations are of either theoretical interest, or practical advantages.

7.1 Building a CMH-like Functionality from CR and OWF

With a closer look, one may notice that the scheme and the security proof in Section 4 did not use the full power of the CMH because the simulator uses the trapdoor only once, i.e. answer the target encapsulation query and return $(t^*||c_1^*, s^*)$ to the adversary. The minimum property we really need are:

- to compute s^* that maps given $t^*||c_1^*$ into id^* selected in advance, and
- random-prefix collision resistance.

Theoretically, such a function can be made from one-way function plus a collision resistant hash function as follows. Let CR be a collision resistant hash function. We use Naor's bit commitment scheme [36]. Let M be the input message to hash. Let m be the length of M and M_i denotes i-th bit of M. Let $R_i \xleftarrow{\$} \{0,1\}^{3k}$ and $R = (R_1, \ldots, R_m)$. Let $G : \{0,1\}^k \to \{0,1\}^{3k}$ be a pseudo-random generator. Finally let s be $s = (s_1, \ldots, s_m)$ where $s_i \xleftarrow{\$} \{0,1\}^k$. We define the CMH-like function $H_R(M, s) : \{0,1\}^m \times \{0,1\}^{km} \to \{0,1\}^k$ as

$$H_R(M, s) = \mathsf{CR}(h_1|| \ldots ||h_m) \text{ where } h_i = M_i \cdot R_i \oplus G(s_i).$$

Note that R is included in the public-key.

We argue that the above function H fulfills the first property as follows. To compute s that matches to a given M and prefixed hash value, first set $R_i = G(s_{0,i}) \oplus G(s_{1,i})$. Then set $h_i = G(s_{0,i})$ and compute $v = \mathsf{CR}(h_1|| \ldots ||h_m)$. The value v is published as the preliminary fixed value. When M is given, set $s_i = s_{M_i,i}$ and $s = (s_1, \ldots, s_m)$. Such s fulfills $v = H_R(M, s)$ as expected and the distribution of R, v and s are computationally indistinguishable from the real one, otherwise the pseudo-randomness of G is broken. To see that this function is random-prefix collision resistant, first observe that each h_i can be opened to both $M_i = 0$ and $M_i = 1$ only with negligible probability due to the statistical binding property of the Naor's bit commitment. Having two distinct h_i clearly contradicts to the collision property of CR. Finally, we note G is constructible from one-way functions and the construction is done.

Nonetheless, this is only of theoretical interest and out of the main scope of this paper since the resulting scheme suffers a huge overhead both in the public-key and ciphertext size.

7.2 Directly Replacing CMH with RPC

As mentioned above, the trapdoor information of CMH is used only in the security proof. The scheme therefore functions correctly even if chameleon hash function CMH is replaced by a collision resistant hash function CR. More precisely, we can use a random prefix collision resistant hash function RPC, which has more relaxed collision resistant property than that of CR as defined in Section 2.7, and replace $\mathsf{CMH.H}(t||c_1, s)$ in $\mathsf{TKEM.Enc}$ and $\mathsf{TKEM.Dec}$ by $\mathsf{RPC}(t||c_1||s)$.

With this modification the current security proof no longer works since the reduction algorithm needs to compute s^* that fulfills $id^* = \mathsf{RPC}(t^*||c_1^*||s^*)$ for id^*, t^*, and c_1^* selected in advance. We circumvent this technical difficulty by allowing the reduction algorithm to access to an oracle, which we call *hash partial preimage oracle*, that outputs such s^* for input (id^*, t^*, c_1^*) whenever it exists. For this idea to work, it is required that the underlying selective-ID IBKEM should remain IND-CPA even against the adversaries equipped with the partial preimage oracle. It would be reasonable when the underlying IBKEM and RPC can be considered as independent. Such a relaxation, which allows the adversary to access to a seemingly irrelevant oracle, has been used in the literature, e.g. [41, 35].

For example, let us take the concrete scheme shown in Section 5.1. The scheme uses the Boneh-Boyen's IBE in the above mentioned modified transformation. The Boneh-Boyen's IBE is selective-ID IND-CPA under the BDDH assumption. To cope with the hash partial preimage oracle, we need to strengthen the underlying assumption to the BDDH with the hash partial preimage oracle. That is, the necessary assumption for the resulting tag-KEM to be secure is that the BDDH assumption holds even for the adversary given access to the hash partial preimage oracle. Assuming that SHA-256 or AES-based hash functions is RPC would be reasonable since they are seemingly irrelevant to the number-theoretic structure of the BDDH problem. On the other hand, using RPC based on the discrete-log assumption such as Pedersen hash would not be acceptable especially when the group is shared with the BDDH instance.

The modified scheme improves the efficiency. All known efficient construction of CMH limit the input length and need CR to compress the input string of arbitrary length. Needing single RPC in the modified scheme thus saves the computation for CMH. Also, the public-key has no additional elements than that of the underlying IBKEM. Note however that the length of the ciphertext gets slightly longer because the randomness s would have to be chosen from larger domain as shown in the following.

The modification applies only to TKEM.Enc and TKEM.Dec as shown below. All other functions remain as described in the following.

$\mathsf{TKEM.Enc}(pk, t, \varphi)$	$\mathsf{TKEM.Dec}(sk, t, c)$
$c_1 \leftarrow \mathsf{IBKEM.EncKey}(\varphi)$	$c_1 \| c_2 \| s \leftarrow c$
$s \xleftarrow{\$} \mathsf{Rand_{RPC}}$; $id \leftarrow \mathsf{RPC}(s\|\|t\|\|c_1)$	$id \leftarrow \mathsf{RPC}(s\|\|t\|\|c_1)$
$c_2 \leftarrow \mathsf{IBKEM.EncID}(\varphi, id)$	$sk[id] \xleftarrow{\$} \mathsf{IBKEM.Extract}(sk, id)$
Return $c \leftarrow c_1\|\|c_2\|\|s$	$K \leftarrow \mathsf{IBKEM.Dec}(sk[id], id, (c_1, c_2))$
	Return K

Let $\mathsf{RPC} : \mathsf{Rand_{RPC}} \times \{0,1\}^* \to \mathit{IDSp}$ be a random prefix collision resistant hash function. For $id \in \mathit{IDSp}$ and $x \in \{0,1\}^*$, define $S_x(id)$ by $S_x(id) = \{s \in \mathsf{Rand_{RPC}} \mid id = \mathsf{RPC}(s||x)\}$. We assume that RPC has a property that for every x the distribution of id is statistically close to uniform when s is taken uniformly. For the sake of this property we assume $|\mathsf{Rand_{RPC}}| \geq |\mathit{IDSp}|$. Having such a property, the random prefix collision resistance is almost the same as the target collision resistance since the adversary has no advantage by choosing t^*. It means that the birthday attack is not applicable and we can expect shorter output length. Based on this observation, we set $|\mathsf{Rand_{RPC}}|$ to 128 bits in our efficiency analysis in Table 3. Let $\mathcal{O}_{\text{p-preimg}}$ be a hash partial preimage oracle for RPC that takes as input $(id, x) \in \mathit{IDSp} \times \{0,1\}^*$ and outputs $s^* \xleftarrow{\$} S_x(id)$. It outputs $\perp$ if $S_x(id)$ is empty. From the above statistical property, $\perp$ is returned only with negligible probability when id is chosen randomly.

Theorem 9. *If the partitioned IBKEM is selective-ID IND-CPA secure with regard to all adversaries that have oracle access to* $\mathcal{O}_{\text{p-preimg}}$ *and* RPC *is random prefix collision resistant, then the above TKEM is IND-CCA secure.*

Proof. The outline of the proof is the same as that of Theorem 3. The following describes the difference only. We first show an adversary $\mathcal{B}$ against the selective-ID IND-CPA security of $\mathcal{IBKEM}$ as follows.

Setup. Given security parameter k, adversary $\mathcal{B}$ chooses RPC and selects the target identity $id^* \xleftarrow{\$} IDSp$. It sends id^* to the challenger and receives a public-key pk and a challenge (K_b^*, c_1^*, c_2^*). Adversary $\mathcal{B}$ now runs $\mathcal{A}$ by giving pk.

Challenge Simulation. At some point, $\mathcal{A}$ outputs a target tag t^*. $\mathcal{B}$ then sends $(id^*, t^*||c_1^*)$ to $\mathcal{O}_{\text{p-preimg}}$. If $\perp$ is returned, abort. Otherwise, $s^* \in S_{t^*||c_1^*}(id^*)$ is obtained. $\mathcal{B}$ sends K_b^* and (c_1^*, c_2^*, s^*) to $\mathcal{A}$.

"Decryption Oracle Simulation" and "Output" are unchanged.

Challenge simulation will be successful with overwhelming probability by the statistical property of RPC. Furthermore, the distribution of selected id^* and s^* are statistically close to the real execution where s^* is selected first. The rest of analysis is unchanged and hence $\mathcal{B}$ simulates $\mathcal{A}$'s view in the IND-CCA experiment statistically close to the real execution unless event COL happens.

If COL does not happen then $\mathcal{B}$ has the same advantage in winning the experiment as $\mathcal{A}$ except for the negligible simulation errors. If COL happens during $\mathcal{A}$'s simulation with noticeable probability, we use $\mathcal{A}$ to build an adversary $\mathcal{C}$ against the random prefix collision resistance property of RPC. Adversary $\mathcal{C}$ will attack RPC by correctly computing the IBKEM part with the secret-key generated by itself and perfectly simulates $\mathcal{A}$'s view in the IND-CCA experiment till $\mathcal{A}$ outputs a collision. □

7.3 Based on Strongly Adaptive-ID IBKEM

In Section 3.2 we introduced the security notion of selective-ID IND-CPA for IBKEMs. For the stronger notion of security against "full-identity" IND-CPA attacks (IND-CPA security) changes the security experiment from Section 3.2 as follows. Instead of committing to the target identity id^* before seeing the public key the adversary now can adaptively select it. Namely, $(pk, sk) \xleftarrow{\$} \mathsf{IBKEM.Kg}(1^k)$ is done at the beginning of the experiment and pk is given to $\mathcal{A}_0$. No changes in anywhere else. We however do not use this notion in this paper and introduce rather stronger one as below to obtain a more efficient transformation.

Roughly, in this stronger case, the adversary is given session-key K_b^* and the first part of ciphertext c_1^* *before* selecting the target identity id^*. Note that K_b^* and c_1^* are independent of id^* in partitioned IBKEMs. Formally, the experiment is as follows.

Experiment $\mathbf{Exp}^{ibkem\text{-id-cpa}^*}_{\mathcal{IBKEM},\mathcal{A}}(k)$

$(pk, sk) \xleftarrow{\$} \mathsf{IBKEM.Kg}(1^k)$; $(K_1^*, \varphi) \xleftarrow{\$} \mathsf{IBKEM.SKey}(pk)$; $K_0^* \xleftarrow{\$} KeySp$; $b \xleftarrow{\$} \{0,1\}$
$c_1^* \leftarrow \mathsf{IBKEM.EncKey}(pk, \varphi)$
$(id^*, St_1) \xleftarrow{\$} \mathcal{A}_1^{\mathsf{IBKEM.Extract}(sk,\cdot)}(pk, K_b^*, c_1^*)$
$c_2^* \leftarrow \mathsf{IBKEM.EncID}(pk, \varphi, id^*)$
$b' \xleftarrow{\$} \mathcal{A}_2^{\mathsf{IBKEM.Extract}(sk,\cdot)}(c_2^*, St_1)$
If $b \neq b'$ then return 0 else return 1

Adversary $\mathcal{A}$ is not allowed to query oracle $\mathsf{IBKEM.Extract}(sk, \cdot)$ for the target identity id^*.

We define the advantage of $\mathcal{A}$ in the experiment as

$$\mathbf{Adv}^{ibkem\text{-id-cpa}^*}_{\mathcal{IBKEM},\mathcal{A}}(k) \;=\; |\Pr[\mathbf{Exp}^{ibkem\text{-id-cpa}^*}_{\mathcal{IBKEM},\mathcal{A}}(k) = 1] - 1/2| \;.$$

A partitioned IBKEM $\mathcal{IBKEM}$ is said to be strongly adaptive-ID IND-CPA (strongly IND-CPA) if the advantage functions $\mathbf{Adv}^{ibkem\text{-id-cpa}^*}_{\mathcal{IBKEM},\mathcal{A}}(k)$ is a negligible function in k for all polynomial-time adversaries $\mathcal{A}$.

The fact that the adversary can adaptively select the target identity id^* depending on K^*_b and c^*_1 is crucial to our definition and also the reason why we call it *strong* adaptive-ID IND-CPA security. In fact, there may exist partitioned IBKEMs that are adaptive-ID IND-CPA in the standard sense of [12] (where id^* has to be provided before receiving challenge ciphertext/key) but not strongly adaptive-ID IND-CPA because of this information given in advance. However, these separating examples will hardly appear in practise since c_1 and K are both required to be independent of id. For concrete schemes (such as [47]) it is an easy exercise to verify that the schemes actually provide this stronger type of security.

We remark that this difficulty does not arise in the case of selective-ID IND-CPA security since there the adversary has to commit to the target identity before even seeing the public key.

When a partitioned strongly adaptive-ID IBKEM is available, we can construct a tag-KEM without any overhead. Indeed, the construction is obtained simply by removing the chameleon hash from the construction in Section 4. For completeness, we show the construction below.

Set the key generation algorithm $\mathsf{TKEM.Kg} = \mathsf{IBKEM.Kg}$. The rest of the functions are constructed as follows.

$\mathsf{TKEM.SKey}(pk)$	$\mathsf{TKEM.Dec}(sk, t, c)$
$(K, \varphi) \xleftarrow{\$} \mathsf{IBKEM.SKey}(pk)$	$c_1 \| \| c_2 \leftarrow c$
Return (K, φ)	$id \leftarrow t \| \| c_1$
	$sk[id] \xleftarrow{\$} \mathsf{IBKEM.Extract}(sk, id)$
$\mathsf{TKEM.Enc}(pk, t, \varphi)$	$K \leftarrow \mathsf{IBKEM.Dec}(sk[id], id, (c_1, c_2))$
$c_1 \leftarrow \mathsf{IBKEM.EncKey}(\varphi)$	Return K
$id \leftarrow t \| \| c_1$	
$c_2 \leftarrow \mathsf{IBKEM.EncID}(\varphi, id)$	
Return $c \leftarrow c_1 \| \| c_2$	

Theorem 10. *If the partitioned IBKEM is strongly adaptive-ID IND-CPA secure, then the above TKEM is IND-CCA secure. In particular,*

$$\mathbf{Adv}^{tkem\text{-cca}}_{\mathcal{TKEM},\mathcal{A}}(k) \leq \mathbf{Adv}^{ibkem\text{-id-cpa}^*}_{\mathcal{IBKEM},\mathcal{B}}(k).$$

The proof can be done by showing the construction of $\mathcal{B}$ from $\mathcal{A}$. Unlike the selective-ID case, $\mathcal{B}$ commits to the target identity after having c^*_1 from the challenger and t^* from $\mathcal{A}$. Hence $\mathcal{B}$ can form $id^* = t^* \| \| c^*_1$ without any problem. The rest of the simulation is the same as that for the proof of Theorem 3 with trivial modifications for removing the chameleon hash.

7.4 Another Approach

This section discusses another approach that might be straightforwardly derived from the \$-rejection property but eventually fails short without assuming further structural property.

Similar to [14, 30] one could try to directly build a key-encapsulation mechanism [22] (KEM) out of an sID partitioned IBKEM using a target-collision resistant hash function TCR as follows. To encapsulate, first compute key and the first part of the ciphertext as $(K, \varphi) \xleftarrow{\$} \mathsf{IBKEM.SKey}(pk)$ and $c_1 \leftarrow \mathsf{IBKEM.EncKey}(\varphi)$. Then, the second part of the ciphertext is computed as $c_2 \leftarrow \mathsf{IBKEM.EncID}(\varphi, id)$, where $id = \mathsf{TCR}(c_1)$ is used to tie the two ciphertexts together. Whereas syntactically this is correct, without assuming further algebraic structure it seems hard to relate security of the KEM to the security of the IBKEM. This is since a simulator for the KEM security experiment interacting with the sID IBKEM challenger has to commit to a target identity before seeing the public key. But in the scheme the target identity depends on the first part of the target ciphertext and hence a stronger (and less natural) security requirement to the IBKEM scheme is needed for a general security reduction. In general, proving that the IBKEM satisfies such a stronger security property seems not easier than providing a direct proof for the transformed KEM.

7.5 Efficiency Comparison

We make an efficiency comparison (with a security parameter of 128 bits) of the IBE to PKE transformations from [18, 13] with ours. The entries in Table 3 refer to the following transformations.

CHK+DLOTP: The CHK transformation instantiated with a one-time signature based on the discrete-log problem. The one-time signature scheme and its security proof is given in Appendix A. This transformation is secure under the DLOG assumption in the group. The overall ciphertext expansion introduced by the CHK transformation consists of the public key (three group elements) plus the signature. Depending on the instantiation of the group this is between 1280 (elliptic curve groups) and $6k$ bits (prime-order subgroups of $\mathbb{Z}_q$).

CHK+hash-tree: The CHK transformation instantiated with a hash-tree based one-time signature, as recommended in [18]. The latter signature scheme has the advantage of having negligible computational cost but has a ciphertext overhead of roughly 65k ($\approx 256^2$). This construction is secure under the assumption that the used hash functions are one-way and collision-resistant.

BK: The BK transformation instantiated with a computationally secure MAC (i.e., CBC-MAC) and a UOWHF-based commitment scheme [13].

Ours+DLCH: Our transformation from Section 4 instantiated with a DLOG-based Chameleon hash. The ciphertext overhead consists of one element in $\mathbb{Z}_{|\mathbb{G}|}$ which can be represented in 256 bits (independent of the group).

Ours+RPC: Our transformation from Section 7.2 based on a RPC-secure hash function.

Ours+strong IBKEM: Our transformation from Section 7.3 applied to a strongly secure IBKEM.

Transformation	Input	Overhead			Additional	Publicly
		Ciphertext	Enc	Dec	assumption	verifiable?
CHK+hash-tree	sID IBE + one-time sig	65k	—	—	CR*	√
CHK+DLOTP	sID IBE + one-time sig	1.2k-6k	2 exp	2 exp	DLOG	√
BK	sID IBE + MAC/COM	704	—	—	MAC*	—
Ours+DLCH §4	sID part. IBKEM + CMH	256	1 exp	—	DLOG	√
Ours §7.2	sID part. IBKEM + RPC hash	128	1 exp	—	RPC	√
Ours §7.3	fID part. IBKEM	—	—	—	strong IBKEM	√

Table 3: Efficiency of the transformations for 128 bits security. *Since OW is basically required in public-key cryptography and implied by CR, we simply write CR here. Similarly, UOWHF that could be implied from MAC is omitted in BK, as well.

8 Extensions

8.1 CCA-secure (Hierarchical) Identity-Based Encryption

Similar to the case of the CHK and BK transformations [19, 13] our transformations from Section 4 generalize to the setting of (hierarchical) identity-based encryption (HIBE) [29]. In particular, any partitioned selective-ID secure k-level HIB-KEM can be transformed into an IND-CCA secure $\ell - 1$-level HIBE.

8.2 Tag-based KEMs

For the CHK and BK transformations [19, 13] from selective-ID secure IBE to CCA-secure PKE it was recently shown in [30] that a more general concept called *tag-based encryption* (TBE) is already sufficient for the transformations to work. TBE can be understood as IBE without the necessity to perform key derivation. In contrast to IBE there exists known instantiations of TBE schemes without pairings, for instance on the linear assumption [30]. We remark that our transformations can also be stated in terms of *partitioned tag-based KEMs* leading to more example instantiations than given in Section 5.

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A CMA-Secure One-time Signature based on DLP

Let $\mathbb{G}$ be a group generated by g where $|\mathbb{G}|$ is a prime. Let $q = |\mathbb{G}|$.

Private-Key: $a, b, c \xleftarrow{\$} \mathbb{Z}_q$

Public-Key: $g_1 = g^a, g_2 = g^b, y = g^c$

Signing: $\sigma = (\sigma_1, \sigma_2)$ where $\sigma_1 \leftarrow \mathbb{Z}_{|\mathbb{G}|}$ and $\sigma_2 = (c - m - a\sigma_1)/b \in \mathbb{Z}_q$ for message $m \in \mathbb{Z}_q$.

Verification: $y \stackrel{?}{=} g^m g_1^{\sigma_1} g_2^{\sigma_2}$

Lemma 11. *The above signature scheme is one-time chosen message secure under the discrete logarithm assumption in* $\mathbb{G}$.

Proof. Let $\mathcal{A}_1$ denote an adversary that launches a one-time chosen message attack against the above scheme. We construct $\mathcal{A}_2$ that solves the discrete-log problem by using $\mathcal{A}_1$ as a black-box. Given a DLP instance $Y = g^X$ in $\mathbb{G}$, $\mathcal{A}_2$ first flips a coin $b \stackrel{\$}{\leftarrow} \{0, 1\}$ and does one of the following.

[**Case** $b = 0$]

- (Setup.) Select $(\sigma_1^*, r, b) \stackrel{\$}{\leftarrow} \mathbb{Z}_q^3$ and set $g_1 = Yg^r$, $g_2 = g^b$, $y = g^c g_1^{\sigma_1^*}$. Give public-key (g_1, g_2, y) to $\mathcal{A}_1$.
- (Signature Simulation.) Given $m^* \in \mathbb{G}$ from $\mathcal{A}_1$, return (σ_1^*, σ_2^*) where $\sigma_2^* = (c - m^*)/b \bmod q$.
- (Output.) Let (m, σ_1, σ_2) denote the successful forgery $\mathcal{A}_1$ outputs. If $\sigma_1 = \sigma_1^*$, abort. Otherwise, output $X = \{(\sigma_2^* - \sigma_2)b + (m^* - m)\}/(\sigma_1 - \sigma_1^*) - r \bmod q$.

[**Case** $b = 1$]

- (Setup.) Select $(\sigma_2^*, r, a) \stackrel{\$}{\leftarrow} \mathbb{Z}_q^3$ and set $g_1 = g^a$, $g_2 = Yg^r$, $y = g^c g_2^{\sigma_2^*}$. Give public-key (g_1, g_2, y) to $\mathcal{A}_1$.
- (Signature Simulation.) Given $m^* \in \mathbb{G}$ from $\mathcal{A}_1$, return (σ_1^*, σ_2^*) where $\sigma_1^* = (c - m^*)/a \bmod q$.
- (Output.) Let (m, σ_1, σ_2) denote the successful forgery $\mathcal{A}_1$ outputs. If $\sigma_2 = \sigma_2^*$, abort. Otherwise, output $X = \{(\sigma_1^* - \sigma_1)a + (m^* - m)\}/(\sigma_2 - \sigma_2^*) - r \bmod q$.

In the case $b = 0$, output X is correct because a successful forgery implies $Y = g^{m^*} g_1^{\sigma_1^*} g_2^{\sigma_2^*} = g^m g_1^{\sigma_1} g_2^{\sigma_2}$, and we have $m^* + (X + r)\sigma_1^* + b\sigma_2^* = m + (X + r)\sigma_1 + b\sigma_2$. Correctness in the case of $b = 1$ can be inspected similarly.

Observe now that the view of $\mathcal{A}_1$ is identical for the cases $b = 0$ and $b = 1$. Hence b is independent of the view of $\mathcal{A}_1$. When $\mathcal{A}_1$ outputs successful forgery (m, σ_1, σ_2), which is not the same as $(m^*, \sigma_1^*, \sigma_2^*)$, at least one of $\sigma_1 \neq \sigma_1^*$ or $\sigma_2 \neq \sigma_2^*$ happens. Hence $\mathcal{A}_2$ outputs X with probability at least $\Pr[\sigma_1 \neq \sigma_1^* | b = 0] + \Pr[\sigma_2 \neq \sigma_2^* | b = 1] = \frac{1}{2}\Pr[\mathcal{A}_1 \text{ outputs forgery.}]$. □

Note that, unlike in our case, more efficient two-base Pedersen CMH [40] does not suit for this purpose since it is not known to be chosen-message secure when it is used as a one-time signature scheme. [48] shows its security in the generic model, which would only be useful when other components already demand this model for their security.

B Threshold Hybrid PKE

In the following, we provide notions on threshold PKE, as well as the proof for proving the threshold Tag-KEM/DEM composition theorem.

B.1 Threshold Public-Key Encryption

Possibly, the most general way of defining threshold PKE would be to follow the definition of conventional PKE, which consists of key generation algorithm, encryption algorithm and decryption algorithm, and re-define key generation and decryption as sets of interactive algorithms executed by each player. Having existing efficient schemes in mind, however, we follow the definition by [46] with details. The definition is not quite general but captures many existing schemes where key generation may be done by a trusted party (in some cases it is actually shared) and the decryption can be done efficiently in a non-interactive way such that each player only needs to broadcast the result of local shared decryption. Nevertheless, such a loss of generality is irrelevant to our results. Appropriate notes will be given when generality is to be considered.

$(pk, vk, sk) \xleftarrow{\$} \mathsf{TPKE.Kg}(n, \nu, k)$**;** A probabilistic key generation algorithm that generates public-key pk, public verification key vk and private-key sk, on input of the number of decryption players n ($n > 1$), a threshold ν ($1 \leq \nu \leq n$), and a security parameter k. $sk = (sk_1, \ldots, sk_n)$ represents n shares of private keys. For each player i ($i \in \{1, \ldots, n\}$), sk_i is given to decrypt the ciphertext share. vk is used to publicly check the validity of the decryption shares.

$C \xleftarrow{\$} \mathsf{TPKE.Enc}(pk, M; r)$**;** A probabilistic encryption algorithm outputs ciphertext C, on input of public key pk, message M and randomness r.

$\sigma_i \leftarrow \mathsf{TPKE.Dec}(pk, sk_i, C)$**;** A (probabilistic) decryption algorithm, takes as input the public key pk, a private sk_i, and a ciphertext C, outputs a decryption share σ_i which can be a special symbol $\perp$ in the case of abnormal operation

$M/\perp \leftarrow \mathsf{TPKE.Com}(pk, vk, C, S)$**;** A combining algorithm takes as input the public key pk, the verification key vk, a ciphertext C, and a set of decryption shares $S \subseteq \{\sigma_1, \ldots, \sigma_n\}$. It outputs a message M or $\perp$.

For correctness, we require that all validly generated decryption shares should be correctly verified with regard to (pk, vk), and for more than ν distinct and valid decryption shares, the encrypted message must be recovered.

Consistency and Robustness. A scheme should be consistent, i.e., for any legitimately generated keys (pk, vk, sk), any ciphertext and its corresponding decryption shares $\{\sigma_1, \ldots, \sigma_n\}$, $\mathsf{TPKE.Com}(pk, vk, C, S)$ outputs the same message $M \neq \perp$ for any $S \subseteq \{\sigma_1, \ldots, \sigma_n\}$ of size $\geq \nu$. Let $\mathcal{A}$ be an adversary that can corrupt up to $\nu - 1$ players. On a corruption, $\mathcal{A}$ is given all the internal state of the corrupted player and through control over the player from that moment. We say a scheme is *robust* if consistency is achieved with overwhelming probability, say $1 - \epsilon$, for any S that contains at least ν decryption shares from honest players in the presence of $\mathcal{A}$. Here, ϵ is a negligible function in the security parameter k (possibly in n and ν, too), and the probability is taken over the coin flips of $\mathcal{A}$. The adversary can be static, i.e., decides which player to be corrupted in advance or adaptive, i.e., corruption occurs depending on the behavior of the players.

Chosen Ciphertext Security. The chosen ciphertext security in threshold scenario is analogous to conventional CCA security [42]. Next we define the security in the following experiment, where adversary $\mathcal{A}$ $(\mathcal{A}_0, \mathcal{A}_1, \mathcal{A}_2)$ is a polynomial-time oracle machine.

Experiment $\mathbf{Exp}^{\text{tpke-cca}}_{\mathcal{TPKE},\mathcal{A}}(k)$

$(I_c, St_0) \xleftarrow{\$} \mathcal{A}_0(1^k)$
$(pk, vk, sk) \xleftarrow{\$} \mathsf{TPKE.Kg}(n, \nu, k)$
$(M_0, M_1, St_1) \xleftarrow{\$} \mathcal{A}_1^{\mathsf{TPKE.Dec}(sk_i,\cdot)_{i\notin I_c}}(pk, vk, \{sk_i\}_{i\in I_c}, St_0)$
$C^* \xleftarrow{\$} \mathsf{TPKE.Enc}(pk, M_b; r)$; $b \xleftarrow{\$} \{0,1\}$
$b' \xleftarrow{\$} \mathcal{A}_2^{\mathsf{TPKE.Dec}(sk_i,\cdot)_{i\notin I_c}}(C^*, St_1)$
If $b \neq b'$ then return 0 else return 1

$I_c \subset \{1, \ldots, n\}$ chosen by $\mathcal{A}$ is a set of corruption players, which cardinality must be less than ν, i.e. the adversary cannot obtain more than $\nu - 1$ secret keys. The adversary $\mathcal{A}$ can have access to oracles which are with regard to a legitimate (pk, vk). $\mathcal{A}$ makes queries $\mathsf{Crrpt}(i)$ if the player i in the corruption set, i.e. $i \in I_c$, which returns sk_i up to $\nu - 1$ times and the random tape of player i. Otherwise, $\mathcal{A}$ has access to decryption oracle $\mathsf{TPKE.Dec}(sk_i, \cdot)_{i\notin I_c}$, and gets the response $\sigma_i \leftarrow \mathsf{TPKE.Dec}(pk, sk_i, C)$. It is prohibited to ask C^* by $\mathcal{A}_2$.

Note that allowing access to the result of shared decryption is sufficient to model active and adaptive adversary since we restrict ourselves to the non-interactive case as mentioned above. An adaptive adversary can corrupt arbitrary player after seeing the result of shared decryption by arbitrary player. In the case of static adversaries, corrupted players will be selected before starting the game. The adversary may locally executes $\mathsf{TPKE.Com}$ to complete the decryption if needed. More complicated interactive case can be handled by treating each round of interaction as a function and allow the adversary to access to it. The key generation can be shared in the same manner if desired.

Then we define the advantage of $\mathcal{A}$ in the above experiment as

$$\mathbf{Adv}^{\text{tpke-cca}}_{\mathcal{TPKE},\mathcal{A}}(k) \;=\; |\Pr[\mathbf{Exp}^{\text{tpke-cca}}_{\mathcal{TPKE},\mathcal{A}}(k) = 1] - 1/2| \;.$$

A threshold public-key encryption $\mathcal{TPKE}$ is said to be indistinguishability secure against chosen-ciphertext attacks (IND-CCA) if the advantage functions $\mathbf{Adv}^{\text{tpke-cca}}_{\mathcal{TPKE},\mathcal{A}}(k)$ is a negligible function in k for all polynomial-time adversaries $\mathcal{A}$.

B.2 Threshold Tag-KEM/DEM Framework

From a threshold tag-KEM and a DEM, we construct a threshold hybrid PKE as follows.

$\mathsf{TPKE.Kg}(n, \nu, k)$	$\mathsf{TPKE.Dec}(pk, sk_i, c)$
$(pk, vk, sk) \xleftarrow{\$} \mathsf{TTKEM.Kg}(n, \nu, k)$	$c \xrightarrow{\text{parse}} (C, \psi)$
Output (pk, vk, sk)	$\sigma_i \leftarrow \mathsf{TTKEM.Dec}(pk, sk_i, C, \psi)$
	Output σ_i
$\mathsf{TPKE.Enc}(pk, m)$	$\mathsf{TPKE.Com}(pk, vk, c, S)$
$(\omega, K) \xleftarrow{\$} \mathsf{TTKEM.SKey}(pk)$	$c \xrightarrow{\text{parse}} (C, \psi)$
$\psi \leftarrow \mathsf{DEM.Enc}(K, m)$	$K \leftarrow \mathsf{TTKEM.Com}(pk, vk, C, S, \psi)$
$C \leftarrow \mathsf{TTKEM.Enc}(pk, \omega, \psi)$	If $K = \perp$, output $\perp$.
Output $c = (C, \psi)$	Otherwise, $m \leftarrow \mathsf{DEM.Dec}(K, \psi)$
	Output m

Since $\mathsf{TPKE.Kg}$ is the same as $\mathsf{TTKEM.Kg}$, if $\mathsf{TTKEM.Kg}$ is shared, so is $\mathsf{TPKE.Kg}$.
We prove the following threshold tag-KEM/DEM composition theorem.

Theorem 12. *The above threshold hybrid PKE scheme is threshold CCA if the tag-KEM $\mathcal{TTKEM}$ is threshold CCA and the $\mathcal{DEM}$ is one-time secure. In particular,*

$$\mathbf{Adv}^{\text{tpke-cca}}_{\mathcal{TPKE},\mathcal{A}}(k) \leq 2\mathbf{Adv}^{\text{ttkem-cca}}_{\mathcal{TTKEM},\mathcal{A}}(k) + \mathbf{Adv}^{dem\text{-ind-ot}}_{\mathcal{DEM},\mathcal{B}}(k)$$

Proof. Outline of the proof follows that of the main theorem of [3]. Let X be the event that $\tilde{b} = b$ happens in experiment $\mathbf{Exp}^{\text{tpke-cca}}_{\mathcal{TPKE},\mathcal{A}}(k)$. Observe that $\mathsf{PKE.Enc}$ is now implemented by

$$(\omega, K) \leftarrow \mathsf{TTKEM.SKey}(pk), \psi \leftarrow \mathsf{DEM.Enc}(K, m_b), C \leftarrow \mathsf{TTKEM.Enc}(pk, \omega, \psi).$$

We first separate the threshold tag-KEM part from the DEM part by replacing K used in $\mathsf{DEM.Enc}$ with a random key $K^* \xleftarrow{\$} \mathit{KeySp}$.

Lemma 13. *Let $\mathbf{Exp}^{\text{tpke-cca}}_{\mathcal{TPKE},\mathcal{A}}(k)'$ and X' denote the modified game and an event in the game that corresponds to the original X in $\mathbf{Exp}^{\text{tpke-cca}}_{\mathcal{TPKE},\mathcal{A}}(k)$. Then we have*

$$\left|\Pr[X] - \Pr[X']\right| \leq 2\,\mathbf{Adv}^{\text{ttkem-cca}}_{\mathcal{TTKEM},\mathcal{A}}(k). \tag{1}$$

Proof. (of Lemma 13) Namely, adversary $\mathcal{A}_{\mathcal{PKE}}$ that acts differently in $\mathbf{Exp}^{\text{tpke-cca}}_{\mathcal{TPKE},\mathcal{A}}(k)$ and $\mathbf{Exp}^{\text{tpke-cca}}_{\mathcal{TPKE},\mathcal{A}}(k)'$ essentially distinguishes the real and random keys used in each game. Hence it achieves the attack goal against $\mathcal{TTKEM}$. Eq.(1) is formally proven by constructing an attacker $\mathcal{A}_{\text{ttkem}}$ for the threshold tag-KEM scheme by using $\mathcal{A}_{\text{tpke}}$ as follows.

Given a public-key, adversary $\mathcal{A}_{\text{ttkem}}$ forwards the public-key to $\mathcal{A}_{\text{tpke}}$. Given m_0 and m_1 from $\mathcal{A}_{\text{tpke}}$, $\mathcal{A}_{\text{ttkem}}$ composes a challenge ciphertext c^* as follows. Assume a random K_δ has been generated ($\delta \xleftarrow{\$} \{0,1\}$) in $\mathbf{Exp}^{\text{ttkem-cca}}_{\mathcal{TTKEM},\mathcal{A}}(k)$. The adversary first obtains challenge key K_δ and computes $\psi^* = \mathsf{DEM.Enc}_{K_\delta}(m_b)$ for $b \xleftarrow{\$} \{0,1\}$. It then sends $t^* = \psi$ to $\mathsf{TTKEM.Enc}$, and receives C^*. Finally c^* is set as $c^* = (C^*, \psi^*)$ and given to $\mathcal{A}_{\text{tpke}}$.

Decryption Oracle Simulation. All the oracle queries from $\mathcal{A}_{\text{tpke}}$ are sent to decryption oracle of $\mathcal{A}_{\text{ttkem}}$ transparently. Indeed, the only difference between the interfaces of decryption oracles in $\mathbf{Exp}^{\text{tpke-cca}}_{\mathcal{TPKE},\mathcal{A}}(k)$ and $\mathbf{Exp}^{\text{ttkem-cca}}_{\mathcal{TTKEM},\mathcal{A}}(k)$ is ψ and t, which are actually the same in this generic construction since ψ is treated as a tag.

Given an answer to a shared decryption query, $\mathcal{A}_{\text{ttkem}}$ decrypts corresponding ψ with K and sends the resulting plaintext to $\mathcal{A}_{\text{tpke}}$. Answers to corruption queries are given to $\mathcal{A}_{\text{tpke}}$ transparently. (This answer is consistent since $\mathsf{DEM.Dec}$ is deterministic.) If $\perp$ is given for any query, $\mathcal{A}_{\text{ttkem}}$ simply sends it to $\mathcal{A}_{\text{tpke}}$. Eventually, $\mathcal{A}_{\text{tpke}}$ outputs $\tilde{b}$. Then $\mathcal{A}_{\text{ttkem}}$ outputs $\tilde{\delta} = 1$ or 0 by following $\tilde{b} = b$ or $\neq b$, respectively.

We analyze the simulation. It is not hard to inspect that oracle queries are simulated perfectly. Regarding the challenge ciphertext, it is the same as that in $\mathbf{Exp}^{\text{tpke-cca}}_{\mathcal{TPKE},\mathcal{A}}(k)$ when $\delta = 1$ and that in $\mathbf{Exp}^{\text{tpke-cca}}_{\mathcal{TPKE},\mathcal{A}}(k)'$ when $\delta = 0$. Hence the view of $\mathcal{A}_{\text{tpke}}$ is perfectly simulated as if it is playing in $\mathbf{Exp}^{\text{tpke-cca}}_{\mathcal{TPKE},\mathcal{A}}(k)$ when $\delta = 1$, and that in $\mathbf{Exp}^{\text{tpke-cca}}_{\mathcal{TPKE},\mathcal{A}}(k)'$ when $\delta = 0$.

Therefore, we have $\Pr[\tilde{b} = b|\delta = 1] = \Pr[X]$ and $\Pr[\tilde{b} = b|\delta = 0] = \Pr[X']$. Since $\mathcal{TTKEM}$ is CCA secure, we have $\left|\Pr[\tilde{\delta} = \delta] - \frac{1}{2}\right| \leq \mathbf{Adv}_{\mathrm{ttkem}}$ and

$$\begin{aligned}\left|\Pr[\tilde{\delta} = \delta] - \frac{1}{2}\right| &= \left|\frac{1}{2}(\Pr[\tilde{\delta} = 1|\delta = 1] - \Pr[\tilde{\delta} = 1|\delta = 0])\right| \\ &= \left|\frac{1}{2}(\Pr[\tilde{b} = b|\delta = 1] - \Pr[\tilde{b} = b|\delta = 0])\right| \\ &= \left|\frac{1}{2}(\Pr[X] - \Pr[X'])\right| \leq \mathbf{Adv}_{\mathrm{ttkem}},\end{aligned}$$

holds. This completes the proof of Lemma 13.

Next, consider the view of $\mathcal{A}_{\mathrm{tpke}}$ in $\mathbf{Exp}^{\text{tpke-cca}}_{\mathcal{TPKE},\mathcal{A}}(k)'$. Only ψ is related to b and everything else are information theoretically independent of b. Therefore, distinguishing b in $\mathbf{Exp}^{\text{tpke-cca}}_{\mathcal{TPKE},\mathcal{A}}(k)'$ essentially means distinguishing the plaintexts in $\mathcal{DEM}$.

Formally, we construct $\mathcal{B}$ from $\mathcal{A}_{\mathrm{tpke}}$ in $\mathbf{Exp}^{\text{tpke-cca}}_{\mathcal{TPKE},\mathcal{A}}(k)'$. $\mathcal{B}$ has to simulate oracles in $\mathbf{Exp}^{\text{tpke-cca}}_{\mathcal{TPKE},\mathcal{A}}(k)'$ in the presence of corrupted players. $\mathcal{B}$ first invokes $\mathsf{PKE.Gen}$ and gives the public-key pk to $\mathcal{A}_{\mathrm{tpke}}$. Given m_0 and m_1 from $\mathcal{A}_{\mathrm{tpke}}$, $\mathcal{B}$ forwards them to the encryption oracle of $\mathcal{DEM}$ and receives a challenge ciphertext ψ^*.

It then creates $c^* = (C^*, \psi^*)$ by executing $\mathsf{TTKEM.SKey}$ and $\mathsf{TTKEM.Enc}$ with ψ as a tag, and sends c^* to $\mathcal{A}_{\mathrm{tpke}}$. For every shared decryption query, $\mathcal{B}$ invokes $\mathsf{TTKEM.Dec}$ with the specified players. This can be done perfectly as $\mathcal{B}$ holds the corresponding sk_i. Corruption queries are also handled correctly by sending the private key (and the randomness used by the simulating party if any).

Eventually, when $\mathcal{A}_{\mathrm{tpke}}$ outputs $\tilde{b}$, $\mathcal{B}$ outputs it as the guessing result. Now, observe that $\mathcal{B}$ runs in polynomial time if $\mathcal{A}_{\mathrm{tpke}}$ does. It is also not hard to see that $\mathcal{B}$ perfectly simulates $\mathbf{Exp}^{\text{tpke-cca}}_{\mathcal{TPKE},\mathcal{A}}(k)'$; the oracle is simulated as just specified, and the key embedded in C^* is independent of the random one chosen by the encryption oracle of $\mathcal{DEM}$ just as intended in $\mathbf{Exp}^{\text{tpke-cca}}_{\mathcal{TPKE},\mathcal{A}}(k)'$. Therefore, we have

$$\left|\Pr[X'] - \frac{1}{2}\right| \leq \mathbf{Adv}^{\textit{dem}\text{-ind-ot}}_{\mathcal{DEM},\mathcal{B}}(k) \tag{2}$$

From Eq.(1) and (2), we have

$$\begin{aligned}\left|(\Pr[X] - \frac{1}{2}) - (\Pr[X'] - \frac{1}{2})\right| &\leq 2\mathbf{Adv}^{\text{ttkem-cca}}_{\mathcal{TTKEM},\mathcal{A}}(k) \\ \mathbf{Adv}^{\text{tpke-cca}}_{\mathcal{TPKE},\mathcal{A}}(k) - \mathbf{Adv}^{\textit{dem}\text{-ind-ot}}_{\mathcal{DEM},\mathcal{B}}(k) &\leq 2\mathbf{Adv}^{\text{ttkem-cca}}_{\mathcal{TTKEM},\mathcal{A}}(k) \\ \mathbf{Adv}^{\text{tpke-cca}}_{\mathcal{TPKE},\mathcal{A}}(k) &\leq 2\mathbf{Adv}^{\text{ttkem-cca}}_{\mathcal{TTKEM},\mathcal{A}}(k) + \mathbf{Adv}^{\textit{dem}\text{-ind-ot}}_{\mathcal{DEM},\mathcal{B}}(k)\end{aligned}$$

□