

Linearization Attacks Against Syndrome Based Hashes

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Abstract. In MyCrypt 2005, Augot, Finiasz, and Sendrier proposed FSB, a family of cryptographic hash functions. The security claim of the FSB hashes is based on a coding theory problem with hard average-case complexity. In the ECRYPT 2007 Hash Function Workshop, new versions with essentially the same compression function but radically different security parameters and an additional final transformation were presented. We show that hardness of average-case complexity of the underlying problem is irrelevant in collision search by presenting a linearization method that can be used to produce collisions in a matter of seconds on a desktop PC for the variant of FSB with claimed 2^{128} security.

Keywords: FSB, Syndrome Based Hashes, Provably Secure Hashes, Hash Function Cryptanalysis, Linearization Attack.

1 Introduction

A number of hash functions have been proposed that are based on “hard problems” from various branches of computer science. Recent proposals in this genre of hash function design include VSH (factoring) [3], LASH (lattice problems) [2], and the topic of this paper, Fast Syndrome Based Hash (FSB), which is based on decoding problems in the theory of error-correcting codes [1, 6].

In comparison to dedicated hash functions designed using symmetric cryptanalysis techniques, “provably secure” hash functions tend to be relatively slow and do not always meet all of criteria traditionally expected of cryptographic hashes. An example of this is VSH, where only collision resistance is claimed, leaving the hash open to various other attacks [8].

Another feature of “provably secure” hash functions is that the proof is often a reduction to a problem with asymptotically hard worst-case or average-case complexity. Worst-case complexity measures the difficulty of solving pathological cases rather than typical cases of the underlying problem. Even a reduction to a problem with hard average complexity, as is the case with FSB, offers only limited security assurance as there still can be an algorithm that easily solves the problem for a subset of the problem space.

This common pitfall of provably secure cryptographic primitives is clearly demonstrated in this paper for FSB – it is shown that the hash function offers minimal pre-image or collision resistance when the message space is chosen in a specific way.

The remainder of this paper is structured as follows. Section 2 describes the FSB compression function. Section 3 gives the basic linearization method for finding pre-images and extends it to “alphabets”. This is followed by an improved collision attack in Section 4 and discussion of attacks based on larger alphabets in Section 5.

Appendix A gives a concrete example of pre-image and collision attacks on a proposed variant of FSB with claimed 128-bit security.

2 The FSB Compression Function

The FSB compression function can be described as follows [1, 6].

Definition 1. Let \mathcal{H} be an $r \times n$ binary matrix. The FSB compression function is a mapping from message vector \mathbf{s} that contains w characters, each satisfying $0 \leq \mathbf{s}_i < \frac{n}{w}$, to an r bit result as follows:

$$\text{FSB}(\mathbf{s}) = \bigoplus_{i=1}^w \mathcal{H}_{(i-1)\frac{n}{w} + \mathbf{s}_i + 1},$$

where \mathcal{H}_i denotes column i of the matrix.

The FSB compression function is operated in Merkle–Damgård mode to process a large message [7, 5]. The exact details of padding and chaining of internal state across compression function iterations are not specified.¹

With most proposed variants of FSB, the character size $\frac{n}{w}$ is chosen to be 2^8 , so that \mathbf{s} can be treated as an array of bytes for practical implementation purposes. See Appendix A for an implementation example.

For the purposes of this paper, we shall concentrate on finding collisions and pre-images in the compression function. These techniques can be easily applied for finding full collisions of the hash function. The choice of \mathcal{H} is taken to be a random binary matrix in this paper, although quasi-cyclic matrices are considered in [6] to reduce memory usage.

The final transformation proposed in [6] does not affect the complexity of finding collisions or second pre-images, although it makes first pre-image search difficult (equal to inverting Whirlpool [9]). Second pre-images can be easily found despite a strong final transform.

The security parameter selection in the current versions of FSB is based primarily on Wagner’s generalized birthday attack [10, 4]. The security claims are summarized in Table 1.

3 Linearization Attack

To illustrate our main attack technique, we shall first consider hashes of messages with binary values in each character: $\mathbf{s}_i \in \{0, 1\}$ for $1 \leq i \leq w$. This message space is a small subset of all possible message blocks.

¹ Ambiguous definitions of algorithms makes experimental cryptanalytic work depend on guess-work on algorithm details. However, the attacks outlined in this paper should work, regardless of the particular details of chaining and padding.

Security	r	w	n	n/w
64-bit	512	512	131072	256
	512	450	230400	512
	1024	2^{17}	2^{25}	256
80-bit	512	170	43520	256
	512	144	73728	512
128-bit	1024	1024	262144	256
	1024	904	462848	512
	1024	816	835584	1024

Table 1. Parameterizations of FSB, as given in [6]. Line 6 (in bold) with claimed 2^{128} security was proposed for practical use. Pre-images and collisions can be found for this variant in a matter of seconds on a desktop PC.

We define a constant vector \mathbf{c} ,

$$\mathbf{c} = \bigoplus_{i=1}^w \mathcal{H}_{(i-1)\frac{n}{w}+1},$$

and an auxiliary $r \times w$ binary matrix \mathbf{A} , whose columns \mathbf{A}_i , $1 \leq i \leq w$ are given by

$$\mathbf{A}_i = \mathcal{H}_{(i-1)\frac{n}{w}+1} \oplus \mathcal{H}_{(i-1)\frac{n}{w}+2}.$$

By considering how the XOR operations cancel each other out, it is easy to see that for messages of this particular type the FSB compression function is entirely linear:

$$\text{FSB}(\mathbf{s}) = \mathbf{A} \cdot \mathbf{s} \oplus \mathbf{c}.$$

Note that in this paper \mathbf{s} and \mathbf{c} and other vectors are column vectors unless otherwise stated.

Furthermore, let us consider the case where $r = w$, and therefore \mathbf{A} is a square matrix. If $\det \mathbf{A} \neq 0$ the inverse exists and we are able to find a pre-image \mathbf{s} from the Hash $\mathbf{h} = \text{FSB}(\mathbf{s})$ simply as

$$\mathbf{s} = \mathbf{A}^{-1} \cdot (\mathbf{h} \oplus \mathbf{c}).$$

If r is greater than w , the technique can still be applied to force given w bits of the final hash to some predefined value. Since the order of the rows is not relevant, we can simply construct a matrix that contains only the given w rows (i.e.. bits of the hash function result) of \mathbf{A} that we are interested in.

3.1 The selection of alphabet in a preimage attack

We note that the selection of $\{0, 1\}$ as the set of allowable message characters (“the alphabet”) is arbitrary. We can simply choose any pair of values for each i so that $\mathbf{s}_i \in \{x_i, y_i\}$ and map each $x_i \mapsto 0$ and $y_i \mapsto 1$, thus creating a binary vector for the attack.

The constant is then given by

$$\mathbf{c} = \bigoplus_{i=1}^w \mathcal{H}_{(i-1)\frac{n}{w}+x_i},$$

and columns of the \mathbf{A} matrix are given by

$$\mathbf{A}_i = \mathcal{H}_{(i-1)\frac{n}{w}+x_i+1} \oplus \mathcal{H}_{(i-1)\frac{n}{w}+y_i+1}.$$

To invert a hash \mathbf{h} we first compute

$$\mathbf{b} = \mathbf{A}^{-1}(\mathbf{h} \oplus \mathbf{c})$$

and then apply the mapping $s_i = x_i + \mathbf{b}_i(y_i - x_i)$ on the binary result \mathbf{b} to obtain a message \mathbf{s} that satisfies $\text{FSB}(\mathbf{s}) = \mathbf{h}$.

3.2 Invertibility of random binary matrices

The binary matrices are essentially random for each arbitrarily chosen alphabet. Since the success of a pre-image attack depends upon the invertibility of the binary matrix \mathbf{A} , we note (without a proof) that the probability that an $n \times n$ random binary matrix has non-zero determinant and is therefore invertible in $\text{GF}(2)$ is given by

$$p = \prod_{i=1}^n (1 - 2^{-i}) \approx 0.28879 \approx 2^{-1.792}$$

when n is even moderately large.

Two trials with two distinct alphabets are on the average enough to find an invertible matrix (total probability for 2 trials is $1 - (1 - p)^2 \approx 0.49418$).

4 Finding collisions when $r = 2w$

We shall expand our approach for producing collisions in $2w$ bits of the hash function result by controlling w message characters. This is twice the number compared to pre-image attack of Section 3.1. The complexity of the attack remains negligible – few simple matrix operations.

Assume that by selection of two distinct alphabets, $\{x_i, y_i\}$ and $\{x'_i, y'_i\}$, there are two distinct linear presentations for FSB, one containing the matrix \mathbf{A} and constant \mathbf{c} and the other one \mathbf{A}' and \mathbf{c}' correspondingly. To find a pair of messages \mathbf{s}, \mathbf{s}' that produces a collision we must find a solution for \mathbf{b} and \mathbf{b}' in the equation

$$\mathbf{A} \cdot \mathbf{b} \oplus \mathbf{c} = \mathbf{A}' \cdot \mathbf{b}' \oplus \mathbf{c}'.$$

This basic collision equation can be manipulated to the form

$$(\mathbf{A} \mid \mathbf{A}') \cdot \begin{pmatrix} \mathbf{b} \\ \mathbf{b}' \end{pmatrix} = \begin{pmatrix} \mathbf{c} \\ \mathbf{c}' \end{pmatrix}.$$

The solution of the inverse $(\mathbf{A} \mid \mathbf{A}')^{-1}$ will allow us to compute the message pair $(\mathbf{b} \mid \mathbf{b}')^T$ that yields the same hash in $2w$ different message bits (since $r = 2w$ yields a square matrix in this case).

$$(\mathbf{A} \mid \mathbf{A}')^{-1} \cdot \begin{pmatrix} \mathbf{c} \\ \mathbf{c}' \end{pmatrix} = \begin{pmatrix} \mathbf{b} \\ \mathbf{b}' \end{pmatrix}.$$

The binary vector $(\mathbf{b} \mid \mathbf{b}')^T$ can then be split into two messages \mathbf{s} and \mathbf{s}' that produce the collision. For $1 \leq i \leq w$ we apply the alphabet mapping as follows:

$$\begin{aligned} \mathbf{s}_i &= x_i + \mathbf{b}_i(y_i - x_i), \\ \mathbf{s}'_i &= x'_i + \mathbf{b}'_i(y'_i - x'_i). \end{aligned}$$

Here x_i, y_i and x'_i, y'_i represent the alphabets for \mathbf{s}_i and \mathbf{s}'_i , respectively.

5 Larger alphabets

Consider an alphabet of cardinality three, $\{x_i, y_i, z_i\}$. We can construct a linear equation in $\text{GF}(2)$ that computes the FSB compression function in this message space by using two columns for each message character \mathbf{s}_i . The linear matrix therefore has size $r \times 2w$. The constant \mathbf{c} is computed as before as:

$$\mathbf{c} = \bigoplus_{i=1}^w \mathcal{H}_{(i-1)\frac{n}{w}+x_i},$$

and the odd and even columns are given by

$$\begin{aligned} A_{2i-1} &= \mathcal{H}_{(i-1)\frac{n}{w}+x_i+1} \oplus \mathcal{H}_{(i-1)\frac{n}{w}+y_i+1}, \\ A_{2i} &= \mathcal{H}_{(i-1)\frac{n}{w}+x_i+1} \oplus \mathcal{H}_{(i-1)\frac{n}{w}+z_i+1}. \end{aligned}$$

The message \mathbf{s} must also be transformed into a binary vector \mathbf{b} of length $2w$ via a selection function v :

$$\begin{array}{cc} \mathbf{s}_i & v(\mathbf{s}_i) \\ x_i & (0, 0) \\ y_i & (1, 0) \\ z_i & (0, 1) \end{array}$$

The binary vector \mathbf{b} is constructed by concatenating the selection function outputs:

$$\mathbf{b} = (v(\mathbf{s}_1) \parallel v(\mathbf{s}_2) \parallel \dots \parallel v(\mathbf{s}_w))^T.$$

We again arrive at a simple linear equation for the FSB compression function:

$$\text{FSB}(\mathbf{s}) = \mathbf{A} \cdot \mathbf{b} \oplus \mathbf{c}.$$

The main difference is that the message space is much larger, $3^w \approx 2^{1.585w}$. This construction is easy to generalize for alphabets of any size: $r \times (k-1)w$ size linear matrix is required for an alphabet of size k . However, we have not found cryptanalytic advantages in mapping hashes back to message spaces with alphabets larger than three.

5.1 Pre-image search

It is easy to see that even if \mathbf{A} is invertible, not all hash results are, since the solution of \mathbf{b} may contain $v(s_i) = (1, 1)$ pairs. These do not map back to the message space in the selection function.

Given a random binary \mathbf{b} , the fraction of valid messages in the message space (alphabet of size 3) is given by $(3/4)^w = 2^{-0.415w}$. Despite this disadvantage, larger alphabets can be useful in attacks. We will illustrate this with an example.

Example. FSB parameters with $w = 64$, $n = 256 \times 64 = 16384$ and $r = 128$ is being used; 64 input bytes are processed into a 128 bit result. What is the complexity of a pre-image attack ? ²

Solution. We will use an alphabet of size 3. Considering both matrix invertibility (Section 3.2) and the alphabet mapping, the probability of successfully mapping the hash back to the alphabet is $0.28879 \times (3/4)^{64} = 2^{-28.4}$. We can precompute 2^{27} inverses \mathbf{A}^{-1} for various message spaces offline, hence speeding up the time required to find an individual pre-image. There are also early-abort strategies that can be used to speed up the search.

Using these techniques, the pre-image search requires roughly 2^{28} steps in this case, compared to the theoretical 2^{128} .

5.2 Collision search

Three-character alphabets can be used in conjunction with the collision attack outlined in Section 4. It is easy to see that it is possible to mix 3-character alphabets with binary alphabets. Each character position s_i that is mapped to a 3-character alphabet requires two columns in the linear matrix, whereas those mapped to a 2-character alphabet require only one column.

Generally speaking, the probability for finding two valid messages in each trial is $(3/4)^{2k} = 2^{-0.830k}$ when k characters in \mathbf{s} and \mathbf{s}' are mapped to 3-character alphabets.

Example. FSB parameters with $w = 170$, $n = 256 \times 170 = 43520$ and $r = 512$ is being used; 170 input bytes are processed into a 512-bit result. What is the complexity of collision search ? ³

Solution. We use a mixed alphabet; $k = 86$ characters are mapped to a 3-character alphabet and the remaining 84 characters are mapped to a binary alphabet. The linear matrix \mathbf{A} therefore has $2 \times 86 + 84 = 256$ columns, and the combined matrix $(\mathbf{A} \mid \mathbf{A}')$ in the collision attack (similarly to 4) has size 512×512 . Success of matrix inversion is $2^{-1.792}$. The probability of success in each trial is $2^{-0.830k-1.792} = 2^{-73.2}$, collision search has complexity of roughly 2^{73} .

² The complexity of a collision attack in this case is negligible, as $r = 2w$ and the technique from Section 4 can be used.

³ These security parameters are proposed for 80-bit security in [6] and reproduced in Table 1.

6 Conclusions

We have shown that Fast Syndrome Based Hashes (FSB) are not secure against pre-image or collision attacks under the proposed security parameters. The attacks have been implemented and collisions for a variant with claimed 128-bit security can be found in less than a second on a low-end PC.

We feel that the claim of “provable security” is hollow in the case of FSB, where the security proof is based on a problem with hard average-case complexity, but which is almost trivially solvable for special classes of messages.

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A Appendix: A Collision and Pre-Image Example

For parameter selection $r = 1024$, $w = 1024$, $n = 262144$, $s = 8192$, $n/w = 256$, the FSB compression function can be implemented in C as follows.

```
typedef unsigned char u8;           // u8 = single byte
typedef unsigned long long u64;     // u64 = 64-bit word

void fsb(u64 h[0x40000][0x10],      // "random" matrix
         u8 s[0x400],              // 1k message block
         u64 r[0x10])              // result
{
    int i, j, idx;

    for (i = 0; i < 0x10; i++)      // zeroise result
        r[i] = 0;
    for (i = 0; i < 0x400; i++)     // process a block
    {
        idx = (i << 8) + s[i];      // index in H
        for (j = 0; j < 0x10; j++)
            r[j] ^= h[idx][j];      // xor over result
    }
}
```

Since the FSB specification does not offer any standard way of defining the “random” matrix \mathcal{H} (or $h[][]$ above), we will do so here using the Data Encryption Standard. Each 64-bit word $h[i][j]$ is created by encrypting the 64-bit input value $(i \ll 4) \wedge j$ under an all-zero 56-bit key (00 00 00 00 00 00 00 00). The input and output values are handled in big-endian fashion. Some of the values are:⁴

Input to DES	Table Index	Value
0x0000000000000000	$h[0x00000][0x0]$	$= 0x8CA64DE9C1B123A7$
0x0000000000000001	$h[0x00000][0x1]$	$= 0x166B40B44ABA4BD6$
0x0000000000000002	$h[0x00000][0x2]$	$= 0x06E7EA22CE92708F$
....		
0x0000000000000010	$h[0x00001][0x0]$	$= 0x5B711BC4CEEBF2EE$
0x0000000000000011	$h[0x00001][0x1]$	$= 0x799A09FB40DF6019$
0x0000000000000012	$h[0x00001][0x2]$	$= 0xAFFA05C77CBE3C45$
....		
0x000000000003FFFFD	$h[0x3FFFF][0xD]$	$= 0x313C4BD8E2F7156A$
0x000000000003FFFFE	$h[0x3FFFF][0xE]$	$= 0x19F32D6B2D9B57F5$
0x000000000003FFFFF	$h[0x3FFFF][0xF]$	$= 0x804DB568319F4F8B$

We shall define two 1024-byte message blocks that produce the same 1024-bit chosen output value in the FSB compression function, hence demonstrating the ease of pre-image and collision search on a variant with claimed 2^{128} security. They were found in less than a second on an iBook G4 laptop.

⁴ Please note that x86 platforms are little-endian. Bi-endian gcc source code for producing pre-images can be downloaded from: http://www.m-js.com/misc/fsb_test.tar.gz

The first message block uses the ASCII alphabet $\{A, C\}$ or $\{0x41, 0x42\}$:

CAACACACCACAACACACACCACAACCCCCACCAACACCAAAACAAACACCAACACCACACCAA
ACACACCCCCAACCCAAAAAACCCACCACCCACCAAACACACCCCCAACCCACACCCAACACCA
AACCCACCCCCAACCCAAACAAAAACCCACAAACACACCACACCCCCACAACCCACACAAA
AACCCACCCCCAACCAACAAAAACAAACACACACACACCCCCAAACCCCCAAAAACCCACAAC
CAAAACAACCCAAACACCAACCCACACCCCCAAACCCAAAAAACACAAACCCCAACAAAACCAA
ACACCCCCCCCCAACAAAAACACCCACCCAACAAAAAACACACCCCCCAACCCACCCCAACA
AAAAACAACAACACCACCCACCCCAACCAAAACCCACCAACCCCAACCCACCCCAACAAAAC
ACCACCCCAACCCACAACCAACCCAACACCAACACCAAAACACACCAAAACACCCAAACACCCC
CAAAACACACACCACCACCAACCAAAAAACCAACACCCCCAAAAAACCCAAACCAACCCCA
CACAACCCCAACCCAACCCAACCAACCACCAAAACCCAACCCCCAAAAACAAACCAACCCCA
AACACCCACAACACCAACACAACAAAAACCAACCCAACCAACCCACCAACCCACACACAAAA
CCACCCCAACCCCCAACCAACCCCAACAACACCAACACCCCAACCCCAACCAAAAAACCCACC
ACAACCCAACACCAACCCCAACAAACCAACCCCAACCCCAACCCCAACCCACACCAACAAAC
CACCCAAAAAACAAACACCAACACACAACCAACCAAAACAAAAAAAACCAAAAAACCCCCAAC
CACCCACCCACAAACAAAAACAAAAAAACCCAAAAAACCAAAACCAACCAACCCCAACCA
CCCACCAAAACAACACCAACACAAAAACCCCAACCCCCCAACCAACCAAAACCAAAACCA

The second message block uses ASCII alphabet {A, H} or {0x41, 0x48}:

[illegible]

The 1024-bit / 128-byte result of compressing either one of these blocks is:

Index	Hex		ASCII
00000000	5468697320697320	6120636f6c6c6973	This is a collis
00000010	696f6e20616e6420	7072652d696d6167	ion and pre-imag
00000020	6520666f72204661	73742053796e6472	e for Fast Syndr
00000030	6f6d652042617365	6420486173682e20	ome Based Hash.
00000040	4172626974726172	79207072652d696d	Arbitrary pre-im
00000050	616765732063616e	20626520666f756e	ages can be foun
00000060	6420696e20612066	72616374696f6e20	d in a fraction
00000070	6f66206120736563	6f6e642120202020	of a second!