

On the security of Identity Based Ring Signcryption Schemes

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Abstract. Signcryption is a cryptographic primitive which offers authentication and confidentiality simultaneously with a cost lower than signing and encrypting the message independently. Ring signcryption enables a user to signcrypt a message along with the identities of a set of potential senders (that includes him) without revealing which user in the set has actually produced the signcryption. Thus a ring signcrypt message has anonymity in addition to authentication and confidentiality. Ring signcryption schemes have no group managers, no setup procedures, no revocation procedures and no coordination: any user can choose any set of users (ring), that includes himself and signcrypt any message by using his private and public key as well as other users (in the ring) public keys, without getting any approval or assistance from them. Ring Signcryption is useful for leaking trustworthy secrets in an anonymous, authenticated and confidential way.

To the best of our knowledge, seven identity based ring signcryption schemes are reported in the literature. Two of them were already proved to be insecure in [12] and [8]. In this paper, we show that four among the remaining five schemes do not provide confidentiality, to be specific, two schemes are not secure against chosen plaintext attack and other two schemes do not provide adaptive chosen ciphertext security. We then propose a new scheme and formally prove the security of the new scheme in the random oracle model. A comparison of our scheme with the only existing correct scheme by Huang et al. shows that our scheme is much more efficient than the scheme by Huang et al.

Keywords: Ring Signcryption, Cryptanalysis, Provable Security, Confidentiality, Chosen Plaintext Attack, Adaptive Chosen Ciphertext Attack, Bilinear Pairing, Random Oracle Model.

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1 Introduction

Identity based cryptography (IBC) was introduced by Shamir [9] in the year 1984. It aims in reducing the over head of public key certification which is inherent in the public key infrastructure (PKI). The public key of a user in IBC is not a random string as in PKI, instead it is an unique identifier of a user such as email id, IP address, social security number etc. The user of an identity based cryptosystem is not required to obtain a certificate for his public key, since his identity is well known in public or available in a public directory. IBC employs a trusted third party, namely the private key generator (PKG). The PKG generates the private key for a user, corresponding to the identity of the user provided at the time of registration with the PKG. Thus, the private key of all users registered with the IBC is known to the PKG.

Signcryption - the cryptographic primitive, proposed by Zheng [13] provides both authenticity and confidentiality with a lower computational cost when compared to signing and encrypting the message independently. Ring signature, was first proposed by Rivest et al. [7], provides authenticity for a message in an anonymous way, i.e. the verifier does not know who has signed the message but he can verify that one of the person from the ring (group). The ring is formed by the signer while signing a message without getting acknowledgment from the ring members. Ring signcryption enables a user to send an authentic message confidentially and anonymously to a specified receiver.

Motivation: Let us consider a scenario, where a member of a cabinet wants to leak a very important and juicy information, regarding the president of the nation to the press. He has to leak the secret in an anonymous way, else he will be black spotted in the cabinet. The press will not accept the information unless it is authenticated by one of the members of the cabinet. Here, if the information is so sensitive and should not be leaked until the authorities in the press receives it, we should have confidential transmission of information. Thus, we require anonymity to safeguard the cabinet member who sends the information, authentication for the authorities in the press to believe the information and confidentiality until the information reaches the hands of the right person in the press. All the three properties are together achieved by a single primitive called “Ring Signcryption”. The first identity based ring signcryption scheme was proposed by Huang et al. [4].

Related Work and Our Contribution: Huang et al.’s scheme [4] was inefficient because the sender has to compute $n + 2$ pairing for signcrypting a message and three pairing operations for unsigncrypting a ring signcryption. Subsequently, identity based ring signcryption schemes were reported in [10, 14, 12, 6, 5, 15] and these papers were attempts to design schemes more efficient than Huang et al.’s [4] scheme.

Among these seven schemes, the security weakness of [12] was shown in [5] and the weakness of [6] was shown in [8]. In this paper, we show that the schemes in [10], [15], [5] and [14] are insecure. Specifically, we show that [5] and [14] does

not withstand adaptive chosen ciphertext attack, [10] and [15] are not secure against chosen plain text attack. This leaves the scheme by Huang et al. [4] as the only correct existing scheme. Then, we propose a new scheme and prove its security formally in a stronger security model. Moreover, our scheme is much more efficient than Huang et al.'s [4] scheme.

2 Preliminaries

2.1 Bilinear Pairing

Let \mathbb{G}_1 be an additive cyclic group generated by P , with prime order q , and \mathbb{G}_2 be a multiplicative cyclic group of the same order q . A bilinear pairing is a map $\hat{e} : \mathbb{G}_1 \times \mathbb{G}_1 \rightarrow \mathbb{G}_2$ with the following properties.

- **Bilinearity.** For all $P, Q, R \in \mathbb{G}_1$,
 - $\hat{e}(P + Q, R) = \hat{e}(P, R)\hat{e}(Q, R)$
 - $\hat{e}(P, Q + R) = \hat{e}(P, Q)\hat{e}(P, R)$
 - $\hat{e}(aP, bQ) = \hat{e}(P, Q)^{ab}$
- **Non-Degeneracy.** There exist $P, Q \in \mathbb{G}_1$ such that $\hat{e}(P, Q) \neq I_{\mathbb{G}_2}$, where $I_{\mathbb{G}_2}$ is the identity in \mathbb{G}_2 .
- **Computability.** There exists an efficient algorithm to compute $\hat{e}(P, Q)$ for all $P, Q \in \mathbb{G}_1$.

2.2 Computational Bilinear Diffie-Hellman Problem (CBDHP)

Definition 1. Given $(P, aP, bP, cP) \in \mathbb{G}_1^4$ for unknown $a, b, c \in \mathbb{Z}_q^*$, the CBDH problem in \mathbb{G}_1 is to compute $\hat{e}(P, P)^{abc} \in \mathbb{G}_2$.

The advantage of any probabilistic polynomial time algorithm \mathcal{A} in solving the CBDH problem in \mathbb{G}_1 is defined as

$$Adv_{\mathcal{A}}^{CBDH} = \Pr [\mathcal{A}(P, aP, bP, cP) = \hat{e}(P, P)^{abc} \mid a, b, c \in \mathbb{Z}_q^*]$$

The CBDH Assumption is that, for any probabilistic polynomial time algorithm \mathcal{A} , the advantage $Adv_{\mathcal{A}}^{CBDH}$ is negligibly small.

2.3 Computation Diffie-Hellman Problem (CDHP)

Definition 2. Given $(P, aP, bP) \in \mathbb{G}_1^3$ for unknown $a, b \in \mathbb{Z}_q^*$, the CDH problem in \mathbb{G}_1 is to compute abP .

The advantage of any probabilistic polynomial time algorithm \mathcal{A} in solving the CDH problem in \mathbb{G}_1 is defined as

$$Adv_{\mathcal{A}}^{CDH} = \Pr [\mathcal{A}(P, aP, bP) = abP \mid a, b \in \mathbb{Z}_q^*]$$

The CDH Assumption is that, for any probabilistic polynomial time algorithm \mathcal{A} , the advantage $Adv_{\mathcal{A}}^{CDH}$ is negligibly small.

2.4 Notations used in this paper

To have a better understanding and to enhance the readability and clarity, we use the following notations throughout the paper.

\mathcal{U}_i - User with identity ID_i .

$\mathcal{U} = \{\mathcal{U}_i\}_{(i=1 \text{ to } n)}$ - Group of users in the ring (including the actual sender).

\mathcal{M} - Message space.

m - Message.

l - Number of bits used to represent m .

Q_i - Public key corresponding to ID_i .

D_i - Private key corresponding to ID_i .

$ID_{\mathbb{S}}$ - Identity of the sender.

$ID_{\mathbb{R}}$ - Identity of the receiver.

$Q_{\mathbb{S}}$ - Public key of the sender.

$Q_{\mathbb{R}}$ - Public key of the receiver.

$D_{\mathbb{S}}$ - Private key of the sender.

$D_{\mathbb{R}}$ - Private key of the receiver.

3 Formal Security Model for Identity Based Ring Signcryption

3.1 Generic Scheme

A generic identity based ring signcryption scheme consists of the following four algorithms.

- **Setup**(κ): Given a security parameter κ , the private key generator (PKG) generates the systems public parameters $params$ and the corresponding master private key msk that is kept secret.
- **Extract**(ID_i): Given a user identity ID_i , the PKG computes the corresponding private key D_i and sends D_i to ID_i via a secure channel.
- **Signcrypt**($m, \mathcal{U}, D_{\mathbb{S}}, ID_{\mathbb{R}}$): This algorithm takes a message $m \in \mathcal{M}$, a receiver with identity $ID_{\mathbb{R}}$, the senders private key $D_{\mathbb{S}}$ and an ad-hoc group of ring members \mathcal{U} with identities $\{ID_1, \dots, ID_n\}$ as input and outputs a ring signcryption C . This algorithm is executed by a sender with identity $ID_{\mathbb{S}} \in \mathcal{U}$. $ID_{\mathbb{R}}$ may or may not be in \mathcal{U} .
- **Unsigncrypt**($C, \mathcal{U}, D_{\mathbb{R}}$): This algorithm takes the ring signcryption C , the ring members (say $\mathcal{U} = \{\mathcal{U}_i\}_{(i=1 \text{ to } n)}$) and the private key $D_{\mathbb{R}}$ of the receiver $ID_{\mathbb{R}}$ as input and produces the plaintext m , if C is a valid ring signcryption of m from the ring \mathcal{U} to $ID_{\mathbb{R}}$ or “Invalid”, if C is an invalid ring signcryption. This algorithm is executed by a receiver $ID_{\mathbb{R}}$.

3.2 Security Notion

The formal security definition of signcryption was given by Baek et al. in [1]. The security requirements for identity based ring signcryption were defined by Huang et al. [4]. We extend the security model given in [4] by incorporating security against insider attacks. The security model is defined as follows.

Definition 3. *An identity based ring signcryption (IRSC) is indistinguishable against adaptive chosen ciphertext attacks (IND-IRSC-CCA2) if there exists no polynomially bounded adversary having non-negligible advantage in the following game:*

1. **Setup Phase:** The challenger \mathcal{C} runs the Setup algorithm with a security parameter κ and sends the system parameters params to the adversary \mathcal{A} and keeps the master private key msk secret.
2. **First Phase:** \mathcal{A} performs polynomially bounded number of queries to the oracles provided to \mathcal{A} by \mathcal{C} . The description of the queries in the first phase are listed below:
 - **Key Extraction query:** \mathcal{A} produces an identity ID_i corresponding to U_i and receives the private key D_i corresponding to ID_i .
 - **Signcryption query:** \mathcal{A} produces a set of users \mathcal{U} , a receiver identity $ID_{\mathbb{R}}$ and a plaintext $m \in_{\mathbb{R}} \mathcal{M}$ to the challenger \mathcal{C} . \mathcal{A} also specifies the sender $U_{\mathbb{S}} \in \mathcal{U}$ whose identity is $ID_{\mathbb{S}}$. Then \mathcal{C} signcrypts m from $ID_{\mathbb{S}}$ to $ID_{\mathbb{R}}$ with $D_{\mathbb{S}}$ and sends the result to \mathcal{A} .
 - **Unsigncryption query:** \mathcal{A} produces a set of users \mathcal{U} , a receiver identity $ID_{\mathbb{R}}$, and a ring signcryption C . \mathcal{C} generates the private key $D_{\mathbb{R}}$ by querying the Key Extraction oracle. \mathcal{C} unsigncrypts C using $D_{\mathbb{R}}$ and returns m if C is a valid ring signcryption from \mathcal{U} to $ID_{\mathbb{R}}$ else outputs “Invalid”.

\mathcal{A} queries the various oracles adaptively, i.e. the current oracle requests may depend on the response to the previous oracle queries.
3. **Challenge:** \mathcal{A} chooses two plaintexts $\{m_0, m_1\} \in \mathcal{M}$ of equal length, a set of n users \mathcal{U} and a receiver identity $ID_{\mathbb{R}}$ and sends them to \mathcal{C} . \mathcal{A} should not have queried the private key corresponding to $ID_{\mathbb{R}}$ in the first phase. \mathcal{C} now chooses a bit $\delta \in_{\mathbb{R}} \{0, 1\}$ and computes the challenge ring signcryption C^* of m_{δ} , and sends C^* to \mathcal{A} .
4. **Second Phase:** \mathcal{A} performs polynomially bounded number of requests just like the first phase, with the restrictions that \mathcal{A} cannot make Key Extraction query on $ID_{\mathbb{R}}$ and should not query for unsigncryption query on C^* . It should be noted that $ID_{\mathbb{R}}$ can be included as a ring member in \mathcal{U} , but \mathcal{A} cannot query the private key of $ID_{\mathbb{R}}$.
5. **Guess:** Finally, \mathcal{A} produces a bit δ' and wins the game if $\delta' = \delta$. The success probability is defined by:

$$\text{Succ}_{\mathcal{A}}^{\text{IND-IRSC-CCA2}}(\kappa) = \frac{1}{2} + \epsilon$$

Here, ϵ is called the advantage for the adversary in the attack.

Note: The difference between the security model for confidentiality in [4] and our model is, we allow the adversary to access the private key of the ring members (selected by the adversary during the challenge phase) and restrict access to the private key of the receiver of the challenge ring signcryption. But in [4], the adversary is not allowed to access the private keys of the ring members and the receiver (of the challenge ring signcryption).

Definition 4. An identity based ring signcryption scheme (IRSC) is said to be existentially unforgeable against adaptive chosen messages attacks (EUF-IRSC-CMA) if no polynomially bounded adversary has a non-negligible advantage in the following game:

1. **Setup Phase:** The challenger runs the Setup algorithm with a security parameter κ and gives the system parameters to the adversary \mathcal{A} .
2. **Training Phase:** \mathcal{A} performs polynomially bounded number of queries as described in First Phase of definition 3. The queries may be adaptive, i.e. the current query may depend on the previous query responses.
3. **Existential Forgery:** Finally, \mathcal{A} produces a new triple $(\mathcal{U}, ID_{\mathbb{R}}, C)$ (i.e. a triple that was not produced by the signcryption oracle), where the private keys of the users in the group \mathcal{U} were not queried in the training phase. \mathcal{A} wins the game if the result of the Unsigncryption $(\mathcal{U}, ID_{\mathbb{R}}, C)$ is not “Invalid”, in other words C is a valid signcrypt of some message $m \in \mathcal{M}$. It should be noted that $ID_{\mathbb{R}}$ can also be member of the ring \mathcal{U} in that case the private key of $ID_{\mathbb{R}}$ should not be queried by \mathcal{A} .

Note: The difference between the security model for unforgeability in [4] and our model is, we do not allow the adversary to access the private key of the ring members (selected by the adversary during the generation of the forgery) and the adversary is given access to the private key of the receiver of the forged ring signcryption. But in [4], the adversary is not allowed to access the private keys of the ring members as well as the receiver (of the forged ring signcryption).

4 Attacks on Various Ring Signcryption Schemes

This section gives an overview of four identity based ring signcryption schemes and the attacks corresponding to them. First we consider Yu et al.’s [10] anonymous signcryption scheme, followed by Fagen Li et al.’s [5] authenticatable anonymous signcryption scheme, next we take up Lijun et al.’s [15] ring signcryption scheme and conclude this section with the review and attack on Zhu et al.’s [14] scheme.

4.1 Overview of Anonymous Signcryption (ASC) Scheme of Yu et al.

Yu et al.’s ASC scheme [10] consists of four algorithms namely: *Setup*, *KeyGen*, *Signcryption* and *Unsigncryption*, which we describe below.

1. **Setup**(κ, l): Here, κ and l are the security parameters.
 - (a) The PKG selects $\mathbb{G}_1, \mathbb{G}_2$ of same order q and a random generator P of \mathbb{G}_1 .
 - (b) Selects the master private key $s \in_R \mathbb{Z}_q^*$.
 - (c) The master public key is computed as $P_{pub} = sP$.
 - (d) Selects three strong public one-way hash functions: $H_1 : \{0, 1\}^* \rightarrow \mathbb{G}_1^*$, $H_2 : \mathbb{G}_2 \rightarrow \{0, 1\}^l$, $H_3 : \{0, 1\}^* \rightarrow \mathbb{Z}_q^*$.
 - (e) Selects an admissible pairing $\hat{e} : \mathbb{G}_1 \times \mathbb{G}_1 \rightarrow \mathbb{G}_2$.
 - (f) The public parameters of the scheme are given by $params = (\mathbb{G}_1, \mathbb{G}_2, \hat{e}, P, P_{pub}, H_1, H_2, H_3, q)$.
2. **KeyGen**(ID_i): Here, ID_i is the identity of the user \mathcal{U}_i . The PKG performs the following.
 - (a) The user public key is computed as $Q_i = H_1(ID_i)$
 - (b) The corresponding private key $D_i = sQ_i$.
 - (c) The PKG sends D_i to the user \mathcal{U}_i via a secure channel.
3. **Signcryption**($\mathcal{U}, m, ID_{\mathbb{R}}, ID_{\mathbb{S}}, D_{\mathbb{S}}$): In order to signcrypt a message m , the sender does the following:
 - (a) Chooses $r \in_R \mathbb{Z}_q^*$ and, computes $R = rP$, $R' = \hat{e}(P_{pub}, Q_{\mathbb{R}})^r$, $t = H_2(R')$ and $c = m \oplus t$.
 - (b) For all $i = 1$ to n and $i \neq \mathbb{S}$, chooses $U_i \in_R \mathbb{G}_1$ and computes $h_i = H_3(m, t, \mathcal{U}, U_i)$.
 - (c) For $i = \mathbb{S}$ chooses $r'_{\mathbb{S}} \in_R \mathbb{Z}_q^*$ and, computes $U_{\mathbb{S}} = r'_{\mathbb{S}}Q_{\mathbb{S}} - \sum_{i=1, i \neq \mathbb{S}}^n (U_i + h_i Q_i)$, $h_{\mathbb{S}} = H_3(m, t, \mathcal{U}, U_{\mathbb{S}})$ and $V = (h_{\mathbb{S}} + r'_{\mathbb{S}})D_{\mathbb{S}}$.
 Finally, the sender outputs the ring signcryption $C = (\mathcal{U}, c, R, h_1, \dots, h_n, U_1, \dots, U_n, V)$.
4. **Unsigncrypt**($C = (\mathcal{U}, c, R, h_1, \dots, h_n, U_1, \dots, U_n, V), D_{\mathbb{R}}$): In order to unsigncrypt a ring signcryption C , the receiver does the following:
 - (a) Computes $t' = H_2(\hat{e}(R, D_{\mathbb{R}}))$ and $m' = c \oplus t'$.
 - (b) For $i = 1$ to n , checks whether $h'_i \stackrel{?}{=} H_3(m', t', \mathcal{U}, U_i)$.
 - (c) Checks whether $\hat{e}(P_{pub}, \sum_{i=1}^n (U_i + h'_i Q_i)) \stackrel{?}{=} \hat{e}(P, V)$.
 If all the n checks in (b) and the check in (c) are true, then output m' as the message, else output "Invalid".

Attack on ASC Scheme of Yu et al.: During the challenge phase of the confidentiality game, the challenger \mathcal{C} receives two messages m_0 and m_1 from the adversary \mathcal{A} . The challenger chooses $\delta \in_R \{0, 1\}$ and produces the challenge ring signcryption C^* using the message m_{δ} and delivers C^* to \mathcal{A} . Upon receipt of $C^* = (\mathcal{U}, c^*, R^*, h_1^*, \dots, h_n^*, U_1^*, \dots, U_n^*, V^*)$, \mathcal{A} does the following to check whether C^* is a signcryption of m_0 or m_1 . (Since \mathcal{A} knows both messages m_0 and m_1 , \mathcal{A} can perform the following computations.)

- Computes $t^* = c^* \oplus m_0$ and checks whether $h_i \stackrel{?}{=} H_3(m_0, t^*, \mathcal{U}, U_i^*)$, for $i = 1$ to n . If all the n checks hold, then C^* is the ring signcryption corresponding to m_0 .

- If the above check does not hold, \mathcal{A} computes $t^* = c^* \oplus m_1$, checks whether $h_i \stackrel{?}{=} H_3(m_1, t^*, \mathcal{U}, U_i^*)$, for $i = 1$ to n . If all the n checks hold then C^* is a valid ring signcryption for message m_1 .
- At least one of the checks should hold *true*, else C^* is an invalid ring signcryption.

Thus, \mathcal{A} distinguishes the ring signcryption without solving any hard problem. Here \mathcal{A} does not interact with the challenger \mathcal{C} after receiving the challenge ring signcryption C^* . Thus, our attack is indeed against the CPA security of the ASC scheme by Yu et al. reported in [10].

Remark: Informally, \mathcal{A} is able to distinguish the ring signcryption because, the key component required to evaluate the hash value h_i is t' and it is available in $c = m_b \oplus t'$. \mathcal{A} knows that m_δ is either m_0 or m_1 because m_0 and m_1 were chosen by \mathcal{A} and submitted to \mathcal{C} during the challenge phase by \mathcal{A} . Hence, \mathcal{A} can find t' without having access to the private key of the receiver and this led to the break in confidentiality (CPA).

4.2 Overview of Authenticatable Anonymous Signcryption Scheme (AASC) of Fagen Li et al.

The AASC scheme of Fagen Li et al. [5] consists of the five algorithms. A secure symmetric key encryption scheme (E, D) is employed in this scheme where, E and D are the secure symmetric key encryption and decryption algorithms respectively.

1. **Setup**(κ): Here, κ is the security parameter.
 - (a) The PKG selects $\mathbb{G}_1, \mathbb{G}_2$ of same order q and a random generator P of \mathbb{G}_1 .
 - (b) Selects the master private key $s \in_R \mathbb{Z}_q^*$.
 - (c) Computes the master public key $P_{pub} = sP$.
 - (d) Selects three strong public one-way hash functions $H_1 : \{0, 1\}^* \rightarrow \mathbb{G}_1$, $H_2 : \mathbb{G}_2 \rightarrow \{0, 1\}^l$, $H_3 : \{0, 1\}^* \rightarrow \mathbb{Z}_q^*$.
 - (e) Selects an admissible pairing $\hat{e} : \mathbb{G}_1 \times \mathbb{G}_1 \rightarrow \mathbb{G}_2$ and a secure symmetric key encryption system (E, D) .
 - (f) The public parameters of the scheme are set to be $params = (\mathbb{G}_1, \mathbb{G}_2, \hat{e}, P, P_{pub}, H_1, H_2, H_3, E, D)$.
2. **Extract**(ID_i): Similar to the **Extract**(ID_i) algorithm in 4.1.
3. **Signcrypt**($\mathcal{U}, m, ID_{\mathbb{R}}, ID_{\mathbb{S}}, D_{\mathbb{S}}$): In order to signcrypt a message m , the sender does the following:
 - (a) Chooses $r \in_R \mathbb{Z}_q^*$, and computes $R = rP$, $k = H_2(\hat{e}(P_{pub}, R)^r)$ and $c = E_k(m)$.
 - (b) For $i = 1$ to n , $i \neq \mathbb{S}$, chooses $a_i \in_R \mathbb{Z}_q^*$, computes $U_i = a_i P$ and $h_i = H_3(c, \mathcal{U}, U_i)$.
 - (c) For $i = \mathbb{S}$, chooses $a_{\mathbb{S}} \in_R \mathbb{Z}_q^*$, computes $U_{\mathbb{S}} = a_{\mathbb{S}} P_{\mathbb{S}} - \sum_{i=1, i \neq \mathbb{S}}^n (U_i + h_i Q_i)$.
 - (d) Computes $h_{\mathbb{S}} = H_3(c, \mathcal{U}, U_{\mathbb{S}})$ and $\sigma = (h_{\mathbb{S}} + a_{\mathbb{S}}) D_{\mathbb{S}}$.

Finally, the sender outputs the ring signcryption as $C = (\mathcal{U}, c, R, U_1, \dots, U_n, \sigma)$.

4. **Unsigncrypt**($C = (\mathcal{U}, c, R, U_1, \dots, U_n, \sigma), D_{\mathbb{R}}$): To unsigncrypt C , the receiver does the following.
 - (a) Computes $k' = H_2(\hat{e}(R, D_{\mathbb{R}}))$ and recover $m' = D_{k'}(c)$.
 - (b) For $i = 1$ to n , computes $h'_i = H_3(c, \mathcal{U}, U_i)$.
 - (c) Accepts C and the message m' if and only if $\hat{e}(P_{pub}, \sum_{i=1}^n (U_i + h'_i Q_i)) \stackrel{?}{=} \hat{e}(P, \sigma)$, else output “Invalid”.
5. **Authenticate**(C): The actual sender $ID_{\mathbb{S}}$ can prove that the message m was indeed signcrypted by him by running this protocol.
 - (a) The sender chooses $x \in_R \mathbb{Z}_q^*$, computes $\mu = \hat{e}(P, \sigma)^x$ and sends μ to the verifier.
 - (b) The verifier chooses $y \in_R \mathbb{Z}_q^*$ and sends it to the sender.
 - (c) The sender computes $v = (x + y)(h_{\mathbb{S}} + a_{\mathbb{S}})$ and returns v to the verifier.
 - (d) The verifier checks whether $\hat{e}(P_{pub}, Q_{\mathbb{S}})^v \stackrel{?}{=} \mu \cdot \hat{e}(P, \sigma)^y$ and accepts if the check holds.

Attack on AASC Scheme of Fagen Li et al.: The attack on AASC scheme is quite tricky one and it shows that the model considered by the authors did not address explicitly the scenario of the attack we propose. On receiving the challenge ring signcryption $C^* = (\mathcal{U}^*, c^*, R^*, U_1^*, \dots, U_n^*, \sigma^*)$, in the challenge phase of the confidentiality game, \mathcal{A} can find the message used for generating C^* . \mathcal{A} knows the private keys of all the users except the receiver $ID_{\mathbb{R}}$ and the members of \mathcal{U}^* (here, \mathcal{U}^* is the group of ad-hoc members in the challenge ring signcryption C^*). Now, \mathcal{A} chooses $\mathcal{U}_E \notin \mathcal{U}^*$ with identity string ID_E for which \mathcal{A} knows the private key D_E . \mathcal{A} performs the following steps to distinguish C^* as, whether it is a signcryption of m_0 or m_1 , during the second phase of oracle queries by performing the following.

- \mathcal{A} forms a new group with η users who are totally different from \mathcal{U}^* . Let the new group be $\mathcal{U}' = \{\mathcal{U}'_1, \dots, \mathcal{U}'_{\eta}\}$, where $\mathcal{U}'_E \in \mathcal{U}'$ and $\mathcal{U}' \neq \mathcal{U}^*$.
- For $i = 1$ to η , $i \neq E$, \mathcal{A} chooses $a_i \in_R \mathbb{Z}_q^*$, computes $U'_i = a_i P$ and $h'_i = H_3(c^*, \mathcal{U}', U'_i)$.
- For $i = E$, \mathcal{A} chooses $a_E \in_R \mathbb{Z}_q^*$, computes $U'_E = a_E Q_E - \sum_{i=1, i \neq E}^{\eta} (U'_i + h'_i Q_i)$.
- \mathcal{A} computes $h'_E = H_3(c^*, \mathcal{U}', U'_E)$ and $\sigma' = (h'_E + a_E) D_E$.
- Now, $C' = (\mathcal{U}', c^*, R^*, U'_1, \dots, U'_{\eta}, \sigma')$ is also a valid ring signcryption on the same message m_{δ} , which was used by \mathcal{C} to generate C^* and C' is entirely different from C^* , since $\mathcal{U}' \neq \mathcal{U}^*$. Thus, \mathcal{A} can legally query the unsigncryption of C' during the second phase of the confidentiality game.
- \mathcal{A} gets the unsigncryption to C' from \mathcal{C} as the message m_b and from this \mathcal{A} concludes correctly whether C^* is the signcryption of m_0 or m_1 .

Distinguishing the ring signcryption after the start of the second phase of interaction and a decryption query leads to a break in CCA2 security of the system. Thus, we claim that the AASC scheme by Fagen Li et al. [5] is not adaptive chosen ciphertext secure.

Remark: In this scheme, ring signcryption is achieved by using the *Encrypt-then-Sign* paradigm, where the signature part is a ring signature algorithm. This scheme lacks the binding between the encryption and signature; any adversary can alter the signature component of any ring signcryption and with the same receiver, i.e., the output of the encryption is alone used as input to for signature generation. This facilitates the adversary to generate a new valid signature and use it with the remaining components of the challenge ring signcryption, which forms a totally different valid ring signcryption. Now, the adversary can make use of the unsigncryption oracle to unsigncrypt the newly formed ring signcryption. Note that since the encryption part is same as the challenge ring signcryption and the signature part is varied, the newly formed ring signcryption yields the same message as in the challenge ring signcryption and this query is legal with respect to the security model..

4.3 Overview of Identity Based Ring Signcryption (IRSC) Scheme of Lijun et al.

The IRSC scheme of Lijun et al. [15] consists of the following four algorithms.

1. **Setup**(κ): Here, κ is the security parameters.
 - (a) The PKG selects $\mathbb{G}_1, \mathbb{G}_2$ of same prime order - q and a random generator P of \mathbb{G}_1 .
 - (b) Selects the master private key $s \in_R \mathbb{Z}_q^*$.
 - (c) The master public key is set to be $P_{pub} = sP$.
 - (d) Selects three cryptographic hash functions $H_1 : \{0, 1\}^* \rightarrow \mathbb{G}_1, H_2 : \{0, 1\}^* \rightarrow \mathbb{Z}_q^*, H_3 : \{0, 1\}^* \rightarrow \mathbb{Z}_q^*$.
 - (e) Selects an admissible pairing $\hat{e} : \mathbb{G}_1 \times \mathbb{G}_1 \rightarrow \mathbb{G}_2$.
 - (f) The public parameters of the scheme are set to be $params = (\mathbb{G}_1, \mathbb{G}_2, \hat{e}, P, P_{pub}, H_1, H_2, H_3, q)$.
2. **KeyGen**(ID_i): Similar to the **Extract**(ID_i) algorithm in 4.1.
3. **Signcrypt**($\mathcal{U}, m, ID_{\mathbb{R}}, ID_{\mathbb{S}}, D_{\mathbb{S}}$): In order to signcrypt the message m the sender does the following:
 - (a) Chooses $r_0 \in_R \mathbb{Z}_q^*$ and computes $R_0 = r_0P, W = r_0P_{pub}$.
 - (b) For $i = 1$ to $n, i \neq \mathbb{S}$, chooses $r_i \in_R \mathbb{Z}_q^*$, computes $R_i = r_iP, h_i = H_2(m || \mathcal{U} || R_i || R_0)$.
 - (c) For $i = \mathbb{S}$, chooses $r_{\mathbb{S}} \in_R \mathbb{Z}_q^*$, computes $R_{\mathbb{S}} = r_{\mathbb{S}}P - \sum_{i=1, i \neq \mathbb{S}}^n (h_i Q_i), h_{\mathbb{S}} = H_2(m || \mathcal{U} || R_{\mathbb{S}} || R_0)$ and $V = h_{\mathbb{S}}D_{\mathbb{S}} + \sum_{i=1}^n r_i P_{pub}$.
 - (d) Computes $y = \hat{e}(W, Q_{\mathbb{R}}), t = H_3(y), c = m \oplus t$.
 Finally the sender outputs the ciphertext as $C = (\mathcal{U}, c, V, R_0, R_1, \dots, R_n)$.
4. **Unsigncrypt**($C = (\mathcal{U}, c, V, R_0, R_1, \dots, R_n), D_{\mathbb{R}}$): In-order to unsigncrypt C , the receiver does the following.

- (a) Computes $t' = H_3(\hat{e}(D_{\mathbb{R}}, R_0))$ and recovers $m' = c \oplus t'$.
 - (b) For $i = 1$ to n , computes $h'_i = H_2(m \| \mathcal{U} \| R_i \| R_0)$.
 - (c) Checks whether $\hat{e}(P_{pub}, \Sigma_{i=1}^n (R_i + h'_i Q_i)) \stackrel{?}{=} \hat{e}(P, V)$.
- If all the n checks in (b) and the check in (c) are true, then output m' as the message, else output “Invalid”.

Attack on IRSC Scheme of Lijun et al.: During the challenge phase of the confidentiality game, the challenger \mathcal{C} receives two messages m_0 and m_1 from the adversary \mathcal{A} . The challenger chooses $\delta \in_R \{0, 1\}$ and generates the challenge ring signcryption C^* using the message m_δ and delivers C^* to \mathcal{A} . Upon receipt of $C^* = (\mathcal{U}, c^*, V^*, R_0^*, R_1^*, \dots, R_n^*)$, \mathcal{A} does the following to distinguish C^* as, whether C^* is the signcryption of m_0 or m_1 . Since \mathcal{A} knows both messages m_0 and m_1 , \mathcal{A} can perform the following computations.

- \mathcal{A} can compute $h_i = H_2(m_0 \| \mathcal{U} \| R_i^* \| R_0^*)$ for $i = 1$ to n . (since R_i^*, R_0^* are known from the ring signcryption C^*).
- Checks whether $\hat{e}(P_{pub}, \Sigma_{i=1}^n (R_i^* + h_i Q_i)) \stackrel{?}{=} \hat{e}(P, V^*)$. If this check holds, then C^* is a valid ring signcryption of m_0 .
- If the above check does not hold, perform the previous two steps with m_0 replaced by m_1 . If the ring signcryption was formed with one of the two messages m_0 or m_1 , any one of the above checks will hold, else the ring signcryption C^* is an invalid one.

Thus, \mathcal{A} can distinguish the challenge signcryption without knowing the key of the receiver in the challenge ring signcryption C^* .

Remark: The intuition behind the attack is, in the ring signcryption proposed by Lijun et al. [15] the ring signcryption can be verified if the message and the corresponding ring signcryption is known. During the confidentiality game the adversary \mathcal{A} knows the message, which is either m_0 or m_1 , with these information \mathcal{A} concludes whether C^* is a ring signcryption of m_0 or m_1 .

4.4 Overview of IRSC Scheme of Zhu et al.

The IRSC scheme of Zhu et al. [14] consists of the following four algorithms.

1. **Setup**(κ, l): Here, κ and l are the security parameters.
 - (a) The PKG selects $\mathbb{G}_1, \mathbb{G}_2$ of same order q and a random generator P of \mathbb{G}_1 .
 - (b) Selects the master private key $s \in_R \mathbb{Z}_q^*$ and computes the master public key to be $P_{pub} = sP$.
 - (c) Selects four cryptographic hash functions $H_1 : \{0, 1\}^* \rightarrow \mathbb{G}_1^*, H_2 : \mathbb{G}_1^* \rightarrow \{0, 1\}^l, H_3 : \{0, 1\}^l \times \mathbb{G}_1 \rightarrow \{0, 1\}^l, H_4 : \{0, 1\}^* \rightarrow \mathbb{Z}_q^*$.
 - (d) Selects an admissible pairing $\hat{e} : \mathbb{G}_1 \times \mathbb{G}_1 \rightarrow \mathbb{G}_2$.
 - (e) The public parameters of the scheme are set to be $params = (\mathbb{G}_1, \mathbb{G}_2, \hat{e}, P, P_{pub}, H_1, H_2, H_3, H_4, q)$.

2. **KeyGen** (ID_i): Similar to the **Extract** (ID_i) algorithm in 4.1.
3. **Signcrypt** ($\mathcal{U}, m, ID_{\mathbb{R}}, ID_{\mathbb{S}}, D_{\mathbb{S}}$): In order to signcrypt the message m , the sender does the following:
 - (a) Chooses $r \in_R \mathbb{Z}_q^*$, $\hat{m} \in_R \mathcal{M}$ and, computes $R_0 = rP$, $R' = \hat{e}(rP_{pub}, Q_{\mathbb{R}})$, $k = H_2(R')$, $c_1 = \hat{m} \oplus k$ and $c_2 = m \oplus H_3(\hat{m}||R_0)$.
 - (b) For $i = 1$ to n , $i \neq \mathbb{S}$, chooses $U_i \in_R \mathbb{G}_1^*$ and computes $h_i = H_4(c_2||U_i)$.
 - (c) For $i = \mathbb{S}$, chooses $r' \in_R \mathbb{Z}_q^*$, computes $U_{\mathbb{S}} = r'Q_{\mathbb{S}} - \sum_{i=1, i \neq \mathbb{S}}^n (U_i + h_i Q_i)$, $h_{\mathbb{S}} = H_4(c_2||U_{\mathbb{S}})$ and $V = (h_{\mathbb{S}} + r')D_{\mathbb{S}}$.
 Finally, outputs the ring signcryption as $C = (\mathcal{U}, R_0, c_1, c_2, U_1, \dots, U_n, V)$.
4. **Unsigncrypt** ($C = (\mathcal{U}, R_0, c_1, c_2, U_1, \dots, U_n, V)$, $D_{\mathbb{R}}$): To unsigncrypt a ring signcryption C , the receiver does the following.
 - (a) For $i = 1$ to n , computes $h'_i = H_4(c_2||U_i)$.
 - (b) Checks whether $\hat{e}(P_{pub}, \sum_{i=1}^n (U_i + h'_i Q_i)) \stackrel{?}{=} \hat{e}(P, V)$, if so, computes $k' = H_2(\hat{e}(R_0, D_{\mathbb{R}}))$, and recovers $\hat{m}' = c_1 \oplus k'$ and $m' = c_2 \oplus H_3(\hat{m}'||R_0)$. Accept m' as the valid message.

Note: The actual scheme in [11] had typos in setup, keygen as well as signcryption algorithms. The definition of the hash function H_3 was inconsistent. Instead, of H_2 , it was written H_1 , instead of H_1 , it was written H_0 and instead of $U_{\mathbb{S}} = r'Q_{\mathbb{S}} - \sum_{i=1, i \neq \mathbb{S}}^n (U_i + h_i Q_i)$, it was written $U_{\mathbb{S}} = r'Q_{\mathbb{S}} - \sum_{i=1, i \neq \mathbb{S}}^n (U_i + h_i Q_{\mathbb{S}})$. We have corrected all of them in our review, in order to maintain the consistency of the scheme.

Attack on IRSC Scheme of Zhu et al.: On receiving the challenge ring signcryption $C^* = (\mathcal{U}^*, R_0^*, c_1^*, c_2^*, U_1^*, \dots, U_n^*, V^*)$, in the challenge phase of the confidentiality game, \mathcal{A} can find the message used for generating C^* . \mathcal{A} knows the private keys of all the users except the receiver $ID_{\mathbb{R}}$ and the members of \mathcal{U}^* (here, \mathcal{U}^* is the group of ad-hoc members in the challenge ring signcryption C^*). Now, \mathcal{A} chooses $U'_E \notin \mathcal{U}^*$ with identity string ID_E for which \mathcal{A} knows the private key D_E . \mathcal{A} performs the following steps to distinguish C^* as, whether it is a signcryption of m_0 or m_1 , during the second phase of oracle queries by performing the following.

- \mathcal{A} forms a new group \mathcal{U}' with η members who are totally different from the users in \mathcal{U}^* present in the challenge ring signcryption. Consider $\mathcal{U}' = \{U'_1, \dots, U'_\eta\}$ and $U'_E \in \mathcal{U}'$ (The private key of U'_E is known to \mathcal{A}).
- Chooses a message m' and computes $c'_2 = c_2^* \oplus m'$.
- For all $i = 1$ to η and $i \neq E$, chooses $U'_i \in_R \mathbb{G}_1^*$ and computes $h'_i = H_4(c'_2||U'_i)$.
- For $i = E$, chooses $r' \in_R \mathbb{Z}_q^*$ and computes $U'_E = r'Q_A - \sum_{i=1}^{\eta} (U'_i + h'_i Q_i)$.
- Computes $h'_E = H_4(c'_2||U'_E)$ and $V' = (r' + h'_E)D_E$.
- Now, $C' = (\mathcal{U}', R_0^*, c_1^*, c'_2, U'_1, \dots, U'_n, V')$ is a valid ring signcryption on message $m_b \oplus m'$.

Now, during the second phase of training, \mathcal{A} requests the unsigncryption of C' to \mathcal{C} . Note that it is legal for \mathcal{A} to ask for unsigncryption of C' because it is derived from C^* and not exactly the challenge ring signcryption C^* . \mathcal{C} responds with $M = m_b \oplus m'$ as the output for the query. \mathcal{A} now obtains $m_b = M \oplus m'$ and thus identifies the message in the challenge ring signcryption C^* .

Remark: This attack is possible due to the same reason as described in the remark for the attack stated in section 4.2.

5 New Ring Signcryption Scheme (New-IBRSC)

In this section, we present a new improved identity based ring signcryption scheme (New-IBRSC), taking into account the attacks carried out in the previous section. New-IBRSC consists of the following four algorithms:

1. **Setup**(κ): This algorithm is executed by the PKG to initialize the system by taking a security parameter κ as input.
 - Selects \mathbb{G}_1 an additive group and \mathbb{G}_2 a multiplicative group, both cyclic with same prime order - q and a random generator P of the group \mathbb{G}_1 .
 - Selects $s \in_R \mathbb{Z}_q^*$ as the master private key and computes the master public key $P_{pub} = sP$.
 - Selects four cryptographic hash functions $H_1 : \{0, 1\}^* \rightarrow \mathbb{G}_1$, $H_2 : \mathbb{G}_2 \rightarrow \{0, 1\}^{|\mathcal{M}|} \times \mathbb{Z}_q^* \times \mathbb{G}_1$, $H_3 : \{0, 1\}^* \rightarrow \mathbb{Z}_q^*$ and $H_4 : \{0, 1\}^{|\mathcal{M}|} \times \mathbb{Z}_q^* \rightarrow \mathbb{Z}_q^*$.
 - Picks a bilinear pairing $\hat{e} : \mathbb{G}_1 \times \mathbb{G}_1 \rightarrow \mathbb{G}_2$ with the appropriate properties specified in section 2.
 - The public parameter of the scheme is $params = (\mathbb{G}_1, \mathbb{G}_2, \hat{e}, P, P_{pub}, H_1, H_2, H_3, H_4, q)$.
2. **Keygen**(ID_i): This algorithm takes ID_i , the identity of a user \mathcal{U}_i as input. The PKG who executes this algorithm computes the private key and public key for the user with identity ID_i as follows:
 - The public key is computed as $Q_i = H_1(ID_i)$
 - The corresponding private key $D_i = sQ_i$.
 - PKG sends D_i to user \mathcal{U}_i via a secure channel.
3. **Signcrypt**($\mathcal{U}, m, ID_{\mathbb{R}}, Q_{\mathbb{R}}, ID_{\mathbb{S}}, D_{\mathbb{S}}$): For signcrypting a message m to the receiver $\mathcal{U}_{\mathbb{R}}$ with public key $Q_{\mathbb{R}}$ the sender with private key $D_{\mathbb{S}}$ and public key $Q_{\mathbb{S}}$ performs the following:
 - Selects n potential senders and forms an ad-hoc group \mathcal{U} , including its own identity $ID_{\mathbb{S}}$.
 - Chooses $w \in_R \mathbb{Z}_q^*$, computes $r = H_4(m, w)$, $U = rP$ and $\alpha = \hat{e}(P_{pub}, Q_{\mathbb{R}})^r$.
 - For $i = 1$ to n , $i \neq \mathbb{S}$, chooses $U_i \in_R \mathbb{G}_1$ and computes $h_i = H_3(m, U_i, \alpha, \mathcal{U}, Q_{\mathbb{R}})$.
 - For $i = \mathbb{S}$, chooses $r_{\mathbb{S}} \in_R \mathbb{Z}_q^*$ and, computes $U_{\mathbb{S}} = r_{\mathbb{S}}Q_{\mathbb{S}} - \sum_{i=1, i \neq \mathbb{S}}^n (U_i + h_i Q_i)$, $h_{\mathbb{S}} = H_3(m, U_{\mathbb{S}}, \alpha, \mathcal{U}, Q_{\mathbb{R}})$ and $V = (h_{\mathbb{S}} + r_{\mathbb{S}})D_{\mathbb{S}}$.
 - Computes $y = (m || w || V) \oplus H_2(\alpha)$.

Finally, the sender outputs the ring signcryption $C = (y, \mathcal{U}, U, U_1, \dots, U_n)$.

4. **Unsigncrypt**($C = (y, \mathcal{U}, U, U_1, \dots, U_n), D_{\mathbb{R}}$): The receiver $\mathcal{U}_{\mathbb{R}}$ with identity $ID_{\mathbb{R}}$ does the following to unsigncrypt the ring signcryption C :
 - Computes $\alpha' = \hat{e}(U, D_{\mathbb{R}})$, retrieves m' , w' and V' as $(m' \| w' \| V') = y \oplus H_2(\alpha')$.
 - The receiver checks whether $U \stackrel{?}{=} H_4(m', w')P$.
 - For $i = 1$ to n , computes $h'_i = H_3(m', U_i, \alpha', \mathcal{U}, Q_{\mathbb{R}})$ and checks whether $\hat{e}(P_{pub}, \Sigma_{i=1}^n (U_i + h'_i Q_i)) \stackrel{?}{=} \hat{e}(P, V')$.
 If all the above checks hold, then the receiver $\mathcal{U}_{\mathbb{R}}$ accepts C as the valid ring signcryption and the message m' as the valid message. If any one of them fail, the receiver returns “Invalid”.

Correctness: We show the correctness of the unsigncryption algorithm here. From the definition of $U_{\mathbb{S}}$ given in the signcryption algorithm, $\Sigma_{i=1}^n (U_i + h'_i Q_i) = r_{\mathbb{S}} Q_{\mathbb{S}} + h'_{\mathbb{S}} Q_{\mathbb{S}}$. Thus,

$$\begin{aligned}
 \text{LHS} &= \hat{e}(P_{pub}, \Sigma_{i=1}^n (U_i + h'_i Q_i)) \\
 &= \hat{e}(sP, r_{\mathbb{S}} Q_{\mathbb{S}} + h'_{\mathbb{S}} Q_{\mathbb{S}}) \\
 &= \hat{e}(P, (r_{\mathbb{S}} + h'_{\mathbb{S}})sQ_{\mathbb{S}}) \\
 &= \hat{e}(P, V') = \text{RHS}
 \end{aligned}$$

Note that the above correctness holds only if $h_i = h'_i$ for $(i = 1 \text{ to } n)$.

6 Security Results for New-IBRSC:

New-IBRSC can be viewed as a signcryption scheme with the signature replaced by the ring signature given in [2]. This composition does not induce any weakness in the anonymity property of the ring signature. The difference between the ring signature in [2] and New-IBRSC is the definition of the hash function H_3 , which is used to compute h_i , for $i = 1$ to n . In New-IBRSC, the two additional components are α and $Q_{\mathbb{R}}$, where α is the session key established and $Q_{\mathbb{R}}$ is the public key of the receiver. The value α is computed as $\hat{e}(P_{pub}, Q_{\mathbb{R}})^r$, which does not provide any clue regarding the sender. Addition of α and $Q_{\mathbb{R}}$ to the hash function H_3 does not reveal any information regarding the identity of the sender. Hence the anonymity proof of New-IBRSC follows from the underlying identity based ring signature [2]. Therefore, we concentrate only on the security against adaptive chosen ciphertext attack (CCA2) and security against chosen message attack (CMA). We formally prove the security of the new identity based ring signcryption scheme (New-IBRSC), indistinguishable under chosen ciphertext attack (IND-New-IBRSC-CCA2) and existentially unforgeable under chosen message and identity attack (EUF-New-IBRSC-CMA) in the random oracle model. We consider the security model given in section 3 to prove the security of the New-IBRSC.

6.1 Confidentiality Proof of New-IBRSC (IND-IBRSC-CCA2):

Theorem 1. *If an IND-IBRSC-CCA2 adversary \mathcal{A} has an advantage ϵ against New-IBRSC scheme, asking q_{H_i} ($i = 1, 2, 3, 4$) hash queries to random oracles*

\mathcal{O}_{H_i} ($i = 1, 2, 3, 4$), q_e extract queries ($q_e = q_{e_1} + q_{e_2}$, where q_{e_1} and q_{e_2} are the number of extract queries in the first phase and second phase respectively), q_{sc} signcryption queries and q_{us} unsigncryption queries, then there exist an algorithm \mathcal{C} that solves the CBDH problem with advantage $\epsilon \left(\frac{1}{q_{H_1} q_{H_2}} \right)$.

Proof: The challenger \mathcal{C} is challenged to solve an instance (P, aP, bP, cP) of the CBDHP. Assume that there is an adversary \mathcal{A} capable of breaking the IND-IBRSC-CCA2 security of New-IBRSC with non-negligible advantage. \mathcal{C} makes use of \mathcal{A} to solve the CBDHP instance. \mathcal{C} simulates the system with the various oracles \mathcal{O}_{H_1} , \mathcal{O}_{H_2} , \mathcal{O}_{H_3} , $\mathcal{O}_{Signcryption}$, $\mathcal{O}_{Unsigncryption}$ and allows \mathcal{A} to make polynomially bounded number of queries, adaptively to these oracles. The game between \mathcal{C} and \mathcal{A} is demonstrated below:

Setup Phase: \mathcal{C} simulates the system by setting up the system parameters in the following way.

- \mathcal{C} chooses the groups \mathbb{G}_1 and \mathbb{G}_2 and the generator $P \in \mathbb{G}_1$ as given in CBDHP instance.
- Sets the master public key $P_{pub} = aP$, here \mathcal{C} does not know a . \mathcal{C} is using the aP value given in the instance of the CBDHP.
- Models the four hash functions as random oracles \mathcal{O}_{H_1} , \mathcal{O}_{H_2} , \mathcal{O}_{H_3} and \mathcal{O}_{H_4} .
- Selects a bilinear pairing $\hat{e} : \mathbb{G}_1 \times \mathbb{G}_1 \rightarrow \mathbb{G}_2$.
- Delivers $\langle \mathbb{G}_1, \mathbb{G}_2, \hat{e}, P, P_{pub} \rangle$ to \mathcal{A} .

First Phase: To handle the oracle queries, \mathcal{C} maintains four lists L_i , ($i = 1, 2, 3, 4$) which keeps track of the responses given by \mathcal{C} to the corresponding oracle (\mathcal{O}_{H_1} , \mathcal{O}_{H_2} , \mathcal{O}_{H_3} , \mathcal{O}_{H_4}) queries. \mathcal{A} adaptively (means that, the input to the current query may depend on the outputs obtained for the previous queries) queries the various oracles in the first phase, which are handled by \mathcal{C} as given below:

\mathcal{O}_{H_1} oracle Query: We will make a simplifying assumption that \mathcal{A} queries the \mathcal{O}_{H_1} oracle with distinct identities in each query. There is no loss of generality due to this assumption, because, if the same identity is repeated, by definition the oracle consults the list L_1 and gives the same response. Thus, we assume that \mathcal{A} asks q_{H_1} distinct queries for q_{H_1} distinct identities. Among this q_{H_1} identities, a random identity has to be selected as target identity and it is done as follows.

\mathcal{C} selects a random index γ , where $1 \leq \gamma \leq q_{H_1}$. \mathcal{C} does not reveal γ to \mathcal{A} . When \mathcal{A} generates the γ^{th} query on ID_γ , \mathcal{C} decides to fix ID_γ as target identity for the challenge phase. Moreover, \mathcal{C} responds to \mathcal{A} as follows:

- If it is the γ^{th} query, then \mathcal{C} sets $Q_\gamma = bP$, returns Q_γ as the response to the query and stores $\langle ID_\gamma, Q_\gamma, * \rangle$ in the list L_1 . Here, \mathcal{C} does not know b . \mathcal{C} is simply using the bP value given in the instance of the CBDHP.
- For all other queries, \mathcal{C} chooses $x_i \in_R Z_q^*$ and sets $Q_i = x_i P$ and stores $\langle ID_i, Q_i, x_i \rangle$ in the list L_1 .

\mathcal{C} returns Q_i to \mathcal{A} . (Note that as the identities are assumed to be distinct, for each query, we create distinct entry and add in the list L_1).

\mathcal{O}_{H_2} oracle Query: When \mathcal{A} makes a query to this oracle with α as input, \mathcal{C} retrieves h_2 from list L_2 and returns h_2 to \mathcal{A} ; else, chooses a new h_2 randomly, stores $\langle \alpha, h_2 \rangle$ in L_2 and returns h_2 to \mathcal{A} .

\mathcal{O}_{H_3} oracle Query: When \mathcal{A} makes a query to this oracle with $(m, U_i, \alpha, \mathcal{U}, Q_{\mathbb{R}})$ as input, \mathcal{C} retrieves $h_i^{(3)}$ from list L_3 and returns $h_i^{(3)}$ to \mathcal{A} ; else, chooses a new $h_i^{(3)} \in_R \mathbb{Z}_q^*$ randomly, stores $\langle m, U_i, \alpha, \mathcal{U}, Q_{\mathbb{R}}, h_i^{(3)} \rangle$ in the list L_3 and returns $h_i^{(3)}$ to \mathcal{A} .

\mathcal{O}_{H_4} oracle Query: When \mathcal{A} makes a query to this oracle with (m, w) as input, \mathcal{C} retrieves r from list L_4 and returns r to \mathcal{A} ; else, chooses $r \in_R \mathbb{Z}_q^*$, stores $\langle m, w, r \rangle$ in L_4 and returns r to \mathcal{A} .

Extract Query: On getting a request for the private key of user \mathcal{U}_i with identity ID_i , \mathcal{C} aborts if $ID_i = ID_{\gamma}$. Else, \mathcal{C} retrieves Q_i, x_i from list L_1 and returns $D_i = x_i aP = aQ_i$.

(**Note:** It is assumed throughout the confidentiality game, \mathcal{A} queries \mathcal{O}_{H_1} oracle with ID_i before querying other oracles with ID_i as input.)

$\mathcal{O}_{\text{Signcryption}}$ Query: \mathcal{A} chooses a message m , a set of n potential senders and forms an ad-hoc group \mathcal{U} by fixing a sender $ID_{\mathbb{S}}$ and a receiver $ID_{\mathbb{R}}$ and sends them to \mathcal{C} . To respond correctly to the signcryption query on the plaintext m chosen by \mathcal{A} , \mathcal{C} does the following:

\mathcal{C} proceeds according to the signcryption algorithm when $ID_{\mathbb{S}} \neq ID_{\gamma}$. This is possible for \mathcal{C} because \mathcal{C} knows the private key $D_{\mathbb{S}}$ of the sender $ID_{\mathbb{S}}$.

If the sender's identity $ID_{\mathbb{S}} = ID_{\gamma}$ (i.e. when \mathcal{C} does not know the private key corresponding to $ID_{\mathbb{S}}$), \mathcal{C} cooks up a response as explained below:

- Chooses $w \in_R \mathbb{Z}_q^*$, computes $r = H_4(m, w)$, $U = rP$ and $\alpha = \hat{e}(P_{\text{pub}}, Q_{\mathbb{R}})^r$.
 - For $i = 1$ to n , $i \neq \mathbb{S}$, chooses $U_i \in_R \mathbb{G}_1$ and queries the oracle \mathcal{O}_{H_3} and obtains the value $h_i^{(3)} = \mathcal{O}_{H_3}(m, U_i, \alpha, \mathcal{U}, Q_{\mathbb{R}})$.
 - For $i = \mathbb{S}$,
 - Chooses $r_{\mathbb{S}}, h_{\mathbb{S}}^{(3)} \in_R \mathbb{Z}_q^*$
 - Computes $U_{\mathbb{S}} = r_{\mathbb{S}}P - h_{\mathbb{S}}^{(3)}Q_{\mathbb{S}} - \sum_{i=1, i \neq \mathbb{S}}^n (U_i + h_i^{(3)}Q_i)$.
 - Adds the tuple $\langle m, U_{\mathbb{S}}, \alpha, \mathcal{U}, Q_{\mathbb{R}}, h_{\mathbb{S}}^{(3)} \rangle$ to the list L_3 .
 - Computes $V = r_{\mathbb{S}}P_{\text{pub}}$
- (**Note:** Here $h_{\mathbb{S}}^{(3)}$ is not computed by \mathcal{C} , instead it is chosen at random and set as the output for the random oracle query $h_{\mathbb{S}}^{(3)} = \mathcal{O}_{H_3}(m, U_{\mathbb{S}}, \alpha, \mathcal{U}, Q_{\mathbb{R}})$. This is possible because the random oracles are manipulated by \mathcal{C}).
- Queries $h^{(2)} = \mathcal{O}_{H_2}(\alpha)$ and computes $y = (m \| w \| V) \oplus h^{(2)}$

Finally, \mathcal{C} outputs the ring signcryption $C = (y, \mathcal{U}, U, U_1, \dots, U_n)$ to \mathcal{A} as the signcryption of m . The signcryption $C = (y, \mathcal{U}, U, U_1, \dots, U_n)$ is considered as valid by \mathcal{A} because C passes the verification tests as shown below:

From the definition of $U_{\mathbb{S}}, \sum_{i=1}^n (U_i + h_i^{(3)}Q_i) = r_{\mathbb{S}}P$. Thus,

$$\begin{aligned}
\hat{e}(P_{pub}, \Sigma_{i=1}^n (U_i + h'_i Q_i)) &= \hat{e}(aP, r_{\mathbb{S}} P) \\
&= \hat{e}(P, r_{\mathbb{S}} aP) \\
&= \hat{e}(P, r_{\mathbb{S}} P_{pub}) \\
&= \hat{e}(P, V')
\end{aligned}$$

$\mathcal{O}_{Unsigncryption}$ Query: Upon receiving an unsigncryption query on a ring signcryption $C = (y, \mathcal{U}, U, U_1, \dots, U_n)$ with $ID_{\mathbb{R}}$ as receiver, \mathcal{C} proceeds as follows:

\mathcal{C} proceeds as per the unsigncryption algorithm, when $ID_{\mathbb{R}} \neq ID_{\gamma}$. Here, \mathcal{C} can directly use the unsigncryption algorithm because, \mathcal{C} knows the private key $D_{\mathbb{R}}$ of the receiver $ID_{\mathbb{R}}$.

If the receiver identity $ID_{\mathbb{R}} = ID_{\gamma}$ (i.e. When \mathcal{C} does not know the private key corresponding to $ID_{\mathbb{R}}$), \mathcal{C} generates the response as explained below:

1. For a given signcryption $C = (y, \mathcal{U}, U, U_1, \dots, U_n)$, a pair (m, α) is said to be a potential pair if $\langle m, U_i, \alpha, \mathcal{U}, Q_{\mathbb{R}} \rangle \in L_3$ for all $i = 1$ to n . Let \overline{M} denote the set of all potential pairs for C .
2. For each pair $(m, \alpha) \in \overline{M}$, the challenger \mathcal{C} performs the following:
 - Retrieves m', w' and V' as $m' \| w' \| V' = y \oplus \mathcal{O}_{H_2}(\alpha)$.
 - Checks whether $m' \stackrel{?}{=} m$ and checks $\mathcal{O}_{H_4}(m', w')P \stackrel{?}{=} U$. If true, then \mathcal{C} obtains the value $h_i^{(3)'} = \mathcal{O}_{H_3}(m, U_i, \alpha, \mathcal{U}, Q_{\mathbb{R}})$, for $i = 1$ to n from the list L_3 and checks whether $\hat{e}(P_{pub}, \Sigma_{i=1}^n (U_i + h_i^{(3)'} Q_i)) \stackrel{?}{=} \hat{e}(P, V)$.
3. The first time when all checks in (2) passes, \mathcal{C} outputs the corresponding m' and halts.
4. If every $(m, \alpha) \in \overline{M}$ obtained in step (1) fails the checks in step (2), then \mathcal{C} outputs “Invalid” and halts.

Challenge Phase: Finally, \mathcal{A} chooses two equal length plaintexts $m_0, m_1 \in \mathcal{M}$, the set of ring members $\mathcal{U}^* = \{ID_i\}_{(i=1 \text{ to } n^*)}$, a sender identity $ID_{\mathbb{S}} \in \mathcal{U}^*$ and a receiver identity $ID_{\mathbb{R}}$ on which \mathcal{A} wants to be challenged and sends them to \mathcal{C} . \mathcal{A} should not have queried the private key corresponding to $ID_{\mathbb{R}}$ in the first phase. \mathcal{C} aborts, if $ID_{\mathbb{R}} \neq ID_{\gamma}$; else, \mathcal{C} chooses a bit $\delta \in_R \{0, 1\}$ and computes the challenge ring signcryption C^* of m_{δ} as follows :

- Sets $U^* = cP$. (Note that the challenger does not know c but uses the cP value available in the instance of CBDHP.)
- Chooses $\{U_i^*\}_{(i=1 \text{ to } n^*)}$ randomly from \mathbb{G}_1 and $y^* \in_R \{0, 1\}^{|\mathcal{M}|} \times \mathbb{Z}_Q^* \times \mathbb{G}_1$ and outputs $C^* = (y^*, \mathcal{U}^*, U^*, U_1^*, \dots, U_{n^*}^*)$.

(Note that C^* is not constructed according to the signcryption algorithm of the scheme but made up of random values.)

Second Phase: On getting the challenge ring signcryption C^* , \mathcal{A} is allowed to interact with \mathcal{C} as in the first phase. This time, \mathcal{A} is not given access to the private key of $ID_{\mathbb{R}}$ and is also restricted from querying the decryption oracle for the ring unsigncryption of C^* .

Guess: At the end of the second phase, \mathcal{A} returns its guess. \mathcal{C} ignores the answer from \mathcal{A} , picks a random tuple $\langle \alpha, h_2 \rangle$ from list L_2 and returns the corresponding α as the solution to the CBDHP instance. Since the challenge ciphertext C^* given to \mathcal{A} is randomly distributed in the ciphertext space, \mathcal{A} cannot gain any advantage in this simulation. Thus, any adversary that has advantage ϵ in the real IND-IBRSC-CCA2 game must necessarily recognize with probability at least ϵ that the challenge ciphertext provided by \mathcal{C} is incorrect. For \mathcal{A} to find that C^* is not a valid ciphertext, \mathcal{A} should have queried the \mathcal{O}_{H_2} oracle with $\alpha = \hat{e}(U^*, D_\gamma)$. Here D_γ is the private key of the target identity and it is $a(Q_\gamma) = abP$. Also, \mathcal{C} has set $U^* = cP$. Hence $\alpha = \hat{e}(U^*, D_\gamma) = \hat{e}(cP, abP) = \hat{e}(P, P)^{abc}$. Therefore, one of the entries in list L_2 should be the value $\hat{e}(P, P)^{abc}$. With probability $\frac{1}{q_{H_2}}$, the value of α chosen by \mathcal{C} from list L_2 will be the solution to CBDHP instance. Now, we assess the probability of success of \mathcal{C} . The events in which \mathcal{C} aborts the IND-IBRSC-CCA2 game are,

1. E_1 - when \mathcal{A} queries the private key of the target identity ID_γ and $Pr[E_1] = \frac{q_{e_1}}{q_{H_1}}$.
2. E_2 - when \mathcal{A} does not choose the target identity ID_γ as the receiver during the challenge and $Pr[E_2] = \left(1 - \frac{1}{q_{H_1} - q_{e_1}}\right)$.

The probability that, \mathcal{C} does not abort the IND-IBRSC-CCA2 game is given by

$$(Pr[\neg E_1 \wedge \neg E_2]) = \left(1 - \frac{q_{e_1}}{q_{H_1}}\right) \left(\frac{1}{q_{H_1} - q_{e_1}}\right) = \frac{1}{q_{H_1}}$$

The probability that, the α chosen randomly from L_2 by \mathcal{C} , being the solution to CBDHP is $\left(\frac{1}{q_{H_2}}\right)$. Therefore, the probability of \mathcal{C} solving CBDHP is given by,

$$Pr[\mathcal{C}(P, aP, bP, cP | a, b, c \in_R \mathbb{Z}_q^*) = \hat{e}(P, P)^{abc}] = \epsilon \left(\frac{1}{q_{H_1} q_{H_2}}\right)$$

Since ϵ is non-negligible, the probability of \mathcal{C} solving CBDHP is also non-negligible. \square

6.2 Unforgeability Proof of New-IBRSC (EUF-IBRSC-CMA):

Theorem 2. *If an EUF-IBRSC-CMA forger \mathcal{A} exists against New-IBRSC scheme, asking q_{H_i} ($i = 1, 2, 3, 4$) hash queries to random oracles H_i ($i = 1, 2, 3, 4$), q_e extract secret key queries, q_{sc} signcryption queries and q_{us} unsigncryption queries, then there exist an algorithm \mathcal{C} that solves the CDHP.*

Proof: The challenger \mathcal{C} is challenged to solve an instance of the CDHP. \mathcal{C} interacts with an adversary \mathcal{A} which is capable of breaking the EUF-IBRSC-CMA security of New-IBRSC, to solve the CDHP instance. On receiving the instance $\langle P, aP, bP \rangle \in \mathbb{G}_1^3$ of the CDHP as input, \mathcal{C} begins the interaction with

\mathcal{A} to compute the value $abP \in \mathbb{G}_1$. \mathcal{C} simulates the system with the various oracles $\mathcal{O}_{H_1}, \mathcal{O}_{H_2}, \mathcal{O}_{H_3}, \mathcal{O}_{H_4}, \mathcal{O}_{\text{Signcryption}}, \mathcal{O}_{\text{Unsigncryption}}$ and allows \mathcal{A} to adaptively ask polynomially bounded number of queries to these oracles.

Setup Phase: \mathcal{C} simulates the system by setting up the system parameters in the following way.

- It takes \mathbb{G}_1 and \mathbb{G}_2 , the two groups as well as the generator $P \in \mathbb{G}_1$ as given in the CDHP instance.
- Sets the master public key $P_{\text{pub}} = aP$.
- Models the four hash functions as random oracles $\mathcal{O}_{H_1}, \mathcal{O}_{H_2}, \mathcal{O}_{H_3}$ and \mathcal{O}_{H_4} .
- Selects a bilinear pairing $\hat{e} : \mathbb{G}_1 \times \mathbb{G}_1 \rightarrow \mathbb{G}_2$.

Training Phase: \mathcal{A} adaptively performs polynomially bounded number of queries to the various oracles in this phase. The queries may be *Hash Queries*, *Extract Queries*, *Signcryption Queries* and *Unsigncryption Queries* with no restrictions, which are handled by \mathcal{C} as in the confidentiality game for New-IBRSC.

Forgery: Finally, \mathcal{A} produces a forged signcryption $C^* = (y^*, \mathcal{U}^*, U^*, U_1^*, \dots, U_n^*)$ on the message m^* (i.e. C^* was not produced by the *Signcryption Oracle* as an output for the ring signcryption query on the message m^* with an ad-hoc set of users \mathcal{U}^* and the receiver $ID_{\mathbb{R}}$), where the private keys of the users who are in the group \mathcal{U}^* were not queried in the training phase. \mathcal{C} aborts if \mathcal{U}^* do not contain the target identity. Else, \mathcal{C} can very well unsigncrypt and verify the validity of the forged ring signcryption C^* (as done in unsigncrypt oracle).

If the ring signature of the forged ring signcryption passes the verification then \mathcal{C} will be able to generate one more valid ring signcryption from $C^* = (y^*, \mathcal{U}^*, U^*, U_1^*, \dots, U_n^*)$ which is named as $C' = (y', \mathcal{U}^*, U^*, U_1^*, \dots, U_n^*)$, using the oracle replay technique and applying extended version of forking lemma [3] applicable for ring signatures. This is achieved by running the turing machine again with the same random tape but with the different hash value. Obviously, \mathcal{A} , who is capable of generating a valid ring signcryption will be able to generate new valid ring signcryption again with the same randomness again. On getting two valid ring signcryptions on m^* , \mathcal{C} will be able to retrieve $D_{\mathbb{S}} = abP$ as explained below:

- Computes $\alpha = \hat{e}(U, D_{\mathbb{R}})$
- Consecutively, V^* and V' are retrieved as $(m^* \| V^*) = y^* \oplus H_2(\alpha)$ and $(m^* \| V') = y' \oplus H_2(\alpha)$.
- Here, $V^* = (h_{\mathbb{S}}^* + r_{\mathbb{S}})D_{\mathbb{S}}$ and $V' = (h_{\mathbb{S}}' + r_{\mathbb{S}})D_{\mathbb{S}}$ (Since they have the same randomness).
- Thus, $V^* - V' = h_{\mathbb{S}}^*D_{\mathbb{S}} - h_{\mathbb{S}}'D_{\mathbb{S}} = (h_{\mathbb{S}}^* - h_{\mathbb{S}}')D_{\mathbb{S}}$.

Since \mathcal{C} knows the hash values $h_{\mathbb{S}}^*$ and $h_{\mathbb{S}}'$, \mathcal{C} can compute $D_{\mathbb{S}}$ as $D_{\mathbb{S}} = (h_{\mathbb{S}}^* - h_{\mathbb{S}}')^{-1}(V^* - V')$. This means, \mathcal{C} can compute abP because $D_{\mathbb{S}} = abP$. In other words, \mathcal{C} is capable of solving CDHP. This is not possible. Hence, New-IBRSC is secure against EUF-CMA. \square

7 Conclusion

As a concluding remark we summarize the work in this paper. Ring signcryption is a primitive which enables a user to transmit authenticated messages anonymously and confidentially. To the best of our knowledge there were seven ring signcryption schemes in the identity based setting. Already it was shown in [8] that [6] was not CCA2 secure and in [5] it was shown by Fagen Li et al. that, [12] was not CCA2 secure. So, five out of seven identity based ring signcryption schemes were believed to be secure till date. We have shown that [10] and [15] does not even provide security against chosen plaintext attack (CPA); [5] and [14] does not provide security against adaptive chosen ciphertext attack (CCA2), by demonstrating attacks on confidentiality of these schemes. This leaves Huang et al.'s [4] scheme as the only secure identity based ring signcryption scheme. We have proposed a new identity based ring signcryption scheme for which we proved the security against chosen ciphertext attack and existential unforgeability in the random oracle model. Also we have compared our scheme with Huang et al.'s scheme below. In the comparison table, n represents the number of members in the ring.

Scheme	Signcryption					Unsigncryption				
	SPM	BP	EXP	\mathbb{G}_2M	PA	SPM	BP	EXP	\mathbb{G}_2M	PA
New-IBRSC	$n + 3$	1	—	—	$2n - 2$	n	3	—	—	$2n - 1$
Scheme in [4]	$2n + 2$	$n + 2$	—	1	$2n$	n	3	—	$n + 1$	n

Table 1: Efficiency Comparison with [4]

SPM - Scalar Point Multiplication, BP - Bilinear Pairing, EXP - Exponentiation in \mathbb{G}_2 , \mathbb{G}_2M - Multiplication of two \mathbb{G}_2 elements and PA - Point Addition.

Scheme	Ciphertext Size
New-IBRSC	$2 \mathcal{M} + (n + 2) \mathbb{G}_1 $
Scheme in [4]	$2 \mathcal{M} + (n + 1) \mathbb{G}_1 + n \mathbb{Z}_q^* $

Table 2: Ciphertext Size Comparison with [4]

Thus, our new identity based ring signcryption scheme (New-IBRSC) is a significant improvement over the scheme proposed by Huang et al. [4]

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