

# Security Weaknesses in Two Certificateless Signcryption Schemes

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**Abstract.** Recently, a certificateless signcryption scheme in the standard model was proposed by Liu et al. in [1]. Another certificateless signcryption scheme in the standard model was proposed by Xie et al. in [2]. Here, we show that the scheme in [1] and [2] are not secure against Type-I adversary.

## 1 Certificateless Signcryption Scheme by Liu et al.[1]

### 1.1 Review of the Scheme

In this section, we review the certificateless signcryption scheme secure against malicious-but-passive KGC attacks in the standard model proposed by Liu et al. The proposed scheme involves three parties: a KGC, a sender with an identity  $U_S$  and a receiver with an identity  $U_R$ . The scheme consists of the following algorithms.

**Setup :** Let  $(\mathbb{G}, \mathbb{G}_T)$  be bilinear groups, where  $|\mathbb{G}| = |\mathbb{G}_T| = p$  for some prime  $p$  and  $g$  be a generator of  $\mathbb{G}$ . Let  $\hat{e} : \mathbb{G} \times \mathbb{G} \rightarrow \mathbb{G}_T$  be the bilinear pairing and  $H : \{0, 1\}^* \rightarrow \mathbb{G}_T$  be the collision resistant hash function. KGC chooses randomly  $\alpha \in \mathbb{Z}_p$  and computes  $g_1 = g^\alpha$ . Additionally, the KGC selects three random values  $g_2, u', v' \in \mathbb{G}$  and two vectors  $\mathcal{U} = (u_i)_n, \mathcal{V} = (v_j)_m$  whose elements are chosen from  $\mathbb{G}$  at random. The system parameters are  $params = (\mathbb{G}, \mathbb{G}_T, \hat{e}, g, g_1, g_2, u', v', \mathcal{U}, \mathcal{V}, H)$  and the master secret key is  $g_2^\alpha$ .

**Partial-Private-Key-Extract :** Let  $u[i]$  denote the  $i^{th}$  bit of an identity  $u \in \{0, 1\}^n$  and  $\hat{u} = \{i | u[i] = 1, i = 1, \dots, n\}$ . The KGC picks  $r \in \mathbb{Z}_p$  uniformly and computes,

$$d_u = (d_{u,1}, d_{u,2}) = (g_2^\alpha (u' \prod u_i)^r, g^r).$$

An entity with identity  $u$  is given  $d_u$  as his partial private key. Therefore, the sender and the receivers partial private keys are,

$$d_S = (d_{S,1}, d_{S,2}) = (g_2^\alpha (u' \prod u_i)^{r_S}, g^{r_S}).$$

$$d_R = (d_{R,1}, d_{R,2}) = (g_2^\alpha (u' \prod u_i)^{r_R}, g^{r_R}).$$

**User-Key-Generate :** An entity with an identity  $u$  chooses randomly a secret value  $x_u \in \mathbb{Z}_p$  and computes a public key,

$$pk_u = \hat{e}(g_1, g_2)^{x_u}$$

**Private-Key-Extract :** An entity with identity  $u$  picks  $r' \in \mathbb{Z}_p$  at random, and computes a private key,

$$sk_u = sk_{u,1}, sk_{u,2} = \left( d_{u,1}^{x_u} (u' \prod u_i)^{r'}, d_{u,2}^{x_u} g^{r'} \right)$$

where  $t = rx_u + r'$ .

**Signcrypt :** To send a message  $M \in \mathbb{G}_T$  to the receiver with public key  $pk_R = \hat{e}(g_1, g_2)^{x_R}$ , the sender picks  $r'' \in \mathbb{Z}_p$  randomly and carries out the following steps.

- Compute  $\sigma_1 = M \cdot pk_R^{r''} = m \cdot \hat{e}(g_1, g_2)^{x_R r''}$ .
- Compute  $\sigma_2 = g^{r''}$ .
- Compute  $\sigma_3 = (u' \prod u_i)^{r''}$ .
- Set  $\sigma_4 = sk_{S,2}$ .
- Compute  $\hat{M} = H(\sigma_1, \sigma_2, \sigma_3, \sigma_4, u_R, pk_R) \in \{0, 1\}^m$ , where  $m[j]$  denotes the  $j^{th}$  bit of  $\hat{M}$  and  $\mathcal{M} = \{j | m[j] = 1, j = 1, 2, \dots, m\}$ .
- Compute  $\sigma_5 = sk_{S,1} \cdot (v' \prod v_i)^{r''}$ .
- Output the ciphertext  $\sigma = (\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5)$ .

**Unsigncrypt :** Upon receiving a ciphertext  $\sigma = (\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5)$ , the receiver decrypts the ciphertext as follows.

- Compute  $\hat{M} = H(\sigma_1, \sigma_2, \sigma_3, \sigma_4, u_R, pk_R) \in \{0, 1\}^m$ , where  $m[j]$  denotes the  $j^{th}$  bit of  $\hat{M}$  and  $\mathcal{M} = \{j | m[j] = 1, j = 1, 2, \dots, m\}$ .
- Check that the equality,

$$\hat{e}(\sigma_5, g) = pk_S \cdot \hat{e}(u' \prod u_i, \sigma_4) \hat{e}(v' \prod v_j, \sigma_2)$$

holds. If not output “Invalid”. Otherwise, compute and output  $M = \sigma_1 \cdot \hat{e}(\sigma_3, sk_{R,2}) / \hat{e}(\sigma_2, sk_{R,1})$

## 1.2 Attack on the Scheme by Liu et al. :

The scheme proposed by Liu et al. in [1] does not provide confidentiality against Type-I adversary. We show the scheme is not even CPA secure against Type-I adversary. The attack can be launched by a Type-I adversary by replacing the public key of the target receiver whose signcryption the adversary wants to designcrypt. This can be achieved in the following way :

During the Type-I confidentiality game,

- The challenger runs the setup and provides the system public parameters to the adversary.
- The adversary has access to all the oracles namely **Partial-Private-Key-Extract**, **Private-Key-Extract**, **Replace-Public-Key**, **Signcrypt** and **Unsigncrypt**.
- The adversary replaces the public key of the receiver (say  $R^*$ ) which he wants to use during the *challengephase* by  $pk_{R^*} = \hat{e}(g, g)^{r^*}$  where  $r^* \in_R \mathbb{Z}_p$ .
- Without asking any further queries the adversary now picks two messages  $\{m_0, m_1\}$  of equal length and a sender identity  $S$  and receiver identity  $R^*$  on which the adversary wishes to be challenged and sends to the challenger.

- The Challenger now picks a random bit  $\delta \in \{0, 1\}$ , cooks up the signcryption  $\sigma^* = (\sigma_1^*, \sigma_2^*, \sigma_3^*, \sigma_4^*, \sigma_5^*)$  of message  $m_\delta$  and sends  $\sigma^*$  to the adversary.
- Now the adversary can get back the key by performing  $m_{\delta'} = \sigma_1^* \hat{e}(\sigma_2, g^{r^*})$  and outputs  $\delta'$  to the challenger.
- Hence the adversary can successfully distinguish the message being signcrypted. This clearly shows that the scheme given by Liu et al. is not CPA secure against Type-I adversary.

## 2 Certificateless Signcryption Scheme by Xie et al.[2]

Since the scheme is available in public medium, we do not review the scheme here.

### Attack on the Scheme

In this section we present a total break of the certificateless signcryption scheme in [2] by Type-I adversary. During the unforgeability game, the adversary knows the full private key of the receiver. Thus, during the training phase, the Type-I forger queries and obtains a ciphertext  $\sigma = \langle c, u, v, w \rangle$  from the signcrypt oracle. Let  $\sigma$  be a signcryption from sender  $ID_A$  to receiver  $ID_B$ , where the private key  $D_B$  corresponding to the receiver is known to the adversary. The adversary performs the following to compute the partial private key  $d_A$  of the sender.

- We know that  $w = x_A h_2 + r_1$ . (It is known that Type-I adversary can replace the public key and hence have access to the sender secret value  $x_A$ .)
- Computes  $g^{r'_1} = \hat{e}(d_B, u)$  and  $m = c \oplus H_3(g^{r'_1}, x_B u)$ .
- Computes  $h_2 = H_2(m, u, g^{r'_1}, x_B u, pk_A, pk_B)$ .
- Computes  $r_1 = w - x_A h_2$ .
- It is now possible to compute  $d_A = v \left( \frac{r_1 - h_2}{r_1} \right)$ .

Hence, a Type-I adversary can find out the partial private key of any legitimate user in the system, which leads to a total break of the system in [2].

### References

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2. Wenjian Xie and Zhang Zhang. Efficient and provably secure certificateless signcryption from bilinear maps. *Cryptology ePrint Archive*, Report 2009/578, 2009.