Adaptive and Concurrent Secure Computation from New Notions of Non-Malleability

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Abstract

We present a unified framework for obtaining general secure computation that achieves adaptive-Universally Composable (UC)-security. Our framework captures essentially all previous results on adaptive concurrent secure computation, both in relaxed models (e.g., quasi-polynomial time simulation), as well as trusted setup models (e.g., the CRS model, the imperfect CRS model). This provides conceptual simplicity and insight into what is required for adaptive and concurrent security, as well as yielding improvements to set-up assumptions and/or computational assumptions. Moreover, using our framework we provide first constructions of concurrent secure computation protocols that are adaptively secure in the timing model, and in the non-uniform simulation model.

Conceptually, our framework can be viewed as an adaptive analogue to the recent work of Lin, Pass and Venkitasubramaniam [STOC '09], who considered only non-adaptive adversaries. Their main insight was that stand-alone non-malleability was sufficient for UC-security. A main conceptual contribution of this work is, quite surprisingly, that it is indeed the case even when considering adaptive security.

A key element in our construction is a commitment scheme that satisfies a new notion of non-malleability. The notion of *concurrent equivocal non-malleable commitments*, intuitively, guarantees that even when a man-in-the-middle adversary observes concurrent equivocal commitments and decommitments, the binding property of the commitments continues to hold for commitments made by the adversary. This notion is stronger than standard notions of concurrent non-malleable commitments which either consider only specific commits (e.g., statistically-binding) or specific scenarios (e.g., the commitment phase and the decommitment phase are executed in a non-overlapping manner). Previously, commitments that satisfy our definition, have been constructed in setup models, but either require existence of stronger encryption schemes such as CCA-secure encryption or require independent "trapdoors" provided by the setup for every pair of parties to ensure non-malleability. We here provide a construction that eliminates these requirements and require *only* a single trapdoor.

1 Introduction

The notion of *secure multi-party computation* allows mutually distrustful parties to securely compute a function on their inputs, such that only the (correct) output is obtained, and no other information is leaked, even if the adversary controls an arbitrary subset of parties. This security is formalized via the real/ideal simulation paradigm, requiring that whatever the adversary can do in a real execution of the protocol, can be simulated by an adversary ("simulator") working in the ideal model, where the parties submit their inputs to a trusted party who then computes and hands back the output. Properly formalizing this intuitive definition and providing protocols to realize it requires care, and has been the subject of a long line of research starting in the 1980s.

In what is recognized as one of the major breakthroughs in cryptography, strong feasibility results were provided, essentially showing that *any function that can be efficiently computed, can be efficiently computed securely,* assuming the existence of enhanced trapdoor permutations (eTDP). However, these results were originally investigated in the *stand-alone setting*, where a single instance of the protocol is ran in isolation. A stronger notion is that of *concurrent security*, which provides the security guarantee even when many different protocol executions are carried out concurrently. We will focus on the strongest (and most widely used) notion of concurrent security, namely universally-composable (UC) security [4]; This notion provides security even when an unbounded number of different protocol executions are ran concurrently in an arbitrary interleaving schedule. This is obviously important for maintaining security in an uncontrolled environment that allows concurrent executions (e.g., the Internet). Moreover, this notion also facilitates modular design and analysis of protocols, by allowing to design and prove secure smaller protocol components, and then compose them to obtain a secure protocol for a complex functionality.

Unfortunately, it turns out that concurrent security is much more challenging than the stand-alone one, and we do not have general feasibility results for concurrently secure computation of every function. In fact, there are lower bounds showing that the standard notion of concurrent security (in particular UC security) cannot be achieved for general functions, unless some trusted setup is assumed [5, 6, 24]. Previous works overcome this by either by using some sort of a trusted setup infrastructure, or by relaxing the definition of security (we will see examples below).

Another aspect of defining secure computation, is the power given to the adversary. A *static* (or non-adaptive) adversary is one who has to decide which parties to corrupt at the onset, before the execution of the protocol begins. A stronger notion is one that considers an *adaptive* adversary, who may corrupt parties at any time, based on its view of the protocol so far. The adaptive setting is much more complex than the static one. The intuitive reason it is so challenging is that the simulator needs to simulate messages from uncorrupted parties, but may later have to explain them if the party got corrupted, according to any possible input that the party holds. On the other hand, in the real protocol execution, the messages should depend in a committing way on the actual input of the party, so that an adversary cannot cheat. Indeed, the techniques for achieving security against adaptive adversaries are generally quite different than the techniques needed to achieve security against static adversaries, and many of the previous results for concurrent secure computation do not readily extend to the adaptive setting. In fact, many of the results allowing general concurrent secure computation (e.g., using a trusted setup) are only given for the static case, and do not work for the adaptive setting.

In this paper we focus on the strongest (and most realistic, in complex environments) notions of security, and study their fundamental power and limitations. The main question we ask is:

> *Under which circumstances is adaptive concurrent security generally feasible?*

In particular, we refine this question to ask:

> *What is the minimum setup required to achieve adaptive concurrent security?*

We address these questions on both a conceptual and technical level. We provide a framework that unifies and generalizes essentially all previous results in the generic adaptive concurrent setting, as well as providing completely new results (constructions with weaker trusted setup requirements, weaker computational assumptions, or in relaxed models of security), conceptual simplicity, and insight into what is required for adaptive and concurrent secure computation. Our main technical tool is a new primitive of equivocal non-malleable commitment. We describe our results in more detail below.

1.1 Our Results

We provide a general framework for achieving adaptive UC-security both with, and without, trusted set-up. With this framework, essentially all general UC-feasibility results for adaptive adversaries follow as simple corollaries, often improving the set-up assumptions and the complexity theoretic assumptions (although we did not make any attempt to improve round complexity); moreover, the framework yields adaptive UC secure computation in new models (such as the timing model). As such, our framework helps in characterizing models in which adaptive UC security is realizable, and also at what cost.

Although technically quite different, conceptually our framework may conceptually be viewed as an adaptive analogue of the work of Lin, Pass and Venkitasubramaniam [23], who study the *static* case. Their main theorem states that assuming the existence of enhanced trapdoor permutations and stand-alone non-malleable commitments, static UC-security is achievable in any model that admits a "UC-puzzle". In this work, we establish an analogous main theorem for the case of *adaptive* UC-security.

First, we introduce the notion of an *Adaptive UC-Puzzle*, that extends the notion of a UC-Puzzle to the adaptive setting. Next, we define a new primitive (which may be of independent interest) called *equivocal non-malleable commitments* or $\mathsf{EQNMCom}$, which are commitments with the property that a man-in-the-middle who observes concurrent equivocal commitments and decommitments, cannot break the binding property of the commitment. This can be viewed as the adaptive analogue of the standalone non-malleable commitments used by [23]. We then present a construction of equivocal non-malleable commitment for any model that admits an adaptive UC-puzzle (so in that sense, requiring this primitive does not introduce an additional complexity-theoretic assumption). Finally, we need a computational assumption that implies adaptively secure oblivious transfer (analogous to the eTDP used by [23], which implies statically secure OT). Towards that end, we use *simulatable public key encryption* [12, 9][1] We thus get:

THEOREM 1 (**Main Theorem** (Informal)). *Assume the existence of a* $t_1(\cdot)$*-round adaptive UC-secure puzzle* Σ *using some setup* $\mathcal{T}$*, the existence of a* $t_2(\cdot)$*-round* $\mathsf{EQNMCom}$ *primitive, and the existence of simulatable public-key encryption scheme. Then, for every* m*-ary functionality* f*, there exists a* $O(t_1(\cdot) + t_2(\cdot))$*-round protocol* Π *using the same set-up* $\mathcal{T}$ *that adaptively, UC-realizes* f.

Complementing the main theorem, we also show that, in previously studied models, adaptive UC-puzzles are easy to construct. In fact, in most models the puzzles from the static case considered in [23], are sufficient. In addition, we provide new constructions for models where adaptive-security has not been previously established. We highlight some results obtained by instantiating our framework below.

Adaptive UC in the "imperfect" string model. Canetti, Pass and Shelat [8] consider adaptive UC security where parties have access to an "imperfect" reference string–called a "sunspot"–that is generated by any arbitrary efficient min-entropy source (obtained e.g., by measurement of some physical phenomenon). The CPS-protocol, however, requires m communicating parties to share m reference strings, each of them generated using fresh entropy. Our results address the case of security against adaptive adversaries. In this setting, we show that a *single* reference string is sufficient for UC and adaptively-secure MPC (regardless of the number of parties m).

[1] This is *almost* the weakest assumption currently known that implies adaptive OT. An even weaker assumption is "trapdoor-simulatable PKE" [9]. We do not currently know how to show that this weaker version is sufficient.

Adaptive UC in the timing model. Dwork, Naor and Sahai [16] introduced the *timing model*, where all players are assumed to have access to clocks with a certain drift. Kalai, Lindell and Prabhakaran [20] subsequently presented a concurrent secure computation protocol in the timing model; whereas the timing model of [16] does not impose a maximal upper-bound on the clock drift, the protocol of [20] requires the clock-drift to be "small"; furthermore, it requires extensive use of delays (roughly $n\Delta$, where Δ is the latency of the network). Finally, [23] showed that UC security against *static* adversaries is possible also in the *unrestricted* timing model (where the clock drift can be "large"); additionally, they reduce the use of delays to only $O(\Delta)$. To the best of our knowledge, our work is the first to consider security against adaptive adversaries in the Timing model. Thus, our work yields the first feasibility results for UC and adaptively-secure MPC in the timing model and, similarly to the results of [23] in the static case, our results hold in the unrestricted timing model.

Adaptive UC with quasi-polynomial simulation. Pass [27] proposed a relaxation of the standard simulation-based definition of security, allowing for a super polynomial-time or Quasi-polynomial simulation (QPS). In the static setting, Prabhakaran and Sahai [29] and Barak and Sahai [2] obtained general multi-party protocols that are concurrently QPS-secure without any trusted set-up, but rely on strong complexity assumptions. We achieve security in the quasi-polynomial simulation model with adaptive corruptions under relatively weak complexity assumptions and we achieve a stronger notion of security, which (in analogy with [27]) requires that indistinguishability of simulated and real executions holds also for all of quasi-polynomial time; in contrast, [2] only achieves indistinguishability w.r.t. distinguishers with running-time smaller than that of the simulator.

Adaptive UC with non-uniform simulation. Lin et al. [23] introduced the non-uniform UC model, where we consider environments that are $\mathcal{PPT}$ machines and ideal-model adversaries that are non-uniform $\mathcal{PPT}$ machines. Using their framework for static adversaries, [23] showed feasibility results for secure MPC in the non-uniform UC model. They introduce two new complexity assumptions, each of which is individually sufficient (along with existence of enhanced trapdoor permutations) to realize secure MPC in the non-uniform UC model. In our work, we rely on the same complexity assumptions as those introduced by [23] (along with the assumption of the existence of simulatable PKE), to show feasibility results for secure MPC in the adaptive, non-uniform UC model.

In addition to these models, our framework also captures adaptive UC in the common reference string (CRS) model [7], uniform reference string (URS) model [7], key registration model [1], tamper-proof hardware model [21], and partially isolated adversaries model [14] (see Section 7).

Beyond the specific instantiations, our framework provides conceptual simplicity, technical insight, and the potential to facilitate "translation" of results in the static setting into corresponding (and much stronger) adaptive security results. For example, in recent work of [17] one of the results –constructing UC protocols using multiple setups when the parties share an arbitrary belief about the setups– can be translated to the adaptive model by replacing (static) puzzles with our notion of adaptive puzzles. Other results may require more work to prove, but again are facilitated by our framework.

1.2 Technical Approach and Insights Gained

There are two basic properties that must be satisfied in order to achieve UC secure computation: (1) concurrent simulation and (2) concurrent non-malleability. The former requirement amounts to providing the simulator with a trapdoor while the latter requirement amounts to establishing independence of executions. The simulation part is usually "easy" to achieve. Consider, for instance, the Common Reference String (CRS) model where the players have access to a public reference string that is ideally sampled from some distribution. Concurrent non-malleability on the other hand is significantly harder to achieve. In this particular case, Canetti, Lindell, Ostrovsky and Sahai [CLOS02] solve the problem by embedding the public-key of a CCA-secure encryption scheme in the CRS, but in general, quite different techniques are employed in each model. In many models, this is achieved by providing a technique which enables the simulator to have different trapdoors for each player, such that the trapdoor for one player does not reveal a trapdoor for another.

Unfortunately, the same phenomena persists in most set-up models: concurrent simulation is easy to achieve, but concurrent non-malleability requires significantly more work, and often stronger set-up and/or stronger computational assumptions. In the static case, Lin, et. al in [23], show that that concurrent simulation is sufficient, i.e., it is sufficient to provide the simulator with a single trapdoor. Once such a trapdoor is established, concurrent non-malleability (and thus UC-security) can be achieved by further relying on a stand-alone non-malleability. Thus, no additional assumptions are required in the case of static-security to establish non-malleability and this allows the framework in [23] to improve significantly on previous results. Furthermore, stand-alone non-malleable protocols exist in the plain model (i.e. assuming no setup). An important question here is whether the same holds in the adaptive case. Unfortunately, in the case of adaptive security, simulator need to be able to "equivocate" and this requires some setup. Most protocols for adaptive security in literature, achieve non-malleability by simply providing an independent trapdoor for every execution.

In this work, we show that, surprisingly, a single trapdoor is sufficient to achieve concurrent non-malleability. Therefore, we establish that even for the case of adaptive security no additional setup (and no additional assumptions) are required to achieve concurrent non-malleability.

1.3 Main Tool: Equivocal Non-Malleable Commitments

We define and construct a new primitive called *equivocal non-malleable commitments* or $\mathsf{EQNMCom}$. Such commitments have previously been defined in the works of [10, 11] but only for the limited case of bounded concurrency and non-interactive commitments. In our setting, we consider the more general case of unbounded concurrency as well as interactive commitments. Intuitively, the property we require from these commitments is that even when a man-in-the-middle receives concurrent equivocal commitments and concurrent equivocal decommitments, the man-in-the-middle cannot break the binding property of the commitment. Thus, the man-in-the-middle receives equivocal commitments and decommitments, but cannot equivocate himself. Formalizing this notions seems to be tricky and has not been considered in literature before.

In existing literature, non-malleability of commitments has been dealt with in two scenarios:

Non-malleability w.r.t commitment:[15, 28, 22] This requires that no adversary that receives a commitment to value v be able to commit to a related value (even without being able to later decommit to this value).

Non-malleability w.r.t decommitment (or opening):[10, 28] This requires that no adversary that receives a commitment and decommitment to a value v be able to commit and decommit to a related value.

While the former is applicable only in the case the of statistically-binding commitments the latter is useful even for statistically-hiding commitments. In this work, we need a definition that ensures independence of commitments schemes that additionally are equivocable and adaptively secure. Equivocability means that there is a way to commit to the protocol without knowing the value being committed to and later open to any value. This essentially implies that the scheme cannot be statistically-binding. Furthermore, as will be evident in our construction, we require non-malleability w.r.t decommitment. Unfortunately, current definitions for non-malleability w.r.t decommitment in literature is defined only in the scenario where the commitment phase and decommitment phases are decoupled, i.e. in a first phase, a man-in-the-middle adversary receives commitments and sends commitments, then, in a second phase, the adversary requests decommitments of the commitments received in the first phase, followed by it decommitting its own commitments. For our construction, we need to define concurrent non-malleability w.r.t decommitments and such a two phase scenario is not applicable as the adversary can arbitrarily and adaptively decide when to obtain decommitments. Furthermore, it is not clear how to extend the traditional definition to the general case, as at any point, only a subset of the commitments received by the adversary could be decommitted and the adversary could selectively decommit based on the values seen so far and hence it is hard to define a "related" value.

We instead propose a new definition, along the lines of *simulation-extractability* that has been defined in the context of constructing non-malleable zero-knowledge proofs. Loosely speaking, an interactive protocol is said to be simulation extractable if for any man-in-the-middle adversary A, there exists a probabilistic polynomial time machine (called the simulator-extractor) that can simulate both the left and the right interaction for A, while outputting a witness for the statement proved by the adversary in the right interaction. Roughly speaking, we say that a tag-based commitment scheme (i.e., commitment scheme that take an identifier—called the tag—as an additional input) is said to be *concurrent non-malleable w.r.t opening* if for every man-in-the-middle adversary A that participates in several interactions with honest committers as a receiver (called *left* interactions) as well as several interactions with honest receivers as a committer (called *right* interactions, there exists a simulator S that can simulate the left interactions, while extracting the commitments made by the adversary in the right interactions (whose identifiers are different from all the left identifiers) before the adversary decommits.

It is not hard to construct such commitments using trusted set-up. The idea here is to provide the simulator with a trapdoor with which it can equivocate as wells as extract the commitments on the right. (by e.g., relying on encryption). However, to ensure non-malleability, most constructions constructions in literature additionally impose CCA-security or provide independent trapdoors for every interaction. Our main technical contribution consists of showing how to construct a concurrent non-malleable commitment scheme in any trusted set-up by providing with simulator with just one trapdoor. In [23], they introduce the notion of a concurrent puzzle with non-adaptive security which essentially captures concurrent simulation requirement using a single trapdoor. Here, we extend the definition to the adaptive case and show how to construct a concurrent non-malleable commitment scheme w.r.t opening using any concurrent puzzle.

Although our main application of equivocal non-malleable commitments is achieving UC-security, these commitments may also be useful for other applications such as concurrent non-malleable zero knowledge secure under adaptive corruptions. We believe that an interesting open question is to explore other applications of equivocal non-malleable commitments and non-malleable commitments with respect to decommitment.

2 Definitions and Background

2.1 Commitment Schemes

Commitment schemes are used to enable a party, known as the *sender*, to commit itself to a value while keeping it secret from the *receiver* (this property is called hiding). Furthermore, in a later stage when the commitment is opened, it is guaranteed that the "opening" can yield only a single value determined in the committing phase (this property is called binding). In this work, we consider commitment schemes that are statistically-binding, namely while the hiding property only holds against computationally bounded (non-uniform) adversaries, the binding property is required to hold against unbounded adversaries. More precisely, a pair of PPT machines $\langle C, R\rangle$ is said to be a commitment scheme if the following two properties hold.

Computational hiding: For every (expected) PPT machine R^*, it holds that, the following ensembles are computationally indistinguishable over $n \in N$.

- $\{\mathsf{sta}^{R^*}_{\langle C,R\rangle}(v_1, z)\}_{n\in N, v_1, v_2\in\{0,1\}^n, z\in\{0,1\}^*}$
- $\{\mathsf{sta}^{R^*}_{\langle C,R\rangle}(v_2, z)\}_{n\in N, v_1, v_2\in\{0,1\}^n, z\in\{0,1\}^*}$

where $\mathsf{sta}^{R^*}_{\langle C,R\rangle}(v,z)$ denotes the random variable describing the output of R^* after receiving a commitment to v using $\langle C, R\rangle$.

Statistical binding: Informally, the statistical-binding property asserts that, with overwhelming probability over the coin-tosses of the receiver R, the transcript of the interaction fully determines the value committed to by the sender. We refer the reader to [18] for more details.

We say that a commitment is *valid* if there exists a unique committed value that a (potentially malicious) committer can open to successfully.

2.2 Adaptive Language-Based Equivocal Commitment Schemes

We assume all our commitment schemes have non-interactive decommitment and is in fact just the randomness of the committer algorithm.

Definition 1 (Language-Based Commitment Schemes:)**.** *Let L be an* NP*-Language and $\mathcal{R}$, the associated* NP*-relation. A language-based commitment scheme (LBCS) for L is commitment scheme $\langle C, R\rangle$ such that:*

Computational hiding: *For every (expected) PPT machine R^*, it holds that, the following ensembles are computationally indistinguishable over $n \in N$.*

- $\{\mathsf{sta}^{R^*}_{\langle C,R\rangle}(x, v_1, z)\}_{n\in N, x\in\{0,1\}^n, v_1,v_2\in\{0,1\}^n, z\in\{0,1\}^*}$
- $\{\mathsf{sta}^{R^*}_{\langle C,R\rangle}(x, v_2, z)\}_{n\in N, x\in\{0,1\}^n, v_1,v_2\in\{0,1\}^n, z\in\{0,1\}^*}$

where $\mathsf{sta}^{R^*}_{\langle C,R\rangle}(x,v,z)$ denotes the random variable describing the output of $R^*(x,z)$ after receiving a commitment to v using $\langle C, R\rangle$.

Computational binding: *The binding property asserts that, there exists an polynomial-time witness-extractor algorithm Ext, such that for any cheating committer C^*, that can decommit a commitment to two different values v_1, v_2 on common input $x \in \{0,1\}^n$, $Ext(x, v_1, v_2)$ outputs w such that $w \in \mathcal{R}(x)$.*

Definition 2 (Language-Based Equivocal Commitments)**.** *Let L be an* NP*-Language and $\mathcal{R}$, the associated* NP*-relation. A language-based commitment scheme $\langle C, R\rangle$ for L is said to be equivocal, if there exists a tuple of algorithms $(\tilde{C}, \mathsf{Adap})$ and such that the following hold:*

Special-Hiding: *For every (expected) PPT machine R^*, it holds that, the following ensembles are computationally indistinguishable over $n \in N$.*

- $\{\mathsf{sta}^{R^*}_{\langle C,R\rangle}(x, v_1, z)\}_{n\in N, x\in L\cap\{0,1\}^n, w\in\mathcal{R}(x), v_1\in\{0,1\}^n, z\in\{0,1\}^*}$
- $\{\mathsf{sta}^{R^*}_{\langle \tilde{C},R\rangle}(x, w, z)\}_{n\in N, x\in L\cap\{0,1\}^n, w\in\mathcal{R}(x), v_1\in\{0,1\}^n, z\in\{0,1\}^*}$ *where $\mathsf{sta}^{R^*}_{\langle \tilde{C},R\rangle}(x,w,z)$ denotes the random variable describing the output of $R^*(x,z)$ after receiving a commitment to v_1 using $\langle \tilde{C}, R\rangle$.*

Equivocability: *Let τ be the transcript of the interaction between R and $\tilde{C}$ on common input $x \in L \cap \{0,1\}^n$ and private input $w \in \mathcal{R}(x)$ and random tape $r \in \{0,1\}^*$ for $\tilde{C}$. Then for any $v \in \{0,1\}^n$,* $\mathsf{Adap}(x, w, r, \tau, v)$ *produces a random tape r' such that (r', v) serves as a valid decommitment for C.*

Definition 3. *A ℓ-*bit simulatable encryption scheme *consists of an encryption scheme* $(\mathsf{Gen}, \mathsf{Enc}, \mathsf{Dec})$ *augmented with* $(\mathsf{oGen}, \mathsf{oRndEnc}, \mathsf{rGen}, \mathsf{rRndEnc})$*. Here,* oGen *and* $\mathsf{oRndEnc}$ *are the oblivious sampling algorithms for public keys and ciphertexts, and* rGen *and* $\mathsf{rRndEnc}$ *are the respective inverting algorithms,* rGen

(resp. $\mathsf{rRndEnc}$*) takes* r_{G} *(resp.* $(\mathrm{PK}, r_{\mathrm{E}}, m)$*) as the trapdoor information. We require that, for all messages* $m \in \{0,1\}^{\ell}$*, the following distributions are computationally indistinguishable:*

$$\{\mathsf{rGen}(r_{\mathrm{G}}), \mathsf{rRndEnc}(\mathrm{PK}, r_{\mathrm{E}}, m), \mathrm{PK}, c \mid (\mathrm{PK}, \mathrm{SK}) = \mathsf{Gen}(1^k; r_{\mathrm{G}}), c = \mathsf{Enc}_{\mathrm{PK}}(m; r_{\mathrm{E}})\}$$
$$\textit{and } \{\hat{r}_{\mathrm{G}}, \hat{r}_{\mathrm{E}}, \hat{\mathrm{PK}}, \hat{c} \mid (\hat{\mathrm{PK}}, \perp) = \mathsf{oGen}(1^k; \hat{r}_{\mathrm{G}}), \hat{c} = \mathsf{oRndEnc}_{\hat{\mathrm{PK}}}(1^k; \hat{r}_{\mathrm{E}})\}$$

It follows from the definition that a trapdoor simulatable encryption scheme is also semantically secure.

2.3 Traditional UC

Environment. The model of execution includes a special entity called the UC-environment (or environment) Z. The environment "manages" the whole execution: it invokes all the parties at the beginning of the execution, generates all inputs and reads all outputs, and finally produces an output for the whole concurrent execution. Intuitively, the environment models the "larger world" in which the concurrent execution takes place (e.g., for a distributed computing task over the Internet, the environment models all the other activities occurring on the Internet at the same time).

Adversarial behavior. The model of execution also includes a special entity called the adversary, that represents adversarial activities that are directly aimed at the protocol execution under consideration. We consider an *adaptive* adversary, who may corrupt any party at any point during the executions, and as a function of what he sees. When a party is corrupted, it shares all its tapes with the adversary and follows the instructions from the adversary for all its future actions.

While honest parties only communicate with the environment through the input/output of the functions they compute, the adversary is also able to exchange messages with the environment in an arbitrary way through out the computation[2]. Furthermore, the adversary controls the scheduling of the delivery of all messages exchanged between parties (messages sent by the environment is delivered directly). Technically, this is modeled by letting the adversary read the outgoing message tapes of all parties and decide whether or not and when (if at all) to deliver the message to the recipient, therefore the communication is asynchronous and lossy. However, the adversary cannot insert messages and claim arbitrary sender identity. In other words, the communication is authenticated.

Protocol execution. The *execution of a protocol* π *with the environment* Z*, adversary* A *and trusted party* $\mathcal{G}$ proceeds as follows. The environment is the first entity activated in the execution, who then activates the adversary, and invokes other honest parties. At the time an honest party is invoked, the environment assigns it a unique identifier, and inquiries the adversary whether it wants to corrupt the party or not. To start an execution of the protocol π, the environment initiates a *protocol execution session*, identified by a session identifier sid, and activates all the participants in that session. An honest party activated starts executing the protocol π thereafter and has access to the trusted party $\mathcal{G}$. We remark that in the UC model, the environment only initiates one protocol execution session.

Invoking parties. The environment invokes an honest party by passing input (invoke, P_i) to it. P_i is the globally unique identity for the party, and is picked dynamically by the environment at the time it is invoked. Immediately after that, the environment notifies the adversary of the invocation of P_i by sending the message (invoke, P_i) to it, who can then choose to corrupt the party by replying ($\mathsf{corrupt}$, P_i). Note that here as the adversary is static, parties are corrupted only when they are "born" (invoked).

[2] Through its interaction with the environment, the adversary is also able to influence the inputs to honest parties indirectly.

Session initiation. To start an execution of protocol π, the environment selects a subset U of parties that has been invoked so far. For each party $P_i \in U$, the environment activates P_i by sending a start-session message ($\mathsf{start\text{-}session}$, P_i, sid, $c_{i,sid}$, $x_{i,sid}$) to it, where sid is a session id that identifies this execution. We remark that in the UC model, the environment starts only one session, and hence all the parties activated have the same session id.

Honest party execution. An honest party P_i, upon receiving ($\mathsf{start\text{-}session}$, P_i, sid, $c_{i,sid}$, $x_{i,sid}$), starts executing its code $c_{i,sid}$ input $x_{i,sid}$. During the execution,

- the environment can read P_i's output tape and at any time may pass additional inputs to P_i;
- according to its code, P_i can send messages (delivered by the adversary) to other parties in the session, in the format (P_i, P_j, s, content)[3], where P_j is the identity of the receiver;
- according to its code, P_i can send input to the trusted party in the format (P_i, $\mathcal{F}$, s, input).

Adversary execution. After activation, the adversary may perform one of the following activities at any time during the execution.

- The adversary can read the outgoing communication tapes of all honest parties and decides to deliver some of the messages.
- A can exchange arbitrary messages with the environment.
- The adversary can read the inputs, outputs, incoming messages of a corrupted party, and instruct the corrupted party for any action.
- The adversary can decide to corrupt any party from the set of honest parties at the moment.

Output. The environment outputs a final result for the whole execution in the end.

In the execution of protocol π with security parameter $n \in N$, environment Z, adversary A and trusted party $\mathcal{G}$, we define $\mathsf{Exec}^{\mathcal{G}}_{\pi,\mathsf{A},\mathsf{Z}}(\mathsf{n})$ to be the random variable describing the output of the environment Z, resulting from the execution of the above procedure.

Let $\mathcal{F}$ be an ideal functionality; we denote by π_{ideal} the protocol accessing $\mathcal{F}$, called as the ideal protocol. In π_{ideal} parties simply interacts with $\mathcal{F}$with their private inputs, and receives their corresponding outputs from the functionality at the end of the computation. Then the ideal model execution of the functionality $\mathcal{F}$is just the execution of the ideal protocol π_{ideal} with environment Z, adversary A' and trusted party $\mathcal{F}$. The output of the execution is thus $\mathsf{Exec}^{\mathcal{F}}_{\pi_{\mathsf{ideal}},\mathsf{A'},\mathsf{Z}}(\mathsf{n})$. On the other hand, the real model execution does not require the aid of any trusted party. Let π be a multi-party protocol implementing $\mathcal{F}$. Then, the real model execution of π is the execution of π with security parameter n, environment Z and adversary A, whose output is the random variable $\mathsf{Exec}_{\pi,\mathsf{A},\mathsf{Z}}(\mathsf{n})$. Additionally, the $\mathcal{G}$-Hybrid model execution of a protocol π is the execution of π with security parameter n, environment Z and adversary A and ideal functionality $\mathcal{G}$.

Security as emulation of a real model execution in the ideal model. Loosely speaking, a protocol securely realizes an ideal functionality if it securely emulates the ideal protocol π_{ideal}. This is formulated by saying that for every adversary A in the real model, there exists an adversary A' (a.k.a. *simulator*) in the ideal model, such that no environment Z can tell apart if it is interacting with A and parties running the protocol, or A' and parties running the ideal protocol π_{ideal}.

[3] The session id in the messages enables the receiver to correctly de-multiplexing a message to its corresponding session, even though the receiver may involve in multiple sessions simultaneously.

Definition 4. (Adaptive UC security) *Let $\mathcal{F}$ and π_{ideal} be defined as above, π be a multi-party protocol in the $\mathcal{G}$-hybrid model. The protocol π is said to realize $\mathcal{F}$ with adaptive UC security in $\mathcal{G}$-hybrid model, if for every uniform $\mathcal{PPT}$adaptive adversary A, there exists a uniform $\mathcal{PPT}$simulator A', such that, for every non-uniform $\mathcal{PPT}$environment Z, the following two ensembles are indistinguishable.*

$$\left\{\mathsf{Exec}^{\mathcal{G}}_{\pi,\mathsf{A},\mathsf{Z}}(\mathsf{n})\right\}_{\mathsf{n}\in\mathsf{N}} \approx \left\{\mathsf{Exec}^{\mathcal{F}}_{\pi_{\mathsf{ideal}},\mathsf{A}',\mathsf{Z}}(\mathsf{n})\right\}_{\mathsf{n}\in\mathsf{N}}$$

Multi-session extension of ideal functionalities Note that the UC model only considers a single session of the protocol execution. (The environment is only allowed to open one session). To consider multiple concurrent executions, we focus on the multi-session extension of ideal functionalities [4, 7]. More specifically, let $\hat{\mathcal{F}}$ be the multi-session extension of $\mathcal{F}$. $\hat{\mathcal{F}}$ runs multiple copies of $\mathcal{F}$, where each copy will be identified by a special "sub-session identifier". Every k parties, trying access $\mathcal{F}$ together, share a sub-session identifier, $ssid$. To compute, each party simply sends its private input together with $ssid$ to $\hat{\mathcal{F}}$. $\hat{\mathcal{F}}$ upon receiving all the inputs, activates the appropriate copy of $\mathcal{F}$ identified by $ssid$ (running within $\hat{\mathcal{F}}$), and forwards the incoming messages to that copy. (If no such copy of $\mathcal{F}$exists then a new copy is invoked and is given that ssid.) Outputs generated by the copies of $\mathcal{F}$are returned to corresponding parties by $\hat{\mathcal{F}}$.

2.4 A Generalized Version of UC

In the UC model, the environment is modeled as a non-uniform $\mathcal{PPT}$ machine and the ideal-model adversary (or simulator) as a (uniform) $\mathcal{PPT}$ machines. We consider a generalized version (in analogy with [27, 29]) where we allow them to be in arbitrary complexity classes. Note, however, that the adversary is still $\mathcal{PPT}$. Additionally, we "strengthen" the definition by allowing the environment to output a bit string (instead of a single bit) at the end of an execution. In the traditional UC definition, it is w.l.o.g. enough for the environment to output a single bit [4]; in our generalized version this no longer holds and we are thus forced to directly consider the more stringent version.

We represent a generalized UC model by a 2-tuple ($\mathcal{C}_{\mathsf{env}}$, $\mathcal{C}_{\mathsf{sim}}$), where $\mathcal{C}_{\mathsf{env}}$ and $\mathcal{C}_{\mathsf{sim}}$ are respectively the classes of machines the environment and the simulator of the general model belong to. We consider only classes, $\mathcal{C}_{\mathsf{env}}$ and $\mathcal{C}_{\mathsf{sim}}$, that are closed under probabilistic polynomial time computation. For a model $(\mathcal{C}_{\mathsf{env}}, \mathcal{C}_{\mathsf{sim}})$, let $cl(\mathcal{C}_{\mathsf{env}}, \mathcal{C}_{\mathsf{sim}})$ denote the complexity class that includes all computations by $\mathcal{PPT}$ oracle Turing machines M with oracle access to $\mathcal{C}_{\mathsf{env}}$ and $\mathcal{C}_{\mathsf{sim}}$.

Definition 5 (($\mathcal{C}_{\mathsf{env}}, \mathcal{C}_{\mathsf{sim}}$)-Adaptive UC adaptive security)**.** *Let $\mathcal{F}$ and π_{ideal} be, as defined above, and π be a multi-party protocol. The protocol π is said to realize $\mathcal{F}$ with $(\mathcal{C}_{\mathsf{env}}, \mathcal{C}_{\mathsf{sim}})$-adaptive UC security, if for every $\mathcal{PPT}$ machine A, there exists a machine $A' \in \mathcal{C}_{\mathsf{sim}}$, such that, for every $Z \in \mathcal{C}_{\mathsf{env}}$, the following two ensembles are indistinguishable w.r.t $\mathcal{C}_{\mathsf{sim}}$.*

$$\left\{\mathsf{Exec}_{\pi,\mathsf{A},\mathsf{Z}}(\mathsf{n})\right\}_{\mathsf{n}\in\mathsf{N}} \approx \left\{\mathsf{Exec}^{\mathcal{F}}_{\pi_{\mathsf{ideal}},\mathsf{A}',\mathsf{Z}}(\mathsf{n})\right\}_{\mathsf{n}\in\mathsf{N}}$$

Using the above notation, traditional UC is equivalent to (n.u.$\mathcal{PPT}$,$\mathcal{PPT}$)-UC-security. We let QPS-UC denote (n.u.$\mathcal{PPT}$,$\mathcal{PQT}$)-UC-security[4] (where $\mathcal{PQT}$ denotes probabilistic quasi-polynomial time algorithms), and Non-uniform UC denote ($\mathcal{PPT}$,n.u.$\mathcal{PPT}$)-UC-security.

3 Equivocal Non-malleable Commitments

In this section, we define Equivocal Non-malleable Commitments. Intuitively, these are equivocal commitments such that even when a man-in-the-middle adversary receives equivocal commitments and openings from a simulator, the adversary himself remains unable to equivocate. Formal definitions are given below.

[4] We mentioned that this is stronger than the notion of QPS security of [27, 29, 2] which only consider indistinguishability w.r.t $\mathcal{PPT}$; we, in analogy with the notion of *strong QPS* of [27] require indistinguishability to hold also w.r.t $\mathcal{PQT}$.

Let $\langle S, R\rangle$ be a commitment scheme, and let $n \in N$ be a security parameter. Consider man-in-the-middle adversaries that are participating in left and right interactions in which $m = \text{poly}(n)$ commitments take place. We compare between a *man-in-the-middle* and a *simulated* execution. In the man-in-the-middle execution, the adversary A is simultaneously participating in m left and right interactions. In the left interactions the man-in-the-middle adversary A interacts with C receiving commitments to values $v_1, \ldots, v_m$, using identities $\mathsf{id_1}, \ldots, \mathsf{id_m}$ of its choice. It must be noted here that values $v_1, \ldots, v_m$ are provided to committer on the left prior to the interaction. In the right interaction A interacts with R attempting to commit to a sequence of related values again using identities of its choice $\tilde{\mathsf{id}}_1, \ldots, \tilde{\mathsf{id}}_m$; $\tilde{v}_i$ is set to the value decommitted by A in the j^{th} right interaction. If any of the right commitments are invalid its committed value is set to $\perp$. For any i such that $\tilde{\mathsf{id}}_i = \mathsf{id_j}$ for some j, set $\tilde{v}_i = \perp$—i.e., any commitment where the adversary uses the same identity as one of the honest committers is considered invalid. Let $\mathsf{MIM}^{A}_{\langle C,R\rangle}(v_1, \ldots, v_m, z)$ denote a random variable that describes the values $\tilde{v}_1, \ldots, \tilde{v}_m$ and the view of A, in the above experiment.

In the simulated execution, a simulator S directly interacts only with receivers on the right and proceeds as follows:

1. Whenever the commitment phase of j^{th} interaction with a receiver on the right is completed, S outputs a value $\tilde{v}_j$ as the alleged committed value in a special-output tape.

2. During the interaction, S may output a partial view for a man-in-the-middle adversary whose right-interactions are identical to S's interaction so far. If the view contains a left interaction where the i^{th} commitment phase is completed and the decommitment is requested, then a value v_i is provided as the decommitment.

3. Finally, S outputs a view and values $\tilde{v}_1, \ldots, \tilde{v}_m$. Let $\mathsf{sim}^{S}_{\langle C,R\rangle}(1^n, v_1, \ldots, v_m, z)$ denote the random variable describing the view output by the simulation and values $\tilde{v}_1, \ldots, \tilde{v}_m$; again, whenever $view$ contains a right interaction i where the identity is the same as any of the left interactions, $\tilde{v}_i$ is set to $\perp$.

Definition 6. *A commitment scheme $\langle C, R\rangle$ is said to be* concurrent non-malleable w.r.t. opening *if for every polynomial $p(\cdot)$, and every probabilistic polynomial-time man-in-the-middle adversary A that participates in at most $m = p(n)$ concurrent executions, there exists a probabilistic polynomial time simulator S such that the following ensembles are computationally indistinguishable over $n \in N$:*

$$\left\{\mathsf{MIM}^{A}_{\langle C,R\rangle}(v_1, \ldots, v_m, z)\right\}_{n\in N, v_1,\ldots,v_m\in\{0,1\}^n, z\in\{0,1\}^*}$$

$$\left\{\mathsf{sim}^{S}_{\langle C,R\rangle}(1^n, v_1, \ldots, v_m, z)\right\}_{n\in N, v_1,\ldots,v_m\in\{0,1\}^n, z\in\{0,1\}^*}$$

We can also consider relaxed notions of concurrent non-malleability: one-many, many-one and one-one secure non-malleable commitments. In a one-one (i.e., a stand-alone secure) non-malleable commitment, we consider only adversaries A that participate in one left and one right interaction; in one-many, A participates in one left and many right, and in many-one, A participates in many left and one right.

In this work, we consider a slight relaxation of this definition and show how to construct a protocol for the same.

Definition 7. *A commitment scheme $\langle C, R\rangle$ is said to be* concurrent non-malleable w.r.t. opening with independent and identically distributed (i.i.d) commitments *if for every polynomial $p(\cdot)$ and polynomial time samplable distribution D, and every probabilistic polynomial-time man-in-the-middle adversary A that participates in at most $m = p(n)$ concurrent executions, there exists a probabilistic polynomial time simulator S such that the following ensembles are computationally indistinguishable over $n \in N$:*

$$\left\{(v_1 \ldots, v_m) \leftarrow D^n : \mathsf{MIM}^{A}_{\langle C,R\rangle}(v_1, \ldots, v_m, z)\right\}_{n\in N, z\in\{0,1\}^*}$$

$$\left\{(v_1 \ldots, v_m) \leftarrow D^n : \mathsf{sim}^S_{\langle C,R\rangle}(1^n, v_1, \ldots, v_m, z)\right\}_{n\in N, z\in\{0,1\}^*}$$

Jumping ahead, the main reason to consider this relaxed definition is because the simulator will employ rewindings and in the rewinding the simulator will need to fix some value for the commitments in the left interactions and if these are chosen uniformly from a fixed distribution, the simulator can use the same. Finally, we consider commitment schemes that are additionally equivocable and in a setup model (i.e. the the simulator additionally can obtain a trapdoor).

Definition 8. *A commitment scheme $\langle C, R\rangle$ is said to be an* equivocal non-malleable commitment scheme *if it is both a language-based equivocal commitment scheme (see Definition 2) and is concurrent non-malleable w.r.t. opening (see Definitions 6 and 7).*

3.1 Adaptive UC-Puzzles

In this section, we give a formal definition of our new abstraction, *Adaptive UC-Puzzles*. Intuitively, an Adaptive UC-Puzzle is a protocol $\langle \mathsf{S}, \mathsf{R}\rangle$ between two players–a *sender* and a *receiver*–and a PPT computable relation $\mathcal{R}$, such that the following two properties hold:

Soundness: No efficient receiver R^* can successfully complete an interaction with S and also obtain a "trapdoor" y, such that $\mathcal{R}(\mathsf{TRANS}, y) = 1$, where TRANS is the transcript of the interaction.

Statistical UC-simulation with adaptive corruptions: For every efficient adversary $\mathcal{A}$ participating in a polynomial number of concurrent executions with receivers R (i.e., $\mathcal{A}$ is acting as a puzzle sender in all these executions) and at the same time communicating with an environment $\mathcal{Z}$, there exists a simulator $\mathcal{S}$ that is able to statistically simulate the view of $\mathcal{A}$ for $\mathcal{Z}$, while at the same time outputting trapdoors to all successfully completed puzzles. Moreover, $\mathcal{S}$ successfully simulates the view even when $\mathcal{A}$ may adaptively corrupt the receivers.

Formally, let $n \in N$ be a security parameter and $\langle \mathsf{S}, \mathsf{R}\rangle$ be a protocol between two parties, the sender S and the receiver R. We consider a **concurrent puzzle execution** for an adversary $\mathcal{A}$. In a **concurrent puzzle execution**, $\mathcal{A}$ exchanges messages with a puzzle-environment $\mathcal{Z} \in \mathcal{C}_{\mathsf{env}}$ and participates as a sender concurrently in $m = \mathrm{poly}(n)$ puzzles with honest receivers $\mathsf{R}_1, \ldots, \mathsf{R}_m$. At the onset of a concurrent execution, $\mathcal{Z}$ outputs a session identifier sid that all receivers in the concurrent puzzle execution receive as input. Thereafter, the puzzle-environment is allowed to exchange messages only with the adversary $\mathcal{A}$. We compare a *real* and an *ideal* execution.

Real execution. In the real execution, the adversary $\mathcal{A}$ on input 1^n, interacts with a puzzle-environment $\mathcal{Z} \in \mathcal{C}_{\mathsf{env}}$ and participates as a sender in m interactions using $\langle \mathsf{S}, \mathsf{R}\rangle$ with honest receivers that receive input sid (decided by $\mathcal{Z}$). The adversary $\mathcal{A}$ is allowed to exchange arbitrary messages with environment $\mathcal{Z}$ when participating in puzzle interactions with the receivers as a sender. In addition $\mathcal{A}$ may adaptively corrupt any of the receivers $\mathsf{R}_1, \ldots, \mathsf{R}_m$ at any point during or after the execution. We assume without loss of generality that, after every puzzle-interaction, $\mathcal{A}$ honestly sends TRANS to $\mathcal{Z}$, where TRANS is the puzzle-transcript. Finally, $\mathcal{Z}$ outputs a string in $\{0,1\}^*$. We denote this by $\mathsf{REAL}_{\mathcal{A},\mathcal{Z}}(n)$.

Ideal execution. Consider $\mathcal{A}' \in \mathcal{C}_{\mathsf{sim}}$ in the ideal-model that has a special output-tape (not accessible by $\mathcal{Z}$). In the ideal execution, $\mathcal{A}'$ on input 1^n interacts with puzzle-environment $\mathcal{Z}$. We denote the output of $\mathcal{Z}$ at the end of the execution by $\mathsf{IDEAL}_{\mathcal{A}',\mathcal{Z}}(n)$.

Definition 9. *Adaptive UC-Puzzle. A pair $(\langle \mathsf{S}, \mathsf{R}\rangle, \mathcal{R})$ is a $(\mathcal{C}_{\mathsf{env}}, \mathcal{C}_{\mathsf{sim}})$-secure Adaptive UC-puzzle for a polynomial time computable relation $\mathcal{R}$ and model $(\mathcal{C}_{\mathsf{env}}, \mathcal{C}_{\mathsf{sim}})$, if the following conditions hold.*

Soundness: *For every malicious PPT receiver $\mathcal{A}$, there exists a negligible function $(\cdot)$ such that the probability that $\mathcal{A}$, after an execution with* S *on common input 1^n, outputs y such that $y \in \mathcal{R}(\mathsf{TRANS})$ where* TRANS *is the transcript of the messages exchanged in the interaction, is at most $()$.*

Statistical Simulatability: *For every adversary $\mathcal{A} \in \mathcal{C}_{\mathsf{env}}$ participating in a* ***concurrent puzzle execution****, there is a simulator $\mathcal{A}' \in \mathcal{C}_{\mathsf{sim}}$ such that for all puzzle-environments $\mathcal{Z} \in \mathcal{C}_{\mathsf{env}}$, the ensembles $\{\mathsf{REAL}_{\mathcal{A},\mathcal{Z}}(n)\}_{n\in\mathbb{N}}$ and $\{\mathsf{IDEAL}_{\mathcal{A}',\mathcal{Z}}(n)\}_{n\in\mathbb{N}}$ are statistically close over $n \in Nat$ and whenever $\mathcal{A}'$ sends a message of the form* TRANS *to $\mathcal{Z}$, it outputs y in its special output tape such that $y \in \mathcal{R}(\mathsf{TRANS})$.*

4 Achieving Adaptive UC-Security

In this section, we give a high-level overview of the construction of an EQNMCom scheme and the proof of Theorem 1, which relies on the existence of an EQNMCom scheme. For the formal construction and analysis of our EQNMCom scheme, see Section 5. For the restatement and formal proof of Theorem 1, see Section 6.

By relying on previous results, the construction of an adaptive UC-secure protocol for realizing any multiparty functionality reduces to the task of constructing a commitment protocol that satisfies the following two properties:

Concurrent equivocability: For every adversary A receiving honest commitments, there needs to be a way to generate commitments in an *online* manner for honest parties before knowing the message being committed to and later be able to decommit to any specified message.

Concurrent extraction: For every adversary A, there needs to be a way to extract the commitments in an *online* manner that are adversarially generated.

Below we introduce the notion of an adaptive UC-puzzle. Then, we show how to construct an equivocal non-malleable commitment scheme based on any adaptive UC-puzzle. Then combining the equivocal non-malleable commitment scheme with a simulatable encryption scheme we show how to obtain a commitment scheme that is concurrently equivocable and concurrently extractable.

4.1 Constructing EQNMCom based on Adaptive UC-Puzzles

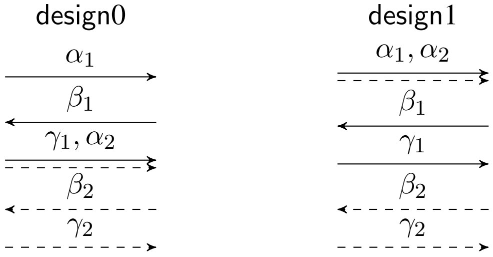
Our protocol is based on the protocol for non-malleable commitments in [22]. However, while the [22] commitment scheme is statistically binding, we must construct non-malleable commitments that are not just *computationally* binding, but are actually *equivocal.* In the protocol from [22], the receiver first sends $f(x)$ for a uniformly chosen x under a one-way function x and this x serves as a trapdoor for the simulator. We here instead, rely on UC-puzzle to achieve this. So we replace Stage 1 with a UC-puzzle where the receiver of the commitment is the sender of the puzzle. Next, since we need an equivocal commitment scheme, we replace the statistically binding commitment sent by the commiter in Stage 2 of the [22] protocol with a language-based equivocal commitment where the NP-language is the one generated by the transcript of the puzzle interaction and witness being the puzzle solution. Informally, this is constructed by relying on a variant of Feige-Shamir's trapdoor commitment.[5] Finally, we replace the WIPOK invocations in Stage 3 with invocations of an *adaptively-secure* (without erasures) WIPOK (see Appendix B) following the work of [25]. More specifically, our protocol proceeds as follows:

[5] Let x be an NP-statement. The sender commits to bit b by running the honest-verifier simulator for Blum's Hamiltonian Circuit protocol [3] on input the statement x and the verifier message b, generating the transcript (a, b, z), and finally outputting a as its commitment. In the decommitment phase, the sender reveals the bit b by providing both b, z.

1. In Stage 1, the Committer and Receiver exchange a UC-puzzle where the Receiver is the sender of the puzzle and the Committer is the receiver of the puzzle. Let x be the transcript of the interaction.

2. In Stage 2, the Committer sends $c = \mathsf{EQCom}^x(v)$, where EQCom is a language-based equivocal commitment scheme as in Definition 2 with common input x.

3. In Stage 3, the Committer proves that c is a valid commitment for v. This is proved by 4ℓ invocations of an adaptively-secure (without erasures) WIPOK (See Appendix B) where the messages are scheduled based on the id (as in [15, 22]). More precisely, there are ℓ rounds, where in round i, the schedule $\mathsf{design}_{\mathsf{id_i}}$ is followed by $\mathsf{design}_{1-\mathsf{id_i}}$ (See Figure 4.1).



The basic idea in [22] that originates from [15], is to construct a protocol where the scheduling of the messages depends on the tag of the commitment. The scheduling ensures that for every right interaction with a tag that is different from the left interaction, there exists a point—called a safe-point—from which we can *rewind* the right interaction (and extract the committed value), without violating the hiding property of the left interaction. It now follows from the hiding property of the left interaction that the values committed to on the right do not depend on the value committed to on the left. To construct a simulator-extractor for our protocol, we unfortunately cannot rely on the above. This is because it will allow to rewind a right interaction safely w.r.t. only one left interaction and we need to deal with an unbounded number of concurrent executions on the left. Instead, we have the simulator simulate the puzzle while extracting the trapdoor, equivocate the left commitments (using the trapdoor) and then rewind a random WIPOK in the right-interaction to obtain the adversary's commitment. This could be problematic since the adversary could gain knowledge of the trapdoor in such rewinds. To avoid this, in all the rewindings we let the simulator follow the honest committer strategy in the left interaction. This is possible since the protocol is adaptively secure and from any point in the interaction the simulator can generate coins for an honest committer in the left-interaction. But to do this, we need to choose a value that the honest committer is committing to (if one has not yet been decommitted). Since we are constructing a commitment scheme for the relaxed notion where all the left commitments are uniformly sampled from D, the simulator in the rewindings just samples a value from D for each left interaction and uses this value. While the simulation itself does not utilize safe-points, proving correctness of the simulation proceeds in hybrids where the left commitments are equivocated one at time and correctness in successive hybrids is proved using safe-points.

4.2 Adaptive UC-secure Commitment Scheme

We now provide the construction of a commitment scheme that is concurrently equivocable and concurrently extractable based on any adaptive UC-puzzle and simulatable encryption scheme. As mentioned earlier, this will imply that adaptive UC-secure computation is feasible in any model where there exists a UC-puzzle. First, we recall the construction of the adaptive UC-secure commitment in the common reference string model (CRS) from [7] to motivate our construction.

In the [7] construction, the CRS contains two strings. The first string consists of a random image $y = f(x)$ of a one-way function f and the second string consists of a public key for a cca-secure encryption scheme. The former allows a simulator to equivocate the commitment when it knows x and the public key allows the simulator to extract committed values from the adversary using its knowledge of the corresponding secret-key. The additional CCA requirement ensures non-malleability.

Our construction follows a similar approach, with the exception that instead of having a common reference string generated by a trusted party, we use the equivocal non-malleable commitment to generate two common-reference strings between every pair of parties: one for equivocation and the other for extraction. This is achieved by running the following "non-malleable" coin-tossing protocol between an initiator and a responder.

1. The initiator commits to a random string r^0 using $\langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle$ to the responder.

2. The responder chooses a random string r^1 and sends to the Initiator.

3. The initiator opens its commitment and reveals r^0.

4. The output of the coin toss is: $r = r^0 \oplus r^1$.

The coin-tossing protocol is run between an initiator and responder and satisfies the following:

- For all interactions where the initiator is honest, there is a way to simulate the coin-toss. This follows directly from the equivocability of the commitment scheme $\langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle$.

- For all interactions where the initiator is controlled by the adversary, the coin-toss generated is uniformly-random. This follows from the simulation-extractability of the commitment scheme.

Using the coin-tossing protocol we construct an adaptive UC-commitment scheme.

First, the sender and receiver interact in two coin-tossing protocols, one where the sender is the initiator, with outcome $coin_1$ and the other, where the receiver is the initiator, with outcome $coin_2$. Let x be the statement that $coin_1$ is in the image of the PRG G. Also let, $\text{PK} = \mathsf{oGen}(coin_2)$ be a public key for the simulatable encryption scheme. To commit to a string β, the sender sends a commitment to β using the non-interactive language-based commitment scheme with statement x along with strings S_0 and S_1 where one of the two strings (chosen at random) is an encryption of decommitment information to β and the other string is outputted by $\mathsf{oRndEnc}$. In fact, this is identical to the construction in [7], with the exception that a simulatable encryption scheme is used instead of a CCA-secure scheme.

Binding follows from the soundness of the adaptive UC-puzzle and hiding follows from the hiding property of the non-interactive commitment scheme and the semantic security of the encryption scheme.

It only remains to show that the scheme is concurrently equivocable and extractable. To equivocate a commitment from a honest committer, the simulator manipulates $coin_1$ (as the honest party is the initiator) so that $coin_1 = G(s)$ for a random string s and then equivocates by equivocating the non-interactive commitment and encrypting the decommitment information for both bits 0 and 1 in S_b and S_{1-b} (where b is chosen at random). To extract a commitment made by the adversary, the simulator manipulates $coin_2$ so that $coin_2 = \mathsf{rGen}(r)$ and $(\text{PK}, \text{SK}) = \mathsf{Gen}(r)$ for a random string r. Then it extracts the decommitment information in the encryptions sent by the adversary using SK.

The procedure described above works only if the adversary does not encrypt the decommitment information for both 0 and 1 even when the simulator is equivocating. We rely on the simulation-extractability of the $\langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle$-scheme to prove this. On a high-level, this follows since, if the coin-toss $coin_1$ cannot be manipulated by the adversary when it is the initiator, then the $coin_1$ is not in the range of G with very high probability and hence the adversary cannot equivocate (equivocating implies a witness can be extracted that proves that $coin_1$ is in the range of G).

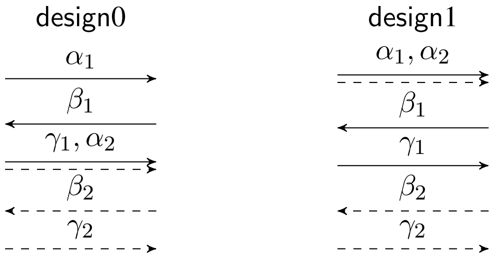
5 The Equivocal Non-Malleable Commitment Scheme $(\mathsf{EQNMCom})$ $\Pi = \langle \mathsf{S_{com}}, \mathsf{R_{com}} \rangle$

We note that the construction presented here is the same as the construction of [15, 22] with the following changes: the statistically-binding commitment is replaced with an equivocal commitment and the special-sound WI proofs are replaced with adaptively-secure WIPOK's. Although the constructions are similar, the analysis here differs significantly from the analysis of the previous constructions of [15, 22] where the fact that the first commitment is statisticall-binding plays a large part in the proof.

The protocol proceeds in the following two stages on common input the identity $\mathsf{id} \in \{0,1\}^\ell$ of the committer, common string x, and security parameter n.

1. In Stage 1, the Committer sends $c = \mathsf{EQCom}^x(v)$, where EQCom is a language-based equivocal commitment scheme as in Definition 2 with common input x.

2. In Stage 2, the Committer proves that c is a valid commitment for v. This is proved by 4ℓ invocations of an adaptively-secure (without erasures) WIPOK (See Appendix B) where the messages are scheduled based on the id (as in [15, 22]). More precisely, there are ℓ rounds, where in round i, the schdule $\mathsf{design_{id_i}}$ is followed by $\mathsf{design_{1-id_i}}$ (See Figure 5).



Commitment Protocol $\Pi = \langle \mathsf{S_{com}}, \mathsf{R_{com}} \rangle$

Common input: An identifier $\mathsf{id} \in \{0,1\}^\ell$ and common input x.

Auxiliary Input for Committer: A string $v \in \{0,1\}^n$.

Stage 1:

C uniformly chooses $r \in \{0,1\}^{\mathrm{poly}(n)}$.

$C \to R$: $c = \mathsf{EQCom}^x(v;r)$.

Stage 2:

$C \to R$: 4ℓ adaptively-secure WIPOK of the statement there exist values v, r such that $c = \mathsf{EQCom}^x(v;r)$ with verifier query of length $2n$, in the following schedule: For $j = 1$ to ℓ do: Execute $\mathsf{design_{id_j}}$ followed by Execute $\mathsf{design_{1-id_j}}$.

Figure 1: Equivocal Non-Malleable Commitment Scheme $\Pi = \langle \mathsf{S_{com}}, \mathsf{R_{com}} \rangle$

5.1 Analysis

In this subsection, we prove that $\Pi = \langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle$ is an equivocal non-malleable commitment scheme when combined with an adaptive UC-puzzle in a preamble phase where the receiver acts the sender and the committer acts as the receiver and the NP-statement x used in $\langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle$ is the transcript of the interaction from the preamble phase. More precisely, consider the following protocol: Let $(\langle \mathsf{S}, \mathsf{R} \rangle, \mathcal{R})$ be a $(\mathcal{C}_{\mathsf{env}}, \mathcal{C}_{\mathsf{sim}})$-secure adaptive UC-puzzle. The protocol $\overline{\Pi}$ proceeds in the following two phases on common input the identity $\mathsf{id} \in \{0,1\}^{\ell}$ of the committer, and private-input string r for the committer and security parameter n.

Preamble Phase: An adaptive UC-Puzzle interaction $\langle \mathsf{S}, \mathsf{R} \rangle$ on input 1^n where $\mathsf{S}_{\mathsf{com}}$ is the receiver and $\mathsf{R}_{\mathsf{com}}$ is the sender. Let $x = \mathsf{TRANS}$ be the transcript of the messages exchanged.

Commitment Phase: The parties run protocol $\langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle$ with common input x and identifier id. S plays the part of sender with input r.

We now show that the protocol $\overline{\Pi}$ is concurrent non-malleable w.r.t opening in the $(\mathcal{C}_{\mathsf{env}}, \mathcal{C}_{\mathsf{sim}})$-model.

THEOREM 2. *Commitment scheme $\overline{\Pi}$ described above is concurrent non-malleable w.r.t. opening with independent and identically distributed (i.i.d) commitments*

Before we prove this theorem, we first show that Π is a language-based equivocal commitment scheme:

Lemma 1. *Commitment scheme $\Pi = \langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle$ shown in Figure 1 is a language-based equivocal commitment scheme.*

Proof. In order to prove the lemma we need to present an equivocator $(\tilde{\mathsf{S}}_{\mathsf{com}}, \mathsf{Adap}_{\mathsf{com}})$ for $\langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle$ and prove that $(\tilde{\mathsf{S}}_{\mathsf{com}}, \mathsf{Adap}_{\mathsf{com}})$ has the required properties listed in Definition 2. Intuitively the equivocator, $\tilde{\mathsf{S}}_{\mathsf{com}}$, will run the equivocator for the commitment scheme EQCom as well as the simulator for the WIPOK. Then, $\mathsf{Adap}_{\mathsf{com}}$ will run $\mathsf{Adap}_{\mathsf{eq}}$ for the EQCom scheme and also will adaptively corrupt the prover and run the simulator for the WIPOK, which produces a simulated view for the prover. By taking a closer look at the simulator for the WIPOK presented in Appendix B we see that, in fact, $\tilde{\mathsf{S}}_{\mathsf{com}}$ simply replaces *every* commitment under EQCom in $\langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle$ (in both Stage 1 and Stage 2) with an equivocal commitment generated by the equivocator, $\tilde{\mathsf{S}}_{\mathsf{eq}}$ for the commitment scheme EQCom. The fact that $\tilde{\mathsf{S}}_{\mathsf{com}}$ has this form will be crucial for the proof of Lemma 3.

We omit the proof that $(\tilde{\mathsf{S}}_{\mathsf{com}}, \mathsf{Adap}_{\mathsf{com}})$ as described above has the desired properties since it follows straightforwardly from the security properties of EQCom and the adaptive security (without erasures) of the WIPOK. □

Now, we turn towards proving Theorem 2.

Proof of Theorem 2: First we describe the simulator and then prove correctness. Let A be a concurrent man-in-the-middle adversary that on input 1^n participates in at most $m(n)$ left-interactions as a receiver, receiving commitments from an honest committer whose values are chosen uniformly from distribution D and at most $m(n)$ right-interactions as a committer.

For simplicity, we assume that the puzzle-simulation is straight-line. Towards the end, we mention how to modify the simulator when the simulation is not straight-line. On a high-level, S internally incorporates A and emulates an execution with A as follows:

1. For all puzzle interactions where A^* controls the sender, S follows the puzzle simulator's strategy to simulate the puzzle and obtains a witness which it stores.

2. For all the messages exchanged by A^* in the right interactions, Sim simply forwards the messages to an external receiver.

3. For every left interaction, Sim internally generates the messages using the simulator for the commitment scheme (that can equivocate the commitment) with the witness w obtained from the puzzle interactions. When a decommitment is requested by A, Sim outputs the current partial view of the transcript of messages exchanged by A in a special-output tape and receives from outside a value v. Internally, it runs the simulator for the stand-alone commitment scheme to decommit to v.

4. Whenever the commitment phase with a receiver is completed on the right, Sim temporary stalls the main-execution and tries to extract the value committed to by A in this interaction. For this, Sim selects a random WIPOK from that interaction and *rewinds* A to the point just before which A receives the challenge-message in the WIPOK. Sim supplies a new challenge message and continues simulation. In this simulation, the right interactions are simulated as before (i.e. honestly). However the left interactions are not simulated as before (i.e. equivocating the commitment phase). Instead they are generated using an honest committer committing to a value v, where v is either the decommitment for that left interaction, if one has been obtained by Sim in the main-execution, or a uniformly chosen sample from D.[6] If in the rewinding, A provides a valid response for the selected WIPOK of the right interaction, then using the special-sound property of the WIPOK, Sim extracts the witness used in the WIPOK, which contains the committed value. If the adversary fails to provide a valid response for the particular WIPOK in the right interaction, Sim cancels the current rewinding and starts a new rewinding by supplying a new challenge.

The proof of correctness of the simulator is expressed in the following lemma.

Lemma 2. *The following ensembles are computationally indistinguishable*

$$\left\{(v_1 \ldots, v_m) \leftarrow D^n : \mathsf{MIM}^A_{\langle C,R\rangle}(v_1, \ldots, v_m, z)\right\}_{n\in N, z\in\{0,1\}^*}$$

$$\left\{(v_1 \ldots, v_m) \leftarrow D^n : \mathsf{sim}^S_{\langle C,R\rangle}(1^n, v_1, \ldots, v_m, z)\right\}_{n\in N, z\in\{0,1\}^*}$$

Proof of Lemma 2. Towards proving this, we consider a sequence of intermediate hybrid experiments $H_0, \ldots, H_m$. In experiment H_i, we consider a simulator Sim^i that knows the values $(v_1, \ldots, v_i)$ being committed to in the first i left interactions. On input z, Sim^i proceeds as follows: It proceeds exactly as Sim with the exception that only the first i left-interactions are equivocated while the rest are simulated using the honest committer algorithm, committing to values $(v_{i+1}, \ldots)$ both in the main-execution as well as in the rewinding. Let $\mathsf{hyb}^i_A(1^n, v_1, \ldots, v_m, z)$ denote the output of Sim^i in H_i. It follows from description that $\mathsf{hyb}^m_A(1^n, v_1, \ldots, v_m, z) = \mathsf{sim}^S_{\langle C,R\rangle}(1^n, v_1, \ldots, v_m, z)$The proof of the Lemma follows from the next two claims using a standard hybrid argument.

Claim 1. *The following ensembles are computationally indistinguishable*

$$\left\{(v_1 \ldots, v_m) \leftarrow D^n : \mathsf{MIM}^A_{\langle C,R\rangle}(v_1, \ldots, v_m, z)\right\}_{n\in N, z\in\{0,1\}^*}$$

$$\left\{(v_1 \ldots, v_m) \leftarrow D^n : \mathsf{hyb}^0_A(1^n, v_1, \ldots, v_m, z)\right\}_{n\in N, z\in\{0,1\}^*}$$

Proof of Claim 1. Recall that in hybrid experiment H_0, the simulator simulates all the commiters in the left interaction using the honest committer algorithm. The only difference from the MIM experiment is that the puzzles are simulated. Assume for contradiction, there exists an adversary A, distinguisher D, polynomial $p(\cdot)$ such that, for inifinitely many n, D distinguishes the ensembles in the claim with probability at least $\frac{1}{p(n)}$. From the definition of the puzzle we have that the distribution of the views in the outputs of $\mathsf{MIM}^A_{\langle C,R\rangle}(v_1, \ldots, v_m, z)$ and $\mathsf{hyb}^0_A(1^n, v_1, \ldots, v_m, z)$ are statistically-close. Furthermore, if the

[6] Sim can generate such messages for any value v, since by adaptive security, Sim can obtain random coins for an honest committer and any value v that is consistent with any partial transcript generated by the equivocator.

value extracted by the simulator in $\mathsf{hyb}_A^0(1^n, v_1, \ldots, v_m, z)$ in each interaction is consistent with the decommitment made by the adversary in the view output by the simulator, then $\mathsf{MIM}^A_{\langle C,R\rangle}(v_1, \ldots, v_m, z)$ and $\mathsf{hyb}_A^0(1^n, v_1, \ldots, v_m, z)$ are statistically-close. Hence, if D distinguishes the distributions, it must be the case that, the values output in both the experiments differs with probability at least $\frac{1}{p(n)}$. This happens, whenever the value output by the simulator in hyb_A^0 is inconsistent with the view output by the simulator. Hence, the Sim^0 obtains two decommitments for the same commitment (one as part of the main-execution and one obtained using the witness extracted) for a commitment made by the adversary in some right-interaction with probability at least $\frac{1}{p(n)}$. With any two valid decommitments, a solution to the puzzle from the preamble phase can be obtained. Consider a slightly altered simulation $\overline{\mathsf{Sim}}^0$ that proceeds exactly like Sim^0 with the exception that all the puzzle interactions in the left interaction are simulated honestly. It follows from the statisical-simulatability of the puzzle that with non-negligible probability, $\overline{\mathsf{Sim}}^0$ extracts a witness for a puzzle in a right interaction where the adversary is a receiver of the puzzle. Hence, if $\overline{\mathsf{Sim}}^0$ runs in $\mathcal{PPT}$, then, $\overline{\mathsf{Sim}}^0$ with adversary A can be used to construct an adversary that violates the soundness of the adapative UC-puzzle.[7] It only remains to argue that $\overline{\mathsf{Sim}}^0$ run in $\mathcal{PPT}$ and then we arrive at a contradiction. Recall that for every right interaction that completes the commitment phase, Sim^0, and hence $\overline{\mathsf{Sim}}^0$ rewinds repeatedly until it obtains a witness for a random WIPOK. We argue that the expected number of restarts for every right- interaction is $O(1)$ and therefore the expected running time of $\overline{\mathsf{Sim}}^0$ is bounded by some polynomial. Fix a particular right-interaction that completes the commitment phase and select a WIPOK. Given the first message of the WIPOK, let p be the probability that over a random challenge-message that A provides a valid response. Since the rewindings are identically distributed to the main-execution, the expected number of restarts required before $\overline{\mathsf{Sim}}^0$ encounters another execution where A provides a valid response is $\frac{1}{p}$. However, note that $\overline{\mathsf{Sim}}^0$ needs to perform the rewinding only with probability p since otherwise the right-interaction does not complete the commitment phase. Therefore, the expected number of restarts for a particular right interaction is $p \times \frac{1}{p} = 1$. □

Claim 2. *The following ensembles are computationally indistinguishable*

$$\left\{(v_1 \ldots, v_m) \leftarrow D^n : \mathsf{hyb}_A^0(1^n, v_1, \ldots, v_m, z)\right\}_{n \in N, z \in \{0,1\}^*}$$

$$\left\{(v_1 \ldots, v_m) \leftarrow D^n : \mathsf{hyb}_A^m(1^n, v_1, \ldots, v_m, z)\right\}_{n \in N, z \in \{0,1\}^*}$$

Proof of Claim 2. Assume for contradiction, there exists an adversary A, distinguisher D, polynomial $p(\cdot)$ such that, for inifinitely many n, D distinguishes the ensembles in the claim with probability at least $\frac{1}{p(n)}$. Then there exists a function $i : \mathcal{N} \to \mathcal{N}$ such that for infinitely many n, D distinguishes the following two ensembles with probability at least $\frac{1}{mp(n)}$.

$$\left\{(v_1 \ldots, v_m) \leftarrow D^n : \mathsf{hyb}_A^{i(n)-1}(v_1, \ldots, v_m, z)\right\}_{n \in N, z \in \{0,1\}^*}$$

$$\left\{(v_1 \ldots, v_m) \leftarrow D^n : \mathsf{hyb}_A^{i(n)}(1^n, v_1, \ldots, v_m, z)\right\}_{n \in N, z \in \{0,1\}^*}$$

Let H'_j denote the experiment that proceeds identically to H_j, with the exception that the simulator performs no rewinding. Let $\mathsf{hyb'}_A^j$ denote the random variable that represents the view output by the simulator in H'_j. It follows from description that $\mathsf{hyb'}_A^j$ is identically distributed to the view in hyb_A^j since the rewindings are conducted independent of the main-execution.

We first claim that the following ensembles are indistinguishable for any function $j : \mathcal{N} \to \mathcal{N}$.

$$\left\{(v_1 \ldots, v_m) \leftarrow D^n : \mathsf{hyb'}_A^{j(n)-1}(v_1, \ldots, v_m, z)\right\}_{n \in N, z \in \{0,1\}^*}$$

[7] Simply forward a random puzzle interaction of A during the straight-line simulation to an external sender of a puzzle execution and then internally obtain two decommitments of A and extract a witness whenever A equivocates

$\left\{(v_1 \ldots, v_m) \leftarrow D^n : \mathsf{hyb'}_A^{j(n)}(1^n, v_1, \ldots, v_m, z)\right\}_{n \in N, z \in \{0,1\}^*}$

This is because the only difference between $H'_{j(n)-1}$ and $H'_{j(n)}$ is that the $j(n)^{th}$ left interaction is equivocated and therefore indistinguishability directly follows from the strong-hiding property of the equivocal commitment.

Recall that if the values extracted by the simulator is always equal to the value decommitted to by the adversary, then the above claim implies that $\mathsf{hyb}_A^{i(n)-1}$ and $\mathsf{hyb}_A^{i(n)}$ are indistinguishable. Therefore, it must be the case that, for infinitely many n, with probability at least $\frac{1}{2mp(n)}$ the value extracted by the simulator is different from the value decommitted to by A. Furthermore, there exists a function $k : \mathcal{N} \to \mathcal{N}$ such that, for infinitely many n, the value extracted by the simulator in the $k(n)^{th}$ right interaction is different from the value decommited to by A in the main-execution with probability at least $\frac{1}{2m^2p(n)}$. Let $\mathsf{hyb}_A^{i,k}$ denote the view output of the simulator in H_i and the value extracted in the k^{th} right interaction. Then there exists a function $k(n)$ such that the probability with which the value output is not the value decommitted in the view jumps by at least $\frac{1}{2m^2p(n)}$ when comparing $\mathsf{hyb}_A^{i(n)-1,k(n)}$ and $\mathsf{hyb}_A^{i(n),k(n)}$ with probability at least $\frac{1}{2m^2p(n)}$ for infinitely many n. Lets say that a $(view, v)$ pair is k- cons if v is the value decommited to by the adversary in k^{th} right-interaction.

We consider the following intermediate hybrid experiments:

Hybrid $\bar{H}_0^k = H_{i-1}$: This experiment proceeds identically to H_{i-1} with the exception that the simulator only extracts the decommitment from the k^{th} right interaction. Define $\overline{\mathsf{hyb}}_A^0$ to be the view output and the value extracted by the simulator, i.e. $\overline{\mathsf{hyb}}_A^0 = \mathsf{hyb}_A^{i-1,k}$.

Hybrid $\bar{H}_1^k$: In the k^{th} right-interaction, we say that a particular WIPOK is a safe WIPOK, if the "safe-point" of this interaction w.r.t i^{th} left interaction corresponds to this WIPOK. The definition of safe-point is analogous and identical to the safe-points defined in [22].[8]The experiment $\bar{H}_1^k$ proceeds identically to $\bar{H}_0^k$ with the exception that it rewinds the adversary to a safe WIPOK in the k^{th} right interaction instead of a choosing a random WIPOK and the i^{th} left interaction. Define $\overline{\mathsf{hyb}}_A^1$ to be the view output and the value extracted by the simulator.

Hybrid $\bar{H}_2^k$: This experiment proceeds identically to $\bar{H}_1^k$ with the exception that the i^{th} left interaction is simulated using *fresh randomness* in each rewinding. In particular, if the next message in the i^{th} left interaction is the first message of a WIPOK sub-protocol, then fresh randomness is used to generate it.[9] Recall that, in the actual simulator and previous hybrids, this is not the case and in the rewinding phase, the randomness of the all the left interactions are fixed. Furthermore, whenever the adversary tries to corrupt the i^{th} left interaction in a rewinding the simulator cuts off the rewinding and restarts. Define $\overline{\mathsf{hyb}}_A^2$ to be the view output and the value extracted by the simulator.

Hybrid $\bar{H}_3^k$: This experiment proceeds identically to $\overline{\mathsf{hyb}}_A^2$ with the exception that in the i^{th} left interaction, the simulator equivocates the commitment both in the main-execution as well as in the rewindings. Again, as in the previous hybrid, a fixed random tape is used for all the left-interactions in the rewindings except the i^{th} interaction where fresh randomness is used in the rewindings. Every rewinding where the adversary tries to corrupt the committer in the i^{th} left-interactions is cancelled. Define $\overline{\mathsf{hyb}}_A^3$ to be the view output and the value extracted by the simulator.

[8] Intuitively, a safe-point ρ of a right interaction, is a point in Δ that lies in between the first two messages α_r and β_r of a WIPOK proof $(\alpha_r, \beta_r, \gamma_r)$ in the right interaction k, such that, when rewinding from ρ to γ_r, if A uses the *same* "scheduling of messages" as in Δ, then the left interaction can be emulated without affecting the hiding property. See [] for more details.

[9] Jumping ahead, this will allow the i^{th} left-interaction to be forwarded externally to a committer, analogous to [15, 23].

Hybrid $\bar{H}_4^k$: The experiment proceeds identically to $\overline{\mathsf{hyb}}_A^3$ with the exception that the i^{th} left interaction is also simulated using a fixed random tape for the committer in all the rewindings. Define $\overline{\mathsf{hyb}}_A^4$ to be the view output and the value extracted by the simulator.

Hybrid $\bar{H}_5^k = H_i$: The experiment proceeds identically to H_i with the exception that the simulator only extracts from the k^{th} right interaction . Define $\overline{\mathsf{hyb}}_A^5$ to be the view output and the value extracted by the simulator, i.e. $\overline{\mathsf{hyb}}_A^5 = \mathsf{hyb}_A^{i,k}$.

Since, the difference in probability that $\mathsf{hyb}_A^{i(n)-1,k(n)}$ and $\mathsf{hyb}_A^{i(n),k(n)}$ are k- cons is at least $\frac{1}{p_1(n)} = \frac{1}{2m^2p(n)}$ for infinitely many n, there must exists a $c \in \{1,2,3,4,5\}$ such that the difference in probability that $\overline{\mathsf{hyb}}_A^{c-1}$ and $\overline{\mathsf{hyb}}_A^{c}$ are k- cons is at least $\frac{1}{5p_1(n)}$ for infinitely many n. We argue below for every c that we arrive at a contradiction if the above statement holds for c.

Comparing $\bar{H}_0^k$ and $\bar{H}_1^k$ In this case, we have that, for infinitely many n,

$$|\Pr[\overline{\mathsf{hyb}}_A^0 \text{ is } k\text{-cons }] - \Pr[\overline{\mathsf{hyb}}_A^1 \text{ is } k\text{-cons }]| \geq \frac{1}{5p_1(n)}$$

Proof Sketch: Since the only difference between $\bar{H}_0^k$ and $\bar{H}_1^k$ is in which WIPOK is rewound to extract the witness, it must be the case that the probability that the witness extracted from a random WIPOK and the specific WIPOK chosen from the safe point is different must be at least $\frac{1}{5p_1(n)}$. Using this fact, we arrive at a contradiction to the soundness of the puzzle.

First, we note that it is possible for a simulator to check if the value extracted in a random WIPOK and a safe WIPOK are the same. Recall that in the left interactions of the main execution in $\bar{H}_1^k$ and $\bar{H}_2^k$, the simulator is equivocating the first i commitments and honestly committing in the rest of them. This in particular means that the value decommitted to in the first i commitments are chosen after the commitment phase. In the rewinding phase, when the simulator tries to extract the witness from a WIPOK, it simulates the left interactions by using the honest committer strategy with a fixed random tape. Consider an experiment E_0^k where the simulator continues the execution until A completes the commitment phase in the kth right-interaction and then cuts off the simulation. Then it extracts the witness from a random WIPOK and the safe WIPOK. If the values are different, the simulator extracts the solution of the puzzle and outputs it. It follows that the simulator outputs the solution of the puzzle with non-negligible probability.

We consider of hybrid experiments and show that in each of them the simulator can output a solution with non-negligible probability and finally arrive at a simulator that violates the soundness of the puzzle.

The first intermediate experiment E_1^k we consider is where the simulator chooses the value to be committed in the first k right interactions before the interaction begins. This modification does not affect the view obtained in the main-execution because all values in the left interactions are chosen independently from distribution D. It also does not affect the rewindings, because the commiters strategy is fixed, i.e. its random tape and commitment are fixed. Therefore, E_1^k and E_0^k proceed identically and the simulator outputs the solution to the puzzle with non-negligible probability in E_1^k as well.

In Lemma 3, we show that, it is possible to construct an honest committers algorithm C^* for $\langle \mathsf{S_{com}}, \mathsf{R_{com}} \rangle$ that knows the witness of the common input statement x and receives a polynomial sequence of strings $s_1, \ldots, s_m$ such that

- The transcript generated by C^* committing to string v, when the strings received as input $s_1, \ldots, s_m$ are uniformly random, is identically distributed to the transcript of an interaction with an honest committer, committing to a value v, and,

- The transcript generated by C^* committing to string v, when the strings received as input $s_1, \ldots, s_m$ are random commitments to 1 using Com, is identically distributed to the transcript generated by an equivocal commitment using the witness for statement x and decommitted to value v.

if the value to be decommitted to is known at the beginning of an execution, then the commitment phase can be generated using an honest-committer's strategy that additionally receives as input a particular sequence of strings that are either uniformly random or commitments to 0 and 1 under $\langle C, R \rangle$. We now observe that in E_1^k, although the simulator is equivocating the first i commitments in the left, the value to be decommitted to is chosen before the execution begins. Consider the experiment E_2^k that proceeds identically to E_1^k with the exception that the simulator generates the equivocal commitments by using the committer strategy C^* that receives as input a sequence of commitments to 1 using Com. This experiment proceeds identically to E_1^k and the simulator extracts the solution to the puzzle with non-negligible probability.

In the next experiment E_3^k, we consider a simulator that proceeds identically to E_2^k with the exception that the sequence of strings received by the honest-committers in the first k left interactions are chosen uniformly at random (as opposed to commitments to 1). It now follows from the pseudo-randomness of the commitments under Com that the simulator extracts the solution to the puzzle in E_3^k with non-negligible probability. Now, observe that experiment E_3^k is identical to experiment H_0 and this violates the soundness of the puzzle interaction. Thus, we arrive at a contradiction. □

Comparing $\bar{H}_1^k$ and $\bar{H}_2^k$ In this case, we have that, for infinitely many n,

$$|\Pr[\overline{\mathsf{hyb}}_A^1 \text{ is } k\text{-cons }] - \Pr[\overline{\mathsf{hyb}}_A^2 \text{ is } k\text{-cons }]| \geq \frac{1}{5p_1(n)}$$

Proof Sketch: We will again show how to construct an adversary that violates the soundness of the puzzle. The proof of this follows identically as in the previous case. We again consider a simulator that cuts off the adversary after the commitment phase on the k^{th} right interaction is completed and then rewinds to extract a witness from the safe WIPOK in two different ways, i.e. as in $\bar{H}_1^k$ and $\bar{H}_2^k$. Again we have that the simulator extracts the solution to the puzzle with non-negligible probability as the value extracted are different with non-negligible probability. We can follow identically as in the previous hybrid, by considering the sequence of hybrid experiments, E_0^k to E_3^k where the left interactions are all honestly generated. Again we have that the simulator extracts the solution of the puzzle in E_3^k and this violates the solution of the puzzle. □

Comparing $\bar{H}_2^k$ and $\bar{H}_3^k$ In this case, we have that, for infinitely many n,

$$|\Pr[\overline{\mathsf{hyb}}_A^2 \text{ is } k\text{-cons }] - \Pr[\overline{\mathsf{hyb}}_A^3 \text{ is } k\text{-cons }]| \geq \frac{1}{5p_1(n)}$$

In this case, we will show that A can be used to violate the special-hiding property of a variant of the commitment scheme.

Proof Sketch: The idea here (that originates from the work in [15], also used in [22], is that the simulation can be carried out even when the i^{th} left interaction is forwarded externally to a committer participating in $\widetilde{\Pi}$ which is a slightly altered version of the protocol Π. The only difference of $\widetilde{\Pi}$ from Π is that $\widetilde{\Pi}$ does not have a fixed scheduling of WIPOKs in the Stage 2 based on the committers identity. Instead, the receiver can request the commiter to provide proofs using WIPOK using designs of its choice. This is analogous to [22]. It was shown in [22], that while rewinding from a safe WIPOK in the k^{th} right-interaction, the messages for the i^{th} left interaction can be received from an external committer, interacting using $\widetilde{\Pi}$. Consider an experiment, where the simulator proceeds identically as in $\bar{H}_3^k$ with the exception that the i^{th} left interaction is forwarded externally to a committer following $\widetilde{\Pi}$ that commits to a value uniformly chosen from D. It now follows that this experiment proceeds identically to $\bar{H}_2^k$ if the external committer is following the honest committer strategy in $\widetilde{\Pi}$, and is identical to $\bar{H}_3^k$ if the external committer is equivocating. Therefore, it is possible to consider an adversary that distinguishes when it receives an honest commitment or a equivocal commitment using $\widetilde{\Pi}$ on the left, by simply extracting the value from the safe WIPOK. This violates the special hiding-property of the $\widetilde{\Pi}$ and thus we arrive at a contradiction. □

Comparing $\bar{H}_3^k$ and $\bar{H}_4^k$ In this case, we have that, for infinitely many n,

$$|\Pr[\overline{\mathsf{hyb}}_A^3 \text{ is } k\text{-cons }] - \Pr[\overline{\mathsf{hyb}}_A^4 \text{ is } k\text{-cons }]| \geq \frac{1}{5p_1(n)}$$

This will follow exactly as with hybrid experiment $\bar{H}_1^k$ and $\bar{H}_2^k$.

Comparing $\bar{H}_4^k$ and $\bar{H}_5^k$ In this case, we have that, for infinitely many n,

$$|\Pr[\overline{\mathsf{hyb}}_A^4 \text{ is } k\text{-cons }] - \Pr[\overline{\mathsf{hyb}}_A^5 \text{ is } k\text{-cons }]| \geq \frac{1}{5p_1(n)}$$

This will follow exactly as with hybrid experiment $\bar{H}_0^k$ and $\bar{H}_1^k$.

□

It only remains to state and prove Lemma 3. First, we need some notation.

Consider the following experiments $\mathsf{expt}^{M,R^*}_{\langle \mathsf{S_{com}}, \mathsf{R_{com}} \rangle}(0, x, v_1, z)$, $\mathsf{expt}^{M,R^*}_{\langle \mathsf{S_{com}}, \mathsf{R_{com}} \rangle}(1, x, v_1, z)$ where probabilistic polynomial time machines R^*, M interact using the equivocal non-malleable commitment protocol $\langle \mathsf{S_{com}}, \mathsf{R_{com}} \rangle$ (defined in Section 5) with common input $x \in L$ and private input $w \in \mathcal{R}(x)$:

Experiment $\mathsf{expt}^{M,R^*}_{\langle \mathsf{S_{com}}, \mathsf{R_{com}} \rangle}(b, x, v_1, z)$**:** R^* plays the part of receiver in the $\langle \mathsf{S_{com}}, \mathsf{R_{com}} \rangle$ protocol and initiates a request for a commitment to v_1. Upon this request, a sequence of $2t(n)$ strings is chosen $(s_1^0, \ldots, s_{t(n)}^0, s_1^1, \ldots, s_{t(n)}^1)$ (for some fixed polynomial $t(\cdot)$) in the following way: If $b = 0$, $(s_1^0, \ldots, s_{t(n)}^0, s_1^1, \ldots, s_{t(n)}^1)$ are chosen uniformly at random. If $b = 1$, $(s_1^0, \ldots, s_{t(n)}^0)$ are chosen to be random commitments to 0 and $(s_1^1, \ldots, s_{t(n)}^1)$ are chosen to be random commitments to 1. A machine running the code of M is initiated with input $(x, w, v_1, (s_1^0, \ldots, s_{t(n)}^0, s_1^1, \ldots, s_{t(n)}^1))$. M interacts with R^* and at any point after M completes the commitment, R^* can request a decommitment from M. Define the output of the experiment $\mathsf{sta}^{M,R^*}_{\langle \mathsf{S_{com}}, \mathsf{R_{com}} \rangle}(b, x, v_1, z)$ to be the output of R^*.

Lemma 3. *There exists a probabilistic polynomial time machine M^* such that for every probabilistic polynomial time adversary R^*, we have that:*

$$\begin{aligned}
&\{\mathsf{sta}^{M,R^*}_{\langle \mathsf{S_{com}}, \mathsf{R_{com}} \rangle}(0, x, v_1, z)\}_{n \in N, x \in L \cap \{0,1\}^n, w \in \mathcal{R}(x), v_1 \in \{0,1\}^n, z \in \{0,1\}^*} \\
\equiv \quad &\{\mathsf{sta}^{R^*}_{\langle \mathsf{S_{com}}, \mathsf{R_{com}} \rangle}(x, v_1, z)\}_{n \in N, x \in L \cap \{0,1\}^n, w \in \mathcal{R}(x), v_1 \in \{0,1\}^n, z \in \{0,1\}^*}
\end{aligned}$$

AND

$$\begin{aligned}
&\{\mathsf{sta}^{M,R^*}_{\langle \mathsf{S_{com}}, \mathsf{R_{com}} \rangle}(1, x, v_1, z)\}_{n \in N, x \in L \cap \{0,1\}^n, w \in \mathcal{R}(x), v_1 \in \{0,1\}^n, z \in \{0,1\}^*} \\
\equiv \quad &\{\mathsf{sta}^{R^*}_{\langle \tilde{\mathsf{S}}_{\mathsf{com}}, R_{\mathsf{com}} \rangle}(x, w, z)\}_{n \in N, x \in L \cap \{0,1\}^n, w \in \mathcal{R}(x), v_1 \in \{0,1\}^n, z \in \{0,1\}^*}
\end{aligned}$$

Proof. On input $(x, w, v_1, (s^0_1, \ldots, s^0_{t(n)}, s^1_1, \ldots, s^1_{t(n)}))$ M^* runs the code of the honest committer $\mathsf{S}_{\mathsf{com}}$ in $\langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle$ with the following exception: Each time $\mathsf{S}_{\mathsf{com}}$ uses the honest sender S_{eq} in the $\langle \mathsf{S}_{\mathsf{eq}}, \mathsf{R}_{\mathsf{eq}} \rangle$ protocol to construct a commitment $c = \mathsf{EQCom}(\alpha)$ to some bit α, M^* does the following: If $\alpha = 0$, M^* runs the code of the honest sender S_{eq}. If $\alpha = 1$, M^* does the following:

1. M^* uses the trapdoor w to compute an adjacency matrix I that corresponds to an isomorphism of the graph $G = \Phi(x)$ as well as the corresponding adjacency matrix H for the Hamiltonian cycle in I chooses an adjacency matrix H for a random Hamiltonian cycle.
2. If $H_{k,j} = 1$, then M^* sets the bit commitment at position (k, j) in $\overline{\mathsf{Com}}_{k,j}$ to be $Com(1)$.
3. If $H_{k,j} = 0$ and $I_{k,j} = 0$, then M^* sets the bit commitment at position (k, j) in $\overline{\mathsf{Com}}_{k,j}$ to be an element from the sequence $s^0_1, \ldots, s^0_{t(n)}$ that has not been used yet.
4. If $H_{k,j} = 0$ and $I_{k,j} = 1$, then M^* sets the bit commitment at position (k, j) in $\overline{\mathsf{Com}}_{k,j}$ to be an element from the sequence $s^1_1, \ldots, s^1_{t(n)}$ that has not been used yet.

To decommit, M^* runs the code of the honest $\mathsf{S}_{\mathsf{com}}$ in $\langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle$.

Recall that equivocal commitments in the $\langle \mathsf{S}_{\mathsf{eq}}, \mathsf{R}_{\mathsf{eq}} \rangle$ scheme are identically distributed to honest commitments to 0 and that the equivocator $(\tilde{\mathsf{S}}_{\mathsf{com}}, \mathsf{Adap}_{\mathsf{com}})$ described in Lemma 1 simply replaces *every* equivocal commitment under EQCom (in both Stage 1 and Stage 2) with an equivocal commitment generated by the equivocator. Therefore, it is clear from inspection that if the $2t(n)$ strings $(s^0_1, \ldots, s^0_{t(n)}, s^1_1, \ldots, s^1_{t(n)})$ are chosen uniformly at random, then R^*'s output is identically distributed to the output of $R^*(x, z)$ after receiving a commitment to v_1 using $\langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle$ and so the random variables $\mathsf{sta}^{M,R^*}_{\langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle}(0, x, v_1, z)$ and $\mathsf{sta}^{R^*}_{\langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle}(x, w, z)$ are identically distributed. On the other hand, if the $2t(n)$ strings $(s^0_1, \ldots, s^0_{t(n)}, s^1_1, \ldots, s^1_{t(n)})$ are commitments to 0 and 1 respectively, then R^*'s output is identically distributed to the output of $R^*(x, z)$ after receiving a commitment to v_1 using $\langle \tilde{\mathsf{S}}_{\mathsf{com}}, R_{\mathsf{com}} \rangle$ and so the random variables $\mathsf{sta}^{M,R^*}_{\langle \tilde{\mathsf{S}}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle}(1, x, v_1, z)$ and $\mathsf{sta}^{R^*}_{\langle \tilde{\mathsf{S}}_{\mathsf{com}}, R_{\mathsf{com}} \rangle}(x, v_1, z)$ are identically distributed. Thus, the lemma is proved. □

This concludes the proof of and Lemma 3 Theorem 2.

Dealing with non-straightline simulatable puzzles. In the proof above, we relied on the fact that the adaptive UC-puzzles were straightline simulatable. Althought for most models considered in this work, this is indeed the case, it is not straight-line for the timing and partially-isolated adversaries model. To deal with such simulation, we just need to follow the approach of [23], where given any adversary A, using the simulatability of the puzzles, construct an adversary $A*$ that is essentially A, with the exception that all puzzle interactions where A is the sender, A^* outputs completed puzzle transcripts with witnesses. The simulator S described above is run on A^* instead of A with the exception that it does not have to simulate puzzles and just use the witness output by A^* to equivocate the left commitments. The rest of the proof essentially follows from the statistical-simulation property. □

6 Proof of Main Theorem

We restate our main theorem and provide the proof below.

THEOREM 3 (**Main Theorem** (restatement)). *Assume the existence of a t_P-round $(\mathcal{C}_{\mathsf{env}}, \mathcal{C}_{\mathsf{sim}})$-secure UC-puzzle in a $\mathcal{G}$-hybrid model, t_C-round* $\mathsf{EQNMCom}$ *protocol secure w.r.t $cl(\mathcal{C}_{\mathsf{env}}, \mathcal{C}_{\mathsf{sim}})$ and the existence of a simulatable PKE scheme. Then, for every "well-formed" functionality $\mathcal{F}$, there exists a $O(t_P + t_C)$-round protocol Π in the $\mathcal{G}$-hybrid model that realizes $\hat{\mathcal{F}}$ with $(\mathcal{C}_{\mathsf{env}}, \mathcal{C}_{\mathsf{sim}})$-adaptive UC-security.*

Functionality $\mathcal{F}_{\mathsf{mcom}}$.

$\mathcal{F}_{\mathsf{mcom}}$ proceeds as follows, running with parties $P_1, \dots, P_n$ and an adversary S:

- Upon receiving input $(\mathsf{commit}, sid, ssid, P_i, P_j, \beta)$ from P_i, where $\beta \in \{0, 1\}$, record the tuple $(ssid, P_i, P_j, \beta)$ and send the message $(\mathsf{receipt}, sid, ssid, P_i, P_j)$ to P_j and S. Ignore any future commit messages with the same $ssid$ from P_i to P_j.
- Upon receiving a value $(\mathsf{reveal}, sid, ssid)$ from P_i: If a tuple $(ssid, P_i, P_j, \beta)$ was previously recorded, then send the message $(\mathsf{reveal}, sid, ssid, P_i, P_j, \beta)$ to P_j and S. Otherwise, ignore.
- Upon receiving a message $(\mathsf{corrupt}{-}P_i, sid)$ from the adversary, send $(ssid, P_i, P_j, \beta)$ to the adversary for each recorded tuple where P_i is the committer. Furthermore, if the adversary now provides a value β', and the receipt output was not yet written to P_j's tape, then change the recorded value to β'.

Figure 2: $\mathcal{F}_{\mathsf{mcom}}$

On a high-level, the compilation proceeds in two steps:

- First, every functionality is compiled into a protocol in the $\mathcal{F}_{\mathsf{mcom}}$-hybrid model. In the $\mathcal{F}_{\mathsf{mcom}}$-hybrid, all parties have access to the ideal commitment functionality called $\mathcal{F}_{\mathsf{mcom}}$ functionality. This step is formalized in the $\mathcal{F}_{\mathsf{mcom}}$-lemma (Lemma 4).
- In the second step, assuming the existence of a UC-*puzzle* and a $\mathsf{EQNMCom}$ protocol, we show that the $\mathcal{F}_{\mathsf{mcom}}$ functionality can be securely realized in the real-model. This step is formalized in the Puzzle-lemma (Lemma 7).

We use the standard definition of the $\mathcal{F}_{\mathsf{mcom}}$ functionality [7], the multi-session extension of $\mathcal{F}_{\mathsf{mcom}}$-functionality. See Figure 3 for the definition.

Next, we provide the $\mathcal{F}_{\mathsf{mcom}}$-Lemma and the Puzzle Lemma. The proof of the main theorem follows using a standard hybrid argument combining the two lemmas.

Lemma 4 ($\mathcal{F}_{\mathsf{mcom}}$-Lemma). *Assume the existence of simulatable PKE secure w.r.t* $\mathcal{C}_{\mathsf{sim}}$. *For every well-formed functionality* $\mathcal{F}$*, there exists a* $O(1)$*-round protocol* Π *in the* $\mathcal{F}_{\mathsf{mcom}}$-hybrid *model, such that, for every adversary* $\mathcal{A} \in \mathcal{C}_{\mathsf{sim}}$ *in the* $\mathcal{F}_{\mathsf{mcom}}$-hybrid *model, there exists an adversary simulator* $\mathcal{A}' \in \mathcal{C}_{\mathsf{sim}}$*, such that for every environment* $\mathcal{Z} \in \mathcal{C}_{\mathsf{env}}$*, the following two ensembles are indistinguishable w.r.t* $cl(\mathcal{C}_{\mathsf{env}}, \mathcal{C}_{\mathsf{sim}})$.

- $\left\{\mathsf{Exec}^{\mathcal{F}_{\mathsf{mcom}}}_{\Pi,\mathcal{A},\mathcal{Z}}(\mathsf{n})\right\}_{\mathsf{n}\in\mathbb{N}}$
- $\left\{\mathsf{Exec}^{\hat{\mathcal{F}}}_{\pi_{\mathsf{ideal}},\mathcal{A}',\mathcal{Z}}(\mathsf{n})\right\}_{\mathsf{n}\in\mathbb{N}}$

The main technical contribution of our work is the following lemma:

Lemma 5 (**Adaptive-Puzzle-Lemma**). *Let* Π' *be a protocol in the* $\mathcal{F}_{\mathsf{mcom}}$-hybrid *model. Assume the existence of a* $(\mathcal{C}_{\mathsf{env}}, \mathcal{C}_{\mathsf{sim}})$*-secure* t_P*-round adaptive puzzle* $\langle \mathsf{S}, \mathsf{R}\rangle$ *in a* $\mathcal{G}$*-hybrid model,* t_C*-round stand-alone* $\mathsf{EQNMCom}$ $\langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}}\rangle$ *secure w.r.t* $cl(\mathcal{C}_{\mathsf{sim}}, \mathcal{C}_{\mathsf{env}})$ *and simulatable PKE scheme secure w.r.t* $\mathcal{C}_{\mathsf{sim}}$*. Then, there exists a* $O(t_P + t_C)$*-round protocol* Π *in the* $\mathcal{G}$*-hybrid such that, for every uniform* $\mathcal{PPT}$ *adversary* $\mathcal{A}$*, there exists a simulator* $\mathcal{A}' \in \mathcal{C}_{\mathsf{sim}}$*, such that, for every environment* $\mathcal{Z} \in \mathcal{C}_{\mathsf{env}}$*, the following two ensembles are indistinguishable over* N *w.r.t* $\mathcal{C}_{\mathsf{sim}}$.

- $\left\{\mathsf{Exec}^{\mathcal{G}}_{\Pi,\mathcal{A},\mathcal{Z}}(\mathsf{n})\right\}_{\mathsf{n}\in\mathbb{N}}$
- $\left\{\mathsf{Exec}^{\mathcal{F}_{\mathsf{mcom}}}_{\Pi',\mathcal{A}',\mathcal{Z}}(\mathsf{n})\right\}_{\mathsf{n}\in\mathbb{N}}$

Proof of the Puzzle Lemma: First, in Figure 6 we construct a protocol $\langle \mathsf{S}, \mathsf{R}\rangle$ that implements the $\mathcal{F}_{\mathsf{mcom}}$-functionality. Next, given any protocol Π' in $\mathcal{F}_{\mathsf{mcom}}$-hybrid model, the protocol Π in the real model is constructed from Π' by instantiating the $\mathcal{F}_{\mathsf{mcom}}$ functionality using our protocol $\langle \mathsf{S}, \mathsf{R}\rangle$. More precisely, all invocations of the $\mathcal{F}_{\mathsf{mcom}}$ functionality with input (sender,$sid, ssid, P_j, \beta$) from an honest party P_i is replaced with an instance of $\langle \mathsf{S}, \mathsf{R}\rangle$between P_i and P_j on identity $\mathsf{id} = (\mathsf{P_i}, \mathsf{sid}, \mathsf{ssid})$. We provide the construction of $\langle \mathsf{S}, \mathsf{R}\rangle$ and then prove correctness.

6.1 The Adaptive Commitment Protocol $\langle \mathsf{S}, \mathsf{R}\rangle$ and the Adaptive UC Simulator

Let $(\langle \mathsf{S}, \mathsf{R}\rangle, \mathcal{R})$ be a $(\mathcal{C}_{\mathsf{env}}, \mathcal{C}_{\mathsf{sim}})$-secure puzzle in the $\mathcal{G}$-hybrid, $\langle \mathsf{S_{com}}, \mathsf{R_{com}}\rangle$ be a $\mathsf{EQNMCom}$ protocol secure w.r.t $cl(\mathcal{C}_{\mathsf{env}}, \mathcal{C}_{\mathsf{sim}})$, $\langle \mathsf{S_{eq}}, \mathsf{R_{eq}}\rangle$ be a non-interactive EQCom protocol secure w.r.t. $cl(\mathcal{C}_{\mathsf{env}}, \mathcal{C}_{\mathsf{sim}})$. $\mathcal{E} = (\mathsf{Gen}, \mathsf{Enc}, \mathsf{Dec}, \mathsf{oGen}, \mathsf{oRndEnc}, \mathsf{rGen}, \mathsf{rRndEnc})$ be a simulatable PKE scheme (as defined by [12]). Let L be a language in NP with witness relation R_L and let G be a pseudo-random generator (which exists based on one-way function which in turn can be based on simulatable PKE). See Figure 6 for a formal description of the protocol $\langle \mathsf{S}, \mathsf{R}\rangle$. An overview of the protocol is given in Section 4.2.

We show that for every adversary $\mathcal{A} \in \mathcal{PPT}$ in the real-model, there exists a simulator $\mathcal{A}' \in \mathcal{C}_{\mathsf{sim}}$ such that no environment $\mathcal{Z} \in \mathcal{C}_{\mathsf{env}}$ can distinguish if it is interacting with $\mathcal{A}$ in the real-model or $\mathcal{A}'$ in the $\mathcal{F}_{\mathsf{mcom}}$-hybrid.

Consider $\mathcal{A}'$ that internally incorporates $\mathcal{A}$ and emulates an execution with $\mathcal{A}$. $\mathcal{A}'$ forwards all messages from $\mathcal{A}$ externally to its intended recipients except messages that are part of any execution using $\langle \mathsf{S}, \mathsf{R}\rangle$, which are instead, dealt with internally. Recall that, since $\mathcal{A}' \in \mathcal{C}_{\mathsf{sim}}$ we have that at the end of every puzzle interaction where $\mathcal{A}$ controls the sender, $\mathcal{A}'$ can obtain a witness to the puzzle transcript. For messages that are part of an execution of $\langle \mathsf{S}, \mathsf{R}\rangle$, $\mathcal{A}'$ does the following:

Simulating the Communication with $\mathcal{Z}$: Every message that $\mathcal{A}'$ receives from the environment $\mathcal{Z}$ is written to $\mathcal{A}$'s input tape. Similarly, every output value that $\mathcal{A}$ writes to its output tape is copied to $\mathcal{A}'$'s own output tape (to be read later by $\mathcal{Z}$).

The Sender is Corrupted and the Receiver is Honest: $\mathcal{A}'$ does the following:

Preamble:

1. $\mathcal{A}'$ simulates the Adaptive UC-Puzzle while playing the part of the receiver, producing transcript TRANS_1 while extracting the trapdoor y
2. $\mathcal{A}'$ honestly plays the part of the sender in the Adaptive UC-Puzzle producing transcript TRANS_2.

Commit Phase:

1. $\mathcal{A}'$ chooses $r_{Gen} \leftarrow \{0,1\}^k$ and computes $(\textsc{pk}, \textsc{sk}) = \mathsf{Gen}(r_{Gen})$, $r = \mathsf{rGen}(r_{Gen})$.
2. $\mathcal{A}'$ uses the simulator for generating equivocal commitments for $\langle \mathsf{S_{com}}, \mathsf{R_{com}}\rangle$ and knowledge of the trapdoor y to send an equivocal commitment in Step 1 of Stage 1.

Protocol $\langle \mathsf{S}, \mathsf{R} \rangle$

Input: The sender S has a bit β to be committed to.

Preamble:

- An adaptive UC-Puzzle interaction $\langle \mathsf{S}, \mathsf{R} \rangle$ on input 1^n where R is the receiver and S is the sender. Let TRANS_1 be the transcript of the messages exchanged.
- An adaptive UC-Puzzle interaction $\langle \mathsf{S}, \mathsf{R} \rangle$ on input 1^n where S is the receiver and R is the sender. Let TRANS_2 be the transcript of the messages exchanged.

Commit phase:

Stage 1: S and R run a coin-tossing protocol to agree on strings $\textsc{pk}$ and CRS:

Coin-toss to generate $\textsc{pk}$:

1. The parties run protocol $\langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle$ with common input TRANS_1. R plays the part of sender with input a random string r^0_{R}.
2. S chooses a random string r^0_{S} and sends to R.
3. R opens its commitment and reveals r^0_{R}.
4. The output of the coin toss is: $r = r^0_{\mathsf{S}} \oplus r^0_{\mathsf{R}}$. S and R run $\mathsf{oGen}(r)$ to obtain public key $\textsc{pk}$.

Coin-toss to generate CRS:

1. The parties run protocol $\langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle$ with common input TRANS_2. S plays the part of sender with input a random string r^1_{S}.
2. R chooses a random string r^1_{R} and sends to S.
3. S opens its commitment and reveals r^1_{S}.
4. The output of the coin-toss is: $x = r^1_{\mathsf{S}} \oplus r^1_{\mathsf{R}}$.

Stage 2:

1. The parties run $\langle \mathsf{S}_{\mathsf{eq}}, \mathsf{R}_{\mathsf{eq}} \rangle$ with common input x to generate a commitment $C = \mathsf{EQCom}^x(\beta; r)$ where S plays the part of S_{eq} with input bit β.
2. S chooses $b \in \{0, 1\}$ at random and sends to R the strings (S_0, S_1) to where:
   - S_b is an encryption of the decommitment information of C (to bit β) under $\textsc{pk}$.
   - S_{1-b} is generated by running $\mathsf{oRndEnc}(\textsc{pk}, r_{\mathsf{Enc}})$ where r_{Enc} is chosen uniformly at random.

Reveal phase:

1. S sends β, b, and the randomness used to generate S_0, S_1 to R.
2. R checks that S_0, S_1 can be reconstructed using β, b and the randomness produced by S.

Figure 3: The Adaptive Commitment Protocol $\langle \mathsf{S}, \mathsf{R} \rangle$

3. Upon receiving the string r_{S}^0, $\mathcal{A}'$ equivocally decommits to $r_{\mathsf{R}}^0 = r \oplus r_{\mathsf{S}}^0$. $\mathcal{A}'$ chooses r_{R}^1 at random and sends to $\mathcal{A}$ on behalf of R.

4. Upon receiving (C, S_0, S_1) from $\mathcal{A}$ in Step 3, $\mathcal{A}'$ computes $m_0 = \mathsf{Dec}_{\mathrm{SK}}(S_0)$ and $m_1 = \mathsf{Dec}_{\mathrm{SK}}(S_1)$. If m_0 is the valid decommitment of C to bit b, $\mathcal{A}'$ sends the message $(\mathsf{commit}, sid, ssid, \mathsf{S}, \mathsf{R}, \beta)$ to the ideal functionality $\mathcal{F}_{\mathsf{mcom}}$ on behalf of S. Otherwise, if m_1 is the valid decommitment of C to bit β, $\mathcal{A}'$ sends the message $(\mathsf{commit}, sid, ssid, \mathsf{S}, \mathsf{R}, \beta)$ to the ideal functionality $\mathcal{F}_{\mathsf{mcom}}$. If both are invalid, $\mathcal{A}'$ chooses a random bit β and sends $(\mathsf{commit}, sid, ssid, \mathsf{S}, \mathsf{R}, \beta)$ to the ideal functionality $\mathcal{F}_{\mathsf{mcom}}$. Additionally, $\mathcal{A}'$ aborts the simulation of R upon $\mathcal{A}$'s decommitment.

Reveal Phase:

1. Upon receiving a valid decommitment from $\mathcal{A}$, $\mathcal{A}'$ sends the message $(\mathsf{reveal}, sid, ssid)$ to $\mathcal{F}_{\mathsf{mcom}}$.

2. Upon receiving an invalid decommitment from $\mathcal{A}$, $\mathcal{A}'$ aborts the simulation of R.

The Sender is Honest and the Receiver is Corrupted: $\mathcal{A}'$ does the following:

Preamble:

1. $\mathcal{A}'$ honestly plays the part of the sender in the Adaptive UC-Puzzle producing transcript TRANS_1.

2. $\mathcal{A}'$ simulates the Adaptive UC-Puzzle while playing the part of the receiver, producing transcript TRANS_2 while extracting the trapdoor y

Commit Phase:

1. $\mathcal{A}'$ chooses $s \leftarrow \{0,1\}^n$ and computes $r = G(s)$.

2. $\mathcal{A}'$ uses the simulator for generating equivocal commitments for $\langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle$ and knowledge of the trapdoor y to send an equivocal commitment in Step 1 of Stage 2. $\mathcal{A}'$ chooses r_{S}^0 at random and sends to $\mathcal{A}$ on behalf of S.

3. Upon receiving the string r_{R}^1, $\mathcal{A}'$ equivocally decommits to $r_{\mathsf{S}}^1 = r \oplus r_{\mathsf{R}}^1$. Note that $x = r_{\mathsf{S}}^1 \oplus r_{\mathsf{R}}^1 = r$.

4. $\mathcal{A}'$ generates a random bit β. Using its knowledge of the trapdoor for common input x the simulator generates an equivocal commitment C for protocol $\langle \mathsf{S}_{\mathsf{eq}}, \mathsf{R}_{\mathsf{eq}} \rangle$.

5. $\mathcal{A}'$ sets S_β to be an encryption under PK of a decommitment of C to 0 and sets $S_{1-\beta}$ to be an encryption under PK of a decommitment of C to 0

6. $\mathcal{A}'$ forwards (C, S_0, S_1) to $\mathcal{A}$.

Reveal Phase: Upon receiving a message $(\mathsf{reveal}, sid, ssid, \mathsf{S}, \mathsf{R}, \beta)$ from the ideal functionality $\mathcal{F}_{\mathsf{mcom}}$, $\mathcal{A}'$ does the following:

1. $\mathcal{A}'$ reveals the decommitment information for C corresponding to bit b and the randomness used to generate the encryption S_v where $v = 1 - \beta$ if $b = 1$ and $v = \beta$ if $b = 0$.

2. The simulator uses $\mathsf{rRndEnc}$ to produce randomness r_{Enc} such that $\mathsf{oRndEnc}(\mathrm{PK}, r_{\mathsf{Enc}}) = S_{1-v}$, and sends r to the adversary.

The Sender and the Receiver are Honest: $\mathcal{A}'$ plays both the part of the honest receiver and the honest sender. When sending messages on behalf of the honest sender, $\mathcal{A}'$ acts as in the case where the sender is honest and the receiver is corrupted. When sending messages on behalf of the honest receiver, $\mathcal{A}'$ acts as in the case where the receiver is honest and the sender is corrupted.

Dealing with Corruptions: When the adversary corrupts the sender S, $\mathcal{A}'$ sends the message $(\mathsf{corrupt}{-}\mathsf{S}, sid)$ to the ideal functionality $\mathcal{F}_{\mathsf{mcom}}$ and receives the value of the bit β. Now, $\mathcal{A}'$ needs to provide $\mathcal{A}$ with the randomness consistent with the (C, S_0, S_1) messages sent on behalf of S as well as the input bit b. $\mathcal{A}'$ does this in the same way as when simulating commitment $(\mathsf{reveal}, sid, ssid, \mathsf{S}, \mathsf{R}, \beta)$ messages in the case of corrupted receiver above.

6.2 Correctness of Simulation

We now proceed to prove correctness of simulation. Recall that the simulator manipulates coin-tosses so that it can equivocate commitments made to the adversary and extract the ones committed to by the adversary. More precisely, for the left-interactions, where the adversary receives commitments, the simulator manipulates the coin-toss to generate the CRS and for the right interactions, where the adversary sends commitments, the simulator manipulates the coin-toss to generate the public-key for the simulatable encryption scheme. In order for the simulation to work successfully, we will require that the adversary not be able to manipulate the other coin-tosses—the coin-toss for generating the public- keys in the left interactions and the coin-toss for generating the CRS in the right interactions. We ensure this property by relying on the non-malleability of the equivocal commitment scheme. In other words, we show that the adversary can never equivocate commitments made using $\langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle$ in those coin-toss interactions.

Towards proving correctness, we consider a series of intermediate hybrid experiments from the real-world to the $\mathcal{F}_{\mathsf{com}}$-hybrid world with the adversary $\mathcal{A}$. Additionally, we define the following property that we maintain as invariant across all hybrids and intuitively, will hold true only if the adversary does not equivocate any of the commitments made using $\langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle$: We say that the adversary $\mathcal{A}$ is *non-abusing* if the following two distributions are indistinguishable

$\mathsf{Expr1}_n(z)$: Emulate a complete execution with adversary $\mathcal{A}(1^n)$, environment $\mathcal{Z}$ with auxiliary input z and all honest parties. In the emulated view, choose at random a $\langle \mathsf{I}_{\mathsf{coin}}, \mathsf{R}_{\mathsf{coin}} \rangle$ interaction where $\mathcal{A}$ controls the initiator $\mathsf{I}_{\mathsf{coin}}$. If $\mathcal{A}$ corrupts the corresponding responder $\mathsf{R}_{\mathsf{coin}}$ before Step 3 of the $\langle \mathsf{I}_{\mathsf{coin}}, \mathsf{R}_{\mathsf{coin}} \rangle$ protocol or fails to complete the interaction, output $(\mathsf{View}_{\mathcal{A}}, \perp)$, where $\mathsf{View}_{\mathcal{A}}$ is the view of the adversary in the simulation. Otherwise, if the $\langle \mathsf{I}_{\mathsf{coin}}, \mathsf{R}_{\mathsf{coin}} \rangle$ interaction completes successfully with outcome r, then $(\mathsf{View}_{\mathcal{A}}, r)$.

$\mathsf{Expr2}_n(z)$: As before, emulate an execution with adversary $\mathcal{A}(1^n)$ environment $\mathcal{Z}$ with auxiliary input z and all honest parties. Choose at random a $\langle \mathsf{I}_{\mathsf{coin}}, \mathsf{R}_{\mathsf{coin}} \rangle$ interaction where $\mathcal{A}$ controls the initiator $\mathsf{I}_{\mathsf{coin}}$ and continue the emulation until the completion of the $\langle \mathsf{I}_{\mathsf{coin}}, \mathsf{R}_{\mathsf{coin}} \rangle$ interaction. If $\mathcal{A}$ corrupts the corresponding responder $\mathsf{R}_{\mathsf{coin}}$ before Step 3 of the $\langle \mathsf{I}_{\mathsf{coin}}, \mathsf{R}_{\mathsf{coin}} \rangle$ protocol, or fails to complete the interaction, output $(\mathsf{View}_{\mathcal{A}}, \perp)$, where $\mathsf{View}_{\mathcal{A}}$ is the view of the adversary in the simulation. Otherwise, if the $\langle \mathsf{I}_{\mathsf{coin}}, \mathsf{R}_{\mathsf{coin}} \rangle$ interaction completes successfully with outcome r, let r_{I} be the string sent by $\mathcal{A}$ in Step 3 of the $\langle \mathsf{I}_{\mathsf{coin}}, \mathsf{R}_{\mathsf{coin}} \rangle$ protocol. Repeat the following:

- Choose string r^* uniformly at random.
- Rewind $\mathcal{A}$ to the point right before Step 2 of the $\langle \mathsf{I}_{\mathsf{coin}}, \mathsf{R}_{\mathsf{coin}} \rangle$ protocol.
- Send string $\mathsf{r}^* \oplus \mathsf{r}_{\mathsf{I}}$ to $\mathcal{A}$ on behalf of $\mathsf{R}_{\mathsf{coin}}$ in Step 2 of the of the $\langle \mathsf{I}_{\mathsf{coin}}, \mathsf{R}_{\mathsf{coin}} \rangle$ protocol.
- Continue simulation until $\mathcal{A}$ decommits. If the adversary fails to decommit or tries to adaptively corrupt the responder, cancel the simulation and start over. Otherwise, let the value decommitted to be $\tilde{\mathsf{r}}_{\mathsf{I}}$

until the $\langle \mathsf{I}_{\mathsf{coin}}, \mathsf{R}_{\mathsf{coin}} \rangle$ interaction completes successfully with outcome $\tilde{\mathsf{r}}$. If $\tilde{\mathsf{r}} \neq \mathsf{r}^*$ then output a special symbol $\perp_{\text{FAIL}}$. Otherwise, output the $(\mathsf{View}_{\mathcal{A}}, \tilde{\mathsf{r}})$.

Remark 1. *If the distributions of* $\mathsf{Expr1}_n(z)$ *and* $\mathsf{Expr2}_n(z)$ *are indistinguishable then it implies that the adversary decommits to the same string* $\mathsf{r_I}$ *with high-probability, i.e. does not equivocate.*

Remark 2. *The experiment* $\mathsf{Expr2}_n(z)$*, in expectation, takes polynomial time to simulate. This is because, even though the simulator rewinds the adversary repeatedly, each rewinding is simulated identically as the main simulation with independent randomness. More formally, if p is the probability with which the adversary decommits successfully from Step 2 of the coin-toss without corrupting the responder, then p is the probability with which the simulator starts rewinding and in expectation rewinds $1/p$ times before it obtains another simulation where the adversary decommits without corrupting the responder. Therefore, in expectation, the simulator performs simulation $p \times 1/p = O(1)$ times. Since each simulation takes at most $poly(n)$ time, in expectation,* $\mathsf{Expr2}_n(z)$ *takes polynomial time to simulate.*

Let $m(\cdot)$ be a function that describes a bound on the maximum number of interactions. The hybrid experiments are as follows:

Hybrid H_0 or the real-world experiment: Since this is the real-world experiment there is no indistinguishability requirement. However, we need to show that $\mathcal{A}$ is non-abusing in H_0. Intuitively, this holds since from the binding property of the commitment scheme $\langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle$ we have that if the adversary equivocates, then we can extract the solution of the adaptive UC-puzzle and this violates the soundness of the puzzle. More formally, we prove the following claim:

Claim 3. *$\mathcal{A}$ is non-abusing in H_0*

Assume for contradiction there exists a distinguisher D and polynomial $p(\cdot)$ such that for infinitely many n, D distinguishes $\mathsf{Expr1}_n(z)$ and $\mathsf{Expr2}_n(z)$ with probability at least $\frac{1}{p(n)}$. Since r^* is chosen uniformly at random in each rewound execution (and thus $\mathsf{r}^* \oplus \mathsf{r_I}$ is also uniformly distributed), if $\mathsf{r}^* = \tilde{\mathsf{r}}$ always, then $\mathsf{Expr1}_n(z)$ and $\mathsf{Expr2}_n(z)$ are identically distributed. Hence if D distinguishes the two experiments with probability $\frac{1}{p(n)}$, it must be the case that $\mathsf{Expr2}_n(z)$ outputs $\perp_{\text{FAIL}}$ with probability at least $\frac{1}{p(n)}$. The proof of Claim 3, will now follow from the following subclaim.

Claim 4. *Let $\mathcal{A}$ be a probabilistic polynomial-time adversary such that* $\mathsf{Expr2}_n(z)$ *outputs $\perp_{\text{FAIL}}$ with $\mathcal{A}$ with non-negligible probability. Then there exists a probabilistic polynomial-time adversary $\overline{\mathcal{A}}$ that violates the soundness of the adaptive UC-puzzle.*

Proof. On a high-level, this essentially follows from the fact that whenever $\mathsf{Expr2}_n(z)$ outputs $\perp_{\text{FAIL}}$, the adversary is equivocating, which in turn means a solution to the adaptive UC-puzzle can be extracted and this violates the soundness condition of the puzzle. More formally, consider $\mathcal{A}$ for which $\mathsf{Expr2}_n(z)$ outputs $\perp_{\text{FAIL}}$ with probability $\frac{1}{p(n)}$ for infinitely many n. Fix an n for which this happens.

On input 1^n and auxiliary input z, $\overline{\mathcal{A}}$ internally incorporates $\mathcal{A}(1^n)$, $\mathcal{Z}(z)$ and all honest parties and begins emulating an execution of hybrid experiment H_0 with the following exceptions: $\overline{\mathcal{A}}$ chooses a random $\langle \mathsf{S}, \mathsf{R} \rangle$ interaction where the adversary controls one of the parties and forwards externally the puzzle interaction where $\mathcal{A}$ controls the receiver. On completion, $\overline{\mathcal{A}}$ chooses the $\langle \mathsf{I}_{\mathsf{coin}}, \mathsf{R}_{\mathsf{coin}} \rangle$ interaction from the same $\langle \mathsf{S}, \mathsf{R} \rangle$ interaction where $\mathcal{A}$ controls the initiator $\mathsf{I}_{\mathsf{coin}}$. After completion of Stage 3 of the $\langle \mathsf{I}_{\mathsf{coin}}, \mathsf{R}_{\mathsf{coin}} \rangle$ interaction, if $\mathcal{A}$ fails to decommit, $\overline{\mathcal{A}}$ outputs $\perp$. Otherwise, it stores the decommitment as r_1. Next, it rewinds to the end of Stage 1 and starts a new emulation (just as in $\mathsf{Expr2}_n$). If the adversary fails to decommit, $\overline{\mathcal{A}}$ outputs $\perp$. Otherwise, it stores the second decommitment as r_2. Finally, if $\mathsf{r}_1 \neq \mathsf{r}_2$, it extracts the witness for the puzzle transcript corresponding to this interaction and outputs the witness. Otherwise it outputs $\perp$. Also, if at any point $\mathcal{A}$ tries to adaptive corrupt the other party, $\overline{\mathcal{A}}$ halts outputting $\perp$.

We claim that with non-negligible probability $\overline{\mathcal{A}}$ outputs a witness of the puzzle, thus violating the soundness of the puzzle. Towards this, consider a modified experiment $\mathsf{Expr2}_n^k(z)$ that proceeds exactly like $\mathsf{Expr2}_n$ with the exception that it chooses the k^{th} coin-toss where the adversary controls the initiator instead of a random interaction. By an averaging argument, it holds that there exists a particular k for which $\mathsf{Expr2}_n^k(z)$ outputs $\bot_{\text{FAIL}}$ with probability at least $q = \frac{1}{p_1(n)p(n)}$ where $p_1(\cdot)$ is a polynomial that bounds the total number of $\langle \mathsf{S}, \mathsf{R}\rangle$ interactions (and hence also the number of coin-toss interactions where $\mathcal{A}$ controls the sender). Using another averaging argument, it holds that with probability $q/2$ over partial transcripts until the end of Stage 1 of the k^{th} coin-toss, the probability $\mathsf{Expr2}_n^k(z)$ outputs $\bot_{\text{FAIL}}$ conditioned on the partial transcript, is at least $q/2$. Fix one such partial transcript τ. We show below that from τ, if two runs are conducted using independent randomness, then the probability that the adversary decommits to two different strings in the two runs is at least $q^2/8$. Using this, we can argue that the probability with which $\overline{\mathcal{A}}$ outputs a witness is at least $\frac{1}{p_1(n)}\frac{q}{2}\frac{q^2}{8}$ and this proves the statement of the claim.

Given τ, let D be the distribution of the decommitments made by the adversary in random continuations from τ. If the adversary fails to decommit or corrupts the other party, the output of D is set to $\bot$. Let D' be the distribution D conditioned on not outputting $\bot$. Let q' be the probability that two independent samples from D are different. Let q'' be the probability that a sample from D is not $\bot$. Since $\mathsf{Expr2}_n^k(z)$ outputs $\bot_{\text{FAIL}}$ with probability at least $q/2$ from τ, it holds that $q'' \times q' \geq q/2$. Now, let r be the sample with maximum probability under D, say q_r. We bound the probability that two samples from D are the same by the expression $q_r^2 + (1-q_r)^2$, which is at most $1 - q'$. This implies that $q_r(1-q_r) > \frac{q'}{2}$. Therefore, the probability that two samples from D are different is at least

$$(q''q_r) \times (q''(1-q_r)) \geq \frac{q''^2 q'}{2} \geq \frac{q''q}{4} \geq \frac{q^2}{8}$$

□

Hybrid H_1: This hybrid proceeds identically to H_0 with the exception that all puzzle-interactions where the honest party plays the part of the receiver are simulated. For every adversary $\mathcal{A}$, we construct another adversary $\overline{\mathcal{A}} \in \mathcal{C}_{\mathsf{sim}}$ that internally emulates $\mathcal{A}$ and simulates puzzles while extracting trapdoors for all puzzles where $\mathcal{A}$ plays the role of sender.

In more detail, an execution in H_1 proceeds identically to the real-execution, with the exception that all parties running $\langle \mathsf{S}, \mathsf{R}\rangle$, instead of participating in the preample phase of $\langle \mathsf{S}, \mathsf{R}\rangle$, receive a simulated puzzle-transcript from $\overline{\mathcal{A}}$. Furthermore, for every puzzle interaction where the party controlled by the adversary is the sender and the receiver is honest, $\overline{\mathcal{A}}$ outputs a witness w corresponding to the simulated puzzle-transcript (in a special-output tape). Additionally, upon adaptive corruption of the receiver in a puzzle interaction, where the sender is controlled by the adversary, $\overline{\mathcal{A}}$ produces random coins for an honest receiver that are consistent with the simulated puzzle-transcript. To construct such an $\overline{\mathcal{A}}$ given $\mathcal{A}$, we rely on the adaptive simulatability of the puzzle in a concurrent puzzle execution. We consider an adversary $\mathcal{A}_{\mathsf{puz}}$ that incorporates $\mathcal{A}$ internally and forwards all puzzle interactions with $\mathcal{A}$ as the sender to external receivers. This $\mathcal{A}_{\mathsf{puz}}$ also simulates all other puzzle interactions interally. All other interactions of $\mathcal{A}$ are forwarded by $\mathcal{A}_{\mathsf{puz}}$ to the puzzle environment that incorporates $\mathcal{A}$ and the other honest parties. Since this can be viewed as a concurrent puzzle execution, there must exist a simulator $\mathcal{A}'_{\mathsf{puz}}$ that simulates all puzzle interactions, outputs a witness w, and successfully simulates adaptive corruptions. Finally, to construct $\overline{\mathcal{A}}$ we incorporate $\mathcal{A}'_{\mathsf{puz}}$ and emulate an execution by forwarding the messages between $\mathcal{A}'_{\mathsf{puz}}$ and the actual parties instead of sending to $\mathcal{Z}_{\mathsf{puz}}$.

The proof of indisintguishability follows identically as in [23] and we omit it. The non-abusing property follows from the statistical indistinguishability of $\mathcal{A}$'s view[10] in H_0 and H_1. Hence we have the following claims.

[10] If $\overline{\mathcal{A}}$ is non-abusing, then just as in proof of Claim 3, we can conclude that $\overline{\mathcal{A}}$ is equivocating in H_1. Then with non-negligible

Claim 5. *The output of $\mathcal{Z}$ in H_0 and H_1 is indistinguishable.*

Claim 6. *$\mathcal{A}$ is non-abusing in H_1*

In subsequent hybrids, the adversary we consider is $\overline{\mathcal{A}}$. However, to avoid confusion in notation, we denote the adversary by $\mathcal{A}$ only.

Hybrid H_2: This hybrid proceeds identically to H_1 with the exception that in all interactions with an honest receiver, the commitments received in Stage 1 are switched to simulated equivocal commitments. More specifically, the protocol $\langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle$ occurring in Step 1 of the two coin-tosses in Stage 1, is modified for interactions where the adversary plays the part of receiver in $\langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle$ in the following ways:

- The first commitment sent by S in the $\langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle$ protocol is replaced by a simulated (equivocal) commitment which can be opened to any value.
- The WIPOK's are replaced with simulated adaptively-secure WIPOK's which can be opened consistently with any valid witness.
- When a decommitment is requested, a value r^0_{R} or r^1_{S} (as appropriate) is chosen uniformly at random and a decommitment to the chosen value is produced.

Note that, in particular, this means that in $\mathsf{Expr2}_n(z)$, commitments produced by $\langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle$ (where the adversary played the role of receiver) will be decommitted to different values in the initial and rewound views.

Claim 7. *The output of $\mathcal{Z}$ in H_1 and H_2 is indistinguishable.*

Claim 8. *$\mathcal{A}$ is non-abusing in H_2*

Proof: We prove both the above claims simultaneously. They follow essentially from the simulation-extractibility property of $\langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle$. Recall from proof of Claim 3 that if the outputs of the two experiments are distinguishable, then it implies that $\mathsf{Expr2}_n(z)$ outputs $\perp_{\text{FAIL}}$. Consider the following adversary $\overline{\mathcal{A}}$ that violates the simulation-extractability property of $\langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle$.

On input 1^n and auxiliary input z, $\overline{\mathcal{A}}$ internally incorporates $\mathcal{A}(1^n)$, $\mathcal{Z}(z)$ and all honest parties and begins emulating an execution of hybrid experiment H_1 with the following exception: $\overline{\mathcal{A}}$ forwards all $\langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle$ interactions that are part of $\langle \mathsf{I}_{\mathsf{coin}}, \mathsf{R}_{\mathsf{coin}} \rangle$ interactions where the adversary controls the receiver are forwarded externally to honest committers on the left and all $\langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle$ interactions that are part of $\langle \mathsf{I}_{\mathsf{coin}}, \mathsf{R}_{\mathsf{coin}} \rangle$ interactions where the adversary controls the sender are forwarded to external receivers on the right. Whenever $\mathcal{A}$ requests a decommitment for the coin-toss interactions on the left, $\overline{\mathcal{A}}$ externally requests a decommitment. For the decommitment phase on the right, $\overline{\mathcal{A}}$ simply forwards the decommitment made by $\mathcal{A}$ in the internal $\langle \mathsf{I}_{\mathsf{coin}}, \mathsf{R}_{\mathsf{coin}} \rangle$ interactions. At the end $\overline{\mathcal{A}}$ outputs $\mathcal{A}$'s view and all the value decommitted to in the right interactions. Observe that when the left commitments are sent by honest committers the view output is identical to view output in H_1 and when the commitments are equivocated, the view is identical to one output in H_2. Furthermore, the simulation proceeds identically to the simulator for the $\langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle$ protocol. Since $\mathsf{Expr2}_n(z)$ outputs $\perp_{\text{FAIL}}$ with non-negligible probability, following the proof of Claim 4, it holds that, when the commitment in the left are equivocated, there exists a particular k for which $\overline{\mathcal{A}}$ equivocates in k^{th} right-interaction with non-negligible probability. This means that, with non-negligible

probability over the random-tapes for $\overline{\mathcal{A}}$ and partial transcripts where $\overline{\mathcal{A}}$ completes a commitment using $\langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle$, it holds that $\overline{\mathcal{A}}$ equivocates with non-negligible probability. This violates the statistical-indistinguishability as for the fixed random tape, $\mathcal{A}$ never equivocates and an unbounded prover, given a partial transcript and random tape, can find the unique value $\mathcal{A}$ decommits to and distinguish from the value decommitted to by $\overline{\mathcal{A}}$.

probability over the partial view of $\mathcal{A}$ after the commitment is made in the k^{th} right interactions, $\mathcal{A}$ decommits to different values with non-negligible probability. However, from the simulation-extractability property of $\langle \mathsf{S_{com}}, \mathsf{R_{com}} \rangle$, it holds that, whenever the left-commitments are equivocated, there is a unique value that any adversary can decommit to after the commitment stage is completed. Thus, we arrive at a contradiction.

Hybrid H_3**:** This hybrid proceeds identically to H_2 with the exception that the protocol $\langle \mathsf{S_{com}}, \mathsf{R_{com}} \rangle$ occurring in Step 1 of the first coin-toss in Stage 1 is modified for interactions where the adversary plays the part of receiver in $\langle \mathsf{S_{com}}, \mathsf{R_{com}} \rangle$ so that instead of sampling a uniformly random r^0_{R} and decommitting to this value, we sample r^0_{R} as follows:

- Using $\mathsf{Gen}(r_{\mathsf{Gen}})$ sample a public-key, secret-key pair (pk, sk).
- Run $\mathsf{rGen}(r_{\mathsf{Gen}})$ to obtain the string s such that $\mathsf{oGen}(s) = \text{PK}$.
- Decommit to $r^0_{\mathsf{R}} = r^0_{\mathsf{S}} \oplus s$.

We show that both the indistinguishability and the non-abusing property reduce to the indistinguishability of random strings s to strings s sampled by running $\mathsf{rGen}(r_{\mathsf{Gen}})$ where r_{Gen} is sampled uniformly at random. Note that a simulator $\overline{\mathcal{A}}$ emulating an exectuion in Hybrid H_3 onward can extract the adversary's committed values by decommitting to $r = r^{\mathsf{S}} \oplus s$ such that $\overline{\mathcal{A}}$ knows the corresponding SK for $\mathsf{oGen}(s) = \text{PK}$ and then decrypting the decommitment information contained in S_0 and/or S_1.

Claim 9. *The output of $\mathcal{Z}$ in H_2 and H_3 is indistinguishable.*

Proof. Assume that there exists a PPT algorithm D that distinguishes the output of $\mathcal{Z}$ in H_2 and H_3 with probability $\frac{1}{p(n)}$ for some polynomial $p(\cdot)$ and infinitely many n. We construct an adversary $\overline{\mathcal{A}}$ that will be able to distinguish strings s chosen uniformly at random from string $s = \mathsf{rGen}(r_{\mathsf{Gen}})$ where r_{Gen} is chosen uniformly at random (and thus breaks the oblivious generation property of the simulatable PKE).

Consider a machine $\overline{\mathcal{A}}$ that on input 1^n and auxiliary input z, participates in an execution with a challenger C and internally incorporates $\mathcal{A}$, $\mathcal{Z}$, and all the honest parties and emulates an interaction in H_2. $\overline{\mathcal{A}}$ receives from C a sequence of values $\{s_1, \ldots, s_{m(n)}\}$ chosen either uniformly at random or chosen such that $s_i = \mathsf{rGen}(r^i_{\mathsf{Gen}})$. $\overline{\mathcal{A}}$ continues the emulation of $\mathcal{A}$ as in H_2 with the difference that in the i-th the commitment protocol $\langle \mathsf{S_{com}}, \mathsf{R_{com}} \rangle$, $\overline{\mathcal{A}}$ decommits to the value $r^0_{\mathsf{R}} = r^0_{\mathsf{S}} \oplus s_i$. At the end of the execution, $\overline{\mathcal{A}}$ runs D on the output of $\mathcal{Z}$ and outputs whatever D outputs.

Note that when the strings $\{s_1, \ldots, s_{m(n)}\}$ are generated via $\mathsf{rGen}(r_{\mathsf{Gen}})$ then the emulation produces a view for $\mathcal{Z}$ that is identical to its view in H_3. On the other hand, when the strings $\{s_1, \ldots, s_{m(n)}\}$ are chosen uniformly at random then the emulation produces a view for $\mathcal{Z}$ that is identical to its view in H_2. Thus, $\overline{\mathcal{A}}$ distinguishes random strings s from strings s sampled by running $\mathsf{rGen}(r_{\mathsf{Gen}})$ where r_{Gen} is sampled uniformly at random with the same probability that D distinguishes the ouput of $\mathcal{Z}$ in H_2 and H_3. This implies that $\overline{\mathcal{A}}$ distinguishes with non-negligible probability, which is a contradiction to the security of the simulatable PKE scheme $\mathcal{E}$ and so the claim is proved. □

Claim 10. *$\mathcal{A}$ is non-abusing in H_3*

Proof. The proof for $\mathcal{A}$ being non-abusing essentially follows from the proof of Claim 9 above. Details follow.

Assume towards contradiction that $\mathcal{A}$ is abusing in H_3. We will construct an adversary $\overline{\mathcal{A}}$ that breaks the security of the simulatable PKE scheme $\mathcal{E}$. Consider the following adversary $\overline{\mathcal{A}}$: On input 1^n and auxiliary input z, $\overline{\mathcal{A}}$ internally incorporates $\mathcal{A}(1^n)$, $\mathcal{Z}(z)$ and all honest parties. Additionally, $\overline{\mathcal{A}}$ receives externally a sequence of $2m$ values, $\{s_1, \ldots, s_{2m(n)}\}$. $\overline{\mathcal{A}}$ begins emulating an execution of hybrid experiment H_3 and chooses a random $\langle \mathsf{I_{coin}}, \mathsf{R_{coin}} \rangle$ interaction where $\mathcal{A}$ controls the initiator $\mathsf{I_{coin}}$. $\overline{\mathcal{A}}$ continues the emulation of $\mathcal{A}$ with the difference that in the i-th commitment protocol $\langle \mathsf{S_{com}}, \mathsf{R_{com}} \rangle$, $\overline{\mathcal{A}}$ decommits to the value $r^0_{\mathsf{R}} = r^0_{\mathsf{S}} \oplus s_i$. After completion of Stage 3 of the $\langle \mathsf{I_{coin}}, \mathsf{R_{coin}} \rangle$ interaction, if $\mathcal{A}$ fails to decommit, $\overline{\mathcal{A}}$ outputs $\perp$. Otherwise, it stores the decommitment as r_1. Next, it rewinds to the end of Stage 1 and starts a new emulation (just as in $\mathsf{Expr2}_n$). Again, in the i-th the commitment protocol $\langle \mathsf{S_{com}}, \mathsf{R_{com}} \rangle$ of the rewound execution, $\overline{\mathcal{A}}$ decommits to the value $r^0_{\mathsf{R}} = r^0_{\mathsf{S}} \oplus s_{m(n)+i}$. If the adversary fails to decommit, $\overline{\mathcal{A}}$ outputs $\perp$. Otherwise, if $\mathcal{A}$ decommits to r_1 where $\mathsf{r}_1 \neq \mathsf{r}_2$, $\overline{\mathcal{A}}$ outputs 1; if $\mathsf{r}_1 = \mathsf{r}_2$, $\overline{\mathcal{A}}$ outputs 0.

Note that when the strings $\{s_1, \ldots, s_{2m(n)}\}$ are generated via $\mathsf{rGen}(r_{\mathsf{Gen}})$ then the emulation produces a view for $\mathcal{A}$ that is identical to its view in $\mathsf{Expr2}_n(z)$ of H_3. On the other hand, when the strings $\{s_1, \ldots, s_{2m(n)}\}$ are chosen uniformly at random then the emulation produces a view for $\mathcal{A}$ that is identical to its view in $\mathsf{Expr2}_n(z)$ of H_2.

Now, since $\mathcal{A}$ is abusing in H_3, it must be the case (see proof of Claim 4) that when $\{s_1, \ldots, s_{2m(n)}\}$ are chosen via $\mathsf{rGen}(r_{\mathsf{Gen}})$ $\overline{\mathcal{A}}$ outputs 1 with non-negligible probability. However, since $\mathcal{A}$ is non-abusing in H_2, it must be the case that when $\{s_1, \ldots, s_{2m(n)}\}$ are chosen uniformly at random $\overline{\mathcal{A}}$ outputs 1 with negligible probability. Thus, $\overline{\mathcal{A}}$ distinguishes random strings s to strings s sampled by running $\mathsf{rGen}(r_{\mathsf{Gen}})$ where r_{Gen} is sampled uniformly at random. This is a contradiction to the security of $\mathcal{E}$ and so the claim is proved. □

Hybrid H_4**:** This hybrid proceeds identically to H_3 with the exception that the protocol $\langle \mathsf{S_{com}}, \mathsf{R_{com}} \rangle$ occurring in Step 1 of the second coin-toss in Stage 1 is modified for interactions where the adversary plays the part of receiver in $\langle \mathsf{S_{com}}, \mathsf{R_{com}} \rangle$ so that instead of sampling a uniformly random r^1_{S} and decommitting to this value, we sample r^1_{S} as follows:

- Sample s uniformly at random and set $r = G(s)$.
- Decommit to $r^1_{\mathsf{S}} = r^1_{\mathsf{R}} \oplus r$.

Claim 11. *The output of $\mathcal{Z}$ in H_3 and H_4 is indistinguishable. Moreover, $\mathcal{A}$ is non-abusing in H_4*

The proof of Claim 11 proceeds analogously to the proofs of Claims 9 and 10. Here we consider an adversary $\overline{\mathcal{A}}$ that receives externally a sequence of strings $\{s_1, \ldots, s_{2m(n)}\}$ which are either uniformly random or generated via the pseudorandom generator G. We show that $\overline{\mathcal{A}}$ perfectly emulates an execution in H_3 (or emulates $\mathsf{Expr2}_n(z)$ in H_3) when the received strings are uniformly random and that $\overline{\mathcal{A}}$ perfectly emulates an execution in H_4 (or emulates $\mathsf{Expr2}_n(z)$ in H_4) when the received strings are pseudorandom. Thus, if the output of $\mathcal{Z}$ in H_3 and H_4 is distinguishable or if $\mathcal{A}$ is abusing in H_4 (and not abusing in H_3), then $\overline{\mathcal{A}}$ distinguishes random and pseudorandom strings. This is a contradiction to the security of the pseudorandom generator G, and so the claim is proved.

Hybrid H_5**:** This hybrid proceeds identically to H_4 with the exception that the protocol $\langle \mathsf{S_{eq}}, \mathsf{R_{eq}} \rangle$ occurring in Step 1 of Stage 2 is modified for interactions in which the adversary plays the part of receiver in the following way: The commitment C is replaced by a simulated (equivocal) commitment which can be opened to both 0 and 1.

We show that both the indistinguishability and the non-abusing property reduce to the special-hiding property of $\langle \mathsf{S_{eq}}, \mathsf{R_{eq}} \rangle$.

Claim 12. *The output of $\mathcal{Z}$ in H_4 and H_5 is indistinguishable. Moreover, $\mathcal{A}$ is non-abusing in H_5.*

Proof. The proof of Claim 12 proceeds analogously to the proofs of Claims 9 and 10. Assume for contradiction there exists an environment $\mathcal{Z}$ that distinguishes the experiments H_4 and H_5. More precisely, there exists D and polynomial $p(\cdot)$ such that D distinguishes the output of $\mathcal{Z}$ in both the experiments. We show given $D, \mathcal{Z}$ and $\mathcal{A}$ how to violate the special-hiding property of the commitment (See Definition 2).

Consider a machine $\overline{\mathcal{A}}$ that on input 1^n and auxiliary input z, internally incorporates $\mathcal{A}$, $\mathcal{Z}$, and all the honest parties and emulates an interaction in H_4. Whenever $\mathcal{A}$ wishes to receive a commitment from an honest receiver to a bit β in Stage 2 of $\langle \mathsf{S}, \mathsf{R} \rangle$, instead of constructing C by emulating the $\langle \mathsf{S}_{\mathsf{eq}}, \mathsf{R}_{\mathsf{eq}} \rangle$ interaction internally, $\overline{\mathcal{A}}$ makes a request externally for a commitment C to bit β. When $\mathcal{A}$ expects a decommitment in the internal emulation $\overline{\mathcal{A}}$ again requests the external committer for a decommitment of C to bit β. Finally, $\overline{\mathcal{A}}$ runs D on the output of $\mathcal{Z}$ and outputs what D outputs.

Observe that when the external committer runs the code of the honest committer S in $\langle \mathsf{S}_{\mathsf{eq}}, \mathsf{R}_{\mathsf{eq}} \rangle$, then the output of $\overline{\mathcal{A}}$ is identically distributed to the output of D in H_4. Similarly, whenever the external committer runs the code of the equivocator in $\langle \mathsf{S}_{\mathsf{eq}}, \mathsf{R}_{\mathsf{eq}} \rangle$, then the output of $\overline{\mathcal{A}}$ is identically distributed to the output of D in H_5. Therefore, D distinguishes honest and simulated commitments, which is a contradiction to the special-hiding property of $\langle \mathsf{S}_{\mathsf{eq}}, \mathsf{R}_{\mathsf{eq}} \rangle$.

The proof for $\mathcal{A}$ being non-abusing essentially follows from above. In this case, $\overline{\mathcal{A}}$ will need to request additional external commitments C so that it can simulate a single rewinding as in $\mathsf{Expr2}_n(z)$. Specifically, $\overline{\mathcal{A}}$ begins emulating an execution of hybrid experiment H_5 and chooses a random $\langle \mathsf{I}_{\mathsf{coin}}, \mathsf{R}_{\mathsf{coin}} \rangle$ interaction where $\mathcal{A}$ controls the initiator $\mathsf{I}_{\mathsf{coin}}$. $\overline{\mathcal{A}}$ continues the emulation of $\mathcal{A}$ with the difference that whenever an equivocal commitment is required in Stage 2 of a $\langle \mathsf{S}, \mathsf{R} \rangle$ protocol where $\mathcal{A}$ plays the reciever, $\overline{\mathcal{A}}$ requests an external commitment C and embeds it in the transcript. After completion of Stage 3 of the $\langle \mathsf{I}_{\mathsf{coin}}, \mathsf{R}_{\mathsf{coin}} \rangle$ interaction, if $\mathcal{A}$ fails to decommit, $\overline{\mathcal{A}}$ outputs $\perp$. Otherwise, it stores the decommitment as r_1. Next, it rewinds to the end of Stage 1 and starts a new emulation (just as in $\mathsf{Expr2}_n$). Again, replacing the equivocal commitment in Stage 2 with an externally supplied commitment. If the adversary fails to decommit, $\overline{\mathcal{A}}$ outputs $\perp$. Otherwise, if $\mathcal{A}$ decommits to r_1 where $\mathsf{r}_1 \neq \mathsf{r}_2$, $\overline{\mathcal{A}}$ outputs 1; if $\mathsf{r}_1 = \mathsf{r}_2$, $\overline{\mathcal{A}}$ outputs 0.

Note that when the external commitments are generated via $\langle \mathsf{S}_{\mathsf{eq}}, \mathsf{R}_{\mathsf{eq}} \rangle$ then the emulation produces a view for $\mathcal{A}$ that is identical to its view in $\mathsf{Expr2}_n(z)$ of H_4. On the other hand, when the external commitments are generated via $\langle \mathsf{S}_{\mathsf{eq}}, \mathsf{R}_{\mathsf{eq}} \rangle$ then the emulation produces a view for $\mathcal{A}$ that is identical to its view in $\mathsf{Expr2}_n(z)$ of H_5.

Now, since $\mathcal{A}$ is abusing in H_5, it must be the case (see proof of Claim 4) that when the external commitments are generated via $\langle \mathsf{S}_{\mathsf{eq}}, \mathsf{R}_{\mathsf{eq}} \rangle$, $\overline{\mathcal{A}}$ outputs 1 with non-negligible probability. However, since $\mathcal{A}$ is non-abusing in H_4, it must be the case that when the external commitments are generated via $\langle \mathsf{S}_{\mathsf{eq}}, \mathsf{R}_{\mathsf{eq}} \rangle$, $\overline{\mathcal{A}}$ outputs 1 with negligible probability. Thus, $\overline{\mathcal{A}}$ distinguishes commitments generated by $\langle \mathsf{S}_{\mathsf{eq}}, \mathsf{R}_{\mathsf{eq}} \rangle$ and commitments generated by $\langle \mathsf{S}_{\mathsf{eq}}, \mathsf{R}_{\mathsf{eq}} \rangle$. This is a contradiction to the special-hiding property of $\langle \mathsf{S}_{\mathsf{eq}}, \mathsf{R}_{\mathsf{eq}} \rangle$ and so the claim is proved. □

Before proving our next claim, we introduce some new experiments: Consider the modified experiment $\mathsf{Expr1}_n^k(z)$ (resp. $\mathsf{Expr2}_n^k(z)$) that proceeds exactly like $\mathsf{Expr1}_n^k(z)$ (resp. $\mathsf{Expr2}_n^k$) with the exception that it chooses the k^{th} coin-toss where the adversary controls the initiator. We note that if $\mathsf{Expr1}_n(z)$ and $\mathsf{Expr2}_n(z)$ are indistinguishable then, for every $1 \leq k \leq m(n)$, $\mathsf{Expr1}_n^k(z)$ and $\mathsf{Expr2}_n^k(z)$ are also indistinguishable.

Note that although $\mathsf{Expr2}_n^k(z)$ runs in expected polynomial time, it may not run in strict polynomial time. This is because the number of rewindings in a given execution may be unbounded. Thus, we define an analogue to experiment $\mathsf{Expr2}_n^k(x)$, called $\mathsf{EffExpr2}_n^k(z, p(\cdot))$, whose run time is bounded. Formally, for any polynomial $p(\cdot)$, we define the following experiment:

$\mathsf{EffExpr2}_n^k(z, p(\cdot))$: The experiment proceeds identically to $\mathsf{Expr2}_n^k(z)$ except that there are at most $p(n)$ rewinding attempts. If after $p(n)$ rewinds, the $\langle \mathsf{I}_{\mathsf{coin}}, \mathsf{R}_{\mathsf{coin}} \rangle$ protocol has not successfully completed, abort the experiment and output $\perp$. Otherwise, output whatever $\mathsf{Expr2}_n^k(z)$ outputs.

The next claim quantifies (as a function of $p(\cdot)$) the distance between the distribution over the output of $\mathsf{EffExpr2}^k$ and the distribution over the output of $\mathsf{Expr2}^k$:

Claim 13. *For every polynomial $p(\cdot)$ and for every $n \in \mathbb{N}$, the statistical distance between the following two probability ensembles is at most $\frac{1}{p(n)}$:*

- $\{\overline{\mathsf{Expr2}}^k_n(z)\}_{z\in\{0,1\}^*}$
- $\{\overline{\mathsf{EffExpr2}}^k_n(z, p(\cdot))\}_{z\in\{0,1\}^*}$

where $\overline{\mathsf{Expr2}}^k_n(z)$ *and* $\overline{\mathsf{EffExpr2}}^k_n(z, p(\cdot))$ *are the outputs of* $\mathsf{Expr2}^k_n(z)$ *and* $\mathsf{EffExpr2}^k_n(z, p(\cdot))$*, respectively.*

Proof. We note that unless an abort occurs in experiment EffExpr2, the random variables $\overline{\mathsf{Expr2}}_n(z)$ and $\overline{\mathsf{EffExpr2}}_n(z, p(\cdot))$ are identically distributed. Thus, the statistical distance can be upperbounded by the probability that EffExpr2 aborts without successful completion of the $\langle \mathsf{I_{coin}}, \mathsf{R_{coin}} \rangle$ protocol in the rewinding stage.

By a standard argument we have that the expected number of rewindings before a successful completion of $\langle \mathsf{I_{coin}}, \mathsf{R_{coin}} \rangle$ in experiment Expr2 is 1. Therefore, by Markov's inequality, the probability that more than $p(n)$ number of rewindings are necessary for successful completion of the $\langle \mathsf{I_{coin}}, \mathsf{R_{coin}} \rangle$ protocol in Expr2 is at most $1/p(n)$. So the probability that $\mathsf{EffExpr2}_n(z, p(\cdot))$ aborts without successful completion of the $\langle \mathsf{I_{coin}}, \mathsf{R_{coin}} \rangle$ protocol in the rewinding stage is at most $1/p(n)$ and the claim is proved. □

We are now ready to prove our correctness claim:

Claim 14. *For every adversary $\mathcal{A}$ in H_5 that successfully commits to a bit β in Stage 2, it is the case that $\mathcal{A}$ encrypts decommitment information for both* 0 *and* 1 *in S_0, S_1 with negligible probability.*

Proof. Assume for contradiction there exists an adversary $\mathcal{A}$ and a value j (where $1 \le j \le m(n)$), such that in the j^{th} $\langle \mathsf{S}, \mathsf{R} \rangle$ interaction where $\mathcal{A}$ plays the part of sender, it is the case that $\mathcal{A}$ encrypts decommitment information for both 0 and 1 in S_0, S_1 probability $1/p(n)$ for some polynomial $p(\cdot)$.

Consider a machine $\overline{\mathcal{A}}$ that on input 1^n, auxiliary input z, and non-uniform advice $p(\cdot)$, participates in a security experiment where $\overline{\mathcal{A}}$ must distinguish a sequence of strings outputted by G from a sequence of random strings. $\overline{\mathcal{A}}$ receives from the external challenger a sequence of strings $\rho_1, \ldots, \rho_{4p(n)}$, internally incorporates $\mathcal{A}$, $\mathcal{Z}$ and all the honest parties and emulates an interaction in H_5. $\overline{\mathcal{A}}$ will choose choose the $\langle \mathsf{I_{coin}}, \mathsf{R_{coin}} \rangle$ interaction corresponding to the j^{th} $\langle \mathsf{S}, \mathsf{R} \rangle$ interaction where $\mathcal{A}$ controls the initiator $\mathsf{I_{coin}}$ of the coin-toss. Call this $\langle \mathsf{I_{coin}}, \mathsf{R_{coin}} \rangle$ interaction the k^{th} $\langle \mathsf{I_{coin}}, \mathsf{R_{coin}} \rangle$ interaction. $\overline{\mathcal{A}}$ will then attempt to fix the outcome of the k^{th} coin toss to some ρ_i for $1 \le i \le 4p(n)$ so that $x = \rho_i$ in the Stage 2 $\langle \mathsf{S_{eq}}, \mathsf{R_{eq}} \rangle$ interaction.

To this end, $\overline{\mathcal{A}}$ runs its internal emulation until $\mathcal{A}$ decommits to some value r^1_{S} in Step 3 of the chosen execution of $\langle \mathsf{S_{com}}, \mathsf{R_{com}} \rangle$ (within the $\langle \mathsf{I_{coin}}, \mathsf{R_{coin}} \rangle$ interaction) and $\langle \mathsf{I_{coin}}, \mathsf{R_{coin}} \rangle$ completes with outcome r. Then $\overline{\mathcal{A}}$ rewinds $\mathcal{A}$ to the point right before $\overline{\mathcal{A}}$ sends r^1_{R} on behalf of the responder and instead sends the value $(r^1)'_{\mathsf{R}} = \rho_1 \oplus r^1_{\mathsf{S}}$. $\overline{\mathcal{A}}$ continues to rewind $\mathcal{A}$ for at most $4p(n)$ times or until the $\langle \mathsf{I_{coin}}, \mathsf{R_{coin}} \rangle$ protocol successfully completes. Note that the execution of $\overline{\mathcal{A}}$, thus far, is distributed identically to an execution of $\mathsf{EffExpr2}^k_n(z, 4p(n))$. Thus, due to the non-abusing property of H_4 and the fact that the outputs of $\mathsf{EffExpr2}^k_n(z, 4p(n))$ and $\mathsf{Expr2}^k_n(z)$ are $1/4p(n)$-close, we have that when the sequence $\rho_1, \ldots, \rho_{4p(n)}$ consists of strings chosen uniformly at random, then with probability at least $1 - 1/4p(n) - \mathrm{neg}(n)$, the $\langle \mathsf{I_{coin}}, \mathsf{R_{coin}} \rangle$ protocol completes successfully in the rewound execution with outcome ρ_i. Thus, it must be the case that when the sequence $\rho_1, \ldots, \rho_{4p(n)}$ consists of pseudorandom strings either:

Case 1: The outcome of the rewound coin toss is $\rho'_i \neq \rho_i$ with probability $\frac{1}{p'(n)}$ for some polynomial $p'(\cdot)$ OR

Case 2: The outcome of the rewound coin toss is ρ_i with with probability $1 - 1/4p(n) - \mathrm{neg}(n)$.

If Case 1 occurs then it is straightforward to see that we can use $\mathcal{A}$ to break the security of the pseudorandom generator G (since $\overline{\mathcal{A}}$ "notices" that the outcome $\rho'_i \neq \rho_i$). Hence, we assume that Case 2 occurs and so we have that with probability at least $1 - 1/4p(n) - \mathrm{neg}(n)$, at the completion of Stage 1 of $\langle \mathsf{S}, \mathsf{R} \rangle$ the common input x is set such that: $x = (r^1)'_{\mathsf{R}} \oplus r^1_{\mathsf{S}} = \rho_i$.

Now, since $\overline{\mathcal{A}}$ in H_5 can decrypt and extract the committed values of the adversary, if the adversary starts to construct commitments in Stage 2 with decommitment information to both 0 and 1 encrypted in S_0, S_1, then $\overline{\mathcal{A}}$ outputs 0; otherwise $\overline{\mathcal{A}}$ outputs 1. When the string ρ_i is pseudorandom, we have by hypothesis and the analysis above, that $\mathcal{A}$ successfully constructs such commitments with probability at least $\frac{1}{p(n)} - \frac{1}{4p(n)} - \mathrm{neg}(n)$. However, when the string ρ_i is truly random, then with all but negligible probability, there does not exist a string s such that $G(s) = \rho_i$ (assuming $G(s)$ has input length n and output length, say, $3n$) and so all commitments using common input $x = \rho_i$ must be *statistically* binding. Thus, when the sequence $\rho_1, \ldots, \rho_{4p(n)}$ consists of truly random strings $\mathcal{A}$ (and in fact even a computationally unbounded adversary) successfully constructs such commitments with only negligible probability. Thus, $\overline{\mathcal{A}}$ distinguishes between a sequence of pseudorandom and random strings with probability at least $\frac{1}{2p(n)}$, which yields a contradiction. □

Hybrid H_6: In this hybrid, Step 2 of Stage 2 of $\langle \mathsf{S}, \mathsf{R} \rangle$ is modified for interactions in which the adversary plays the part of receiver in the following way: Decommitment information for both bits 0 and 1 is encrypted in S_0, S_1.

Indistinguishability and the statistical binding property will be reduced to the security properties of the simulatable PKE scheme. Note that to reduce to the indistinguishability of encryptions $\mathsf{Enc}(\mathrm{PK}^*, m, r)$ of a specified message m and strings generated at random via $\mathsf{oRndEnc}(\mathrm{PK}^*, r_{\mathsf{Enc}})$, we need to ensure that the outcome of the first coin-toss in Stage 1 of $\langle \mathsf{S}, \mathsf{R} \rangle$ yields the target public key PK^*. Fixing the outcome of the coin-toss will require rewinding the adversary and in order to guarantee that the rewinding strategy is succesfull, we will rely on the fact that the adversary is non-abusing. More specifically, we consider the intermediate hybrids $H_6^0, H_6^1, \ldots, H_6^{m(n)}$ where $H_6^0 = H_5$ and H_6^i is the hybrid where the $m(n) - i + 1$-th through $m(n)$-th commitments in interactions $\langle \mathsf{S}, \mathsf{R} \rangle$ where the adversary plays the part of the receiver, are constructed such that decommitment information to both 0 and 1 is encrypted in strings S_0, S_1 of Stage 2.

Before proving our next claim, we introduce some new experiments: Consider the modified experiment $\mathsf{Expr1}_n^k(z)$ (resp. $\mathsf{Expr2}_n^k(z)$) that proceeds exactly like $\mathsf{Expr1}_n^k(z)$ (resp. $\mathsf{Expr2}_n^k$) with the exception that it chooses the k^{th} coin-toss, where the order of the coin-tosses is determined by the order in which $\mathcal{A}$ decommits in Step 3 and where the adversary controls the initiator. Moreover, in $\mathsf{Expr2}_n^k$ the rewinding is repeated until *both* of the following hold:

- The $\langle \mathsf{I}_{\mathsf{coin}}, \mathsf{R}_{\mathsf{coin}} \rangle$ protocol successfully completes.
- The decommitment corresponding to the rewound coin-toss is again the k-th decommitment of the experiment.

We note that if $\mathsf{Expr1}_n(z)$ and $\mathsf{Expr2}_n(z)$ are indistinguishable then, for every $1 \leq k \leq m(n)$, $\mathsf{Expr1}_n^k(z)$ and $\mathsf{Expr2}_n^k(z)$ are also indistinguishable.

Note that although $\mathsf{Expr2}_n^k(z)$ runs in expected polynomial time, it may not run in strict polynomial time. This is because the number of rewindings in a given execution may be unbounded. Thus, we define an analogous experiment to $\mathsf{Expr2}_n^k(x)$, called $\mathsf{EffExpr2}_n^k(z, p(\cdot))$, which has a bounded run time. Formally, for any polynomial $p(\cdot)$, we define the following experiment:

$\mathsf{EffExpr2}_n^k(z, p(\cdot))$: The experiment proceeds identically to $\mathsf{Expr2}_n^k(z)$ except that there are at most $p(n)$ rewinding attempts. If after $p(n)$ rewinds, the $\langle \mathsf{I}_{\mathsf{coin}}, \mathsf{R}_{\mathsf{coin}} \rangle$ protocol has not successfully completed or the corresponding decommitment is not the k^{th} decommitment, abort the experiment and output $\perp$. Otherwise, output whatever $\mathsf{Expr2}_n^k(z)$ outputs.

The next claim quantifies (as a function of $p(\cdot)$) the distance between the distribution over the output of $\mathsf{EffExpr2}^k$ and the distribution over the output of $\mathsf{Expr2}^k$:

Claim 15. *For every polynomial $p(\cdot)$ and for every $n \in \mathbb{N}$, the statistical distance between the following two probability ensembles is at most $\frac{1}{p(n)}$:*

- $\{\overline{\mathsf{Expr2}}_n^k(z)\}_{z\in\{0,1\}^*}$
- $\{\overline{\mathsf{EffExpr2}}_n^k(z, p(\cdot))\}_{z\in\{0,1\}^*}$

where $\overline{\mathsf{Expr2}}_n^k(z)$ and $\overline{\mathsf{EffExpr2}}_n^k(z, p(\cdot))$ are the outputs of $\mathsf{Expr2}_n^k(z)$ and $\mathsf{EffExpr2}_n^k(z, p(\cdot))$, respectively.

Proof. We note that unless an abort occurs in experiment EffExpr2, the random variables $\overline{\mathsf{Expr2}}_n(z)$ and $\overline{\mathsf{EffExpr2}}_n(z, p(\cdot))$ are identically distributed. Thus, the statistical distance can be upperbounded by the probability that EffExpr2 aborts without successful completion of the $\langle \mathsf{I_{coin}}, \mathsf{R_{coin}} \rangle$ protocol in the rewinding stage.

By a standard argument we have that the expected number of rewindings before a successful completion of $\langle \mathsf{I_{coin}}, \mathsf{R_{coin}} \rangle$ in experiment Expr2 is 1. Therefore, by Markov's inequality, the probability that more than $p(n)$ number of rewindings are necessary for successful completion of the $\langle \mathsf{I_{coin}}, \mathsf{R_{coin}} \rangle$ protocol in Expr2 is at most $1/p(n)$. So the probability that $\mathsf{EffExpr2}_n(z, p(\cdot))$ aborts without successful completion of the $\langle \mathsf{I_{coin}}, \mathsf{R_{coin}} \rangle$ protocol in the rewinding stage is at most $1/p(n)$ and the claim is proved. □

We are now ready to prove indistinguishability of the Hybrid experiments:

Claim 16. *For $1 \le k \le m(n)$ the output of $\mathcal{Z}$ in H_6^{k-1} and H_6^k is indistinguishable.*

Note that Claim 16 immediately implies that the output of $\mathcal{Z}$ in H_5 and H_6 is indistinguishable. We now proceed to prove Claim 16.

Proof. Assume for contradiction there exists an adversary $\mathcal{A}$, an environment $\mathcal{Z}$, a value k (where $1 \le k \le m(n)$), a distinguisher D and a polynomial $p(\cdot)$ such that for infinitely many n, D distinguishes the output of $\mathcal{Z}$ in H_6^{k-1} and H_6^k with probability $1/p(n)$ for some polynomial $p(\cdot)$.

Consider a machine $\overline{\mathcal{A}}_k$ that on input 1^n, auxiliary input z, and non-uniform advice $p(\cdot)$, participates in a security experiment for the simulatable PKE scheme $\mathcal{E}$: $\overline{\mathcal{A}}_k$ receives externally a sequence of values $\{r_1^*, \ldots, r_{4p(n)}^*\}$ such that for each $1 \le i \le 4p(n)$, $\mathsf{oGen}(r_i^*) = \mathrm{PK}_i^*$, internally incorporates $\mathcal{A}(1^n)$, $\mathcal{Z}(z)$ and all the honest parties and emulates an interaction in H_6. Intuitively, $\overline{\mathcal{A}}_k$ will embed one of the challenge public keys and ciphertexts from the external security experiment in Stage 2 of the $m(n) - i + 1$-th execution of the coin tossing protocol $\langle \mathsf{I_{coin}}, \mathsf{R_{coin}} \rangle$ (where coin-tosses are ordered according to the order of the decommitments in Step 3). To this end, on input bit β, $\overline{\mathcal{A}}_k$ will play the role of sender and interact with $\mathcal{A}$ in $\langle \mathsf{S_{eq}}, \mathsf{R_{eq}} \rangle$ using common input x. $\overline{\mathcal{A}}_k$ runs the equivocator for $\langle \mathsf{S_{eq}}, \mathsf{R_{eq}} \rangle$ to construct an equivocal commitment C which it can decommitment to both 0 and 1. $\overline{\mathcal{A}}_k$ chooses $b \in \{0, 1\}$ at random and sets S_b to be an encryption of a decommitment to β. Next, $\overline{\mathcal{A}}_k$ sets the message m in the external experiment to be a correct decommitment to bit $1 - \beta$ and receives challenge ciphertexts $\{S_{1-b}^1, \ldots, S_{1-b}^{4p(n)}\}$ (one for each challenge public key). $\overline{\mathcal{A}}_k$ must distinguish whether the ciphertexts $\{S_{1-b}^1, \ldots, S_{1-b}^{4p(n)}\}$ are all encryptions of a decommitment to $1 - \beta$ or whether the ciphertexts $\{S_{1-b}^1, \ldots, S_{1-b}^{4p(n)}\}$ are all outputted by $\mathsf{oRndEnc}(\mathrm{PK}_i^*, r_{\mathsf{Enc}}^i)$ where r_{Enc}^i is chosen uniformly at random.

Next, $\overline{\mathcal{A}}_i$ runs its internal emulation until $\mathcal{A}$ decommits in the $m(n) - i + 1$-th $\langle \mathsf{S_{com}}, \mathsf{R_{com}} \rangle$ interaction in the first coin toss of Stage 1 to some value r_{R}^0 and the $\langle \mathsf{I_{coin}}, \mathsf{R_{coin}} \rangle$ interaction completes. Then $\overline{\mathcal{A}}_k$ rewinds $\mathcal{A}$ to the point right before $\overline{\mathcal{A}}_i$ sends r_{S}^0 and instead sends the value $\overline{r}_{\mathsf{S}}^0 = r_1^* \oplus r_{\mathsf{R}}^0$. $\overline{\mathcal{A}}_k$ continues to rewind $\mathcal{A}$ for at most $4p(n)$ times or until the $\langle \mathsf{I_{coin}}, \mathsf{R_{coin}} \rangle$ protocol successfully completes and the decommitment corresponding to the rewound coin-toss is the k-th decommitment of the rewound execution. Note that up to the $m(n) - i + 1$-th $\langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle$ interaction, the execution of $\overline{\mathcal{A}}_k$ is distributed identically to an execution of $\mathsf{EffExpr2}_n^k(z, 4p(n))$ in H_5.

Thus, due to the non-abusing property of $\mathcal{A}$ in H_5, and the fact that the output of $\mathsf{EffExpr2}_n^k(z, 4p(n))$ and $\mathsf{Expr2}_n^k(z)$ are statistically $1/4p(n)$-close, we have that with probability at least $1 - 1/4p(n) - \mathrm{neg}(n)$, the outcome of the coin toss in the rewound execution is $r_i^* = r_i^* \oplus r_{\mathsf{R}}^0 \oplus r_{\mathsf{R}}^0 = \overline{r}_{\mathsf{S}}^0 \oplus r_{\mathsf{R}}^0$ (for some i). Thus, since $\mathsf{oGen}(r_i^*) = \mathrm{PK}_i^*$, $\overline{\mathcal{A}}_k$ can embed its challenge ciphertext S_{1-b}^i (which is encrypted under public key PK_i^*) in Stage 2 of $\langle \mathsf{S}, \mathsf{R} \rangle$ for the $m(n) - i + 1$-th commitment.

For the $m(n) - i + 2$-th through $m(n)$-th commitments, $\overline{\mathcal{A}}_i$ chooses $b \in \{0, 1\}$ uniformly at random, sets S_b to be an encryption of the decommitment information corresponding to β and sets S_{1-b} to be an encryption of the decommitment information corresponding to $1 - \beta$.

Finally, $\overline{\mathcal{A}}_k$ runs D on the output of $\mathcal{Z}$ and outputs whatever D outputs. Note that if $\{S_{1-b}^1, \ldots, S_{1-b}^{4p(n)}\}$ are all outputted by $\mathsf{oRndEnc}(\mathrm{PK}_i^*, r_{\mathsf{Enc}}^i)$, then the output of $\mathcal{Z}$ is at least $1/4p(n) + \mathrm{neg}(n)$ close to the output of $\mathcal{Z}$ in H_6^{k-1}. On the other hand, if $\{S_{1-b}^1, \ldots, S_{1-b}^{4p(n)}\}$ are encryptions of a decommitment to $1 - \beta$ under $\mathrm{PK}_1^*, \ldots, \mathrm{PK}_{4p(n)}^*$, then the output of $\mathcal{Z}$ is at least $1/4p(n) + \mathrm{neg}(n)$ close to the output of $\mathcal{Z}$ in H_6^k. Thus, the difference between the probability that D outputs 1 in the first case and D outputs 1 in the second case is at least $1/p(n) - 1/4p(n) - \mathrm{neg}(n) - 1/4p(n) - \mathrm{neg}(n) = 1/2p(n) - \mathrm{neg}(n)$. So $\overline{\mathcal{A}}_k$ distinguishes between encryptions of decommitment to $1 - \beta$ and obliviously generated ciphertexts with non-negligible probability. This yields a contradiction to the security of $\mathcal{E}$ and so the claim is proved. □

Claim 17. *For every adversary $\mathcal{A}$ in H_5 that successfully commits to a bit β in Stage 2, it is the case that $\mathcal{A}$ encrypts decommitment information for both 0 and 1 in S_0, S_1 with negligible probability.*

Proof. Since $\mathcal{A}$'s commitments can be extracted in H_5 and H_6, this follows immediately from the indistinguishability of the output of $\mathcal{Z}$ in H_5 and H_6. □

7 Puzzle Instantiations

By Theorem 3, it suffices to present an adaptive UC puzzle in a given model to demonstrate feasibility of adaptive and UC secure computation. We first give some brief intuition on the construction of adaptive UC-puzzles in various models. Formal constructions and proofs follow.

In the Common reference string (CRS) model, the Uniform reference string (URS) model and the Key registration model the puzzles are identical to the ones presented in [23] for the static case, where the puzzle interactions essentially consists of a call to the corresponding ideal setup functionalities. Thus, in these models, the simulator is essentially handed the trapdoor for the puzzle via its simulation of the ideal functionality and the puzzles are non-interactive. In the Timing model and the Partially Isolated Adversaries model, we rely on essentially the same puzzles as [23], however, we need to modify the simulator to accommodate adaptive corruption by the adversary (see Section 7.7 for more details).

Constructing adaptive UC-puzzles in the Sunspots model is less straightforward and so we give more detail here. Simulated reference strings r in the Sunspots model have Kolmogorov complexity smaller than k. Thus, as in [23], the puzzle sender and receiver exchange 4 strings (v_1, c_2, v_2, c_2). We then let Φ' denote the statement that c_1, c_2 are commitments to messages p_1, p_2 such that (v_1, p_1, v_2, p_2) is an accepting transcript of a Universal argument of the statement $\Phi = \mathsf{KOL}(r) \leq k$. Note that since we require *statistical* and *adaptive* simulation of puzzles, the commitment scheme used must be both statistically-hiding and "obliviously samplable" (i.e. there is a way to generate strings that are statistically indistinguishable from commitments, without "knowing" the committed value). See Section 7.6 for details.

7.1 Adaptive UC in the Common Reference String (CRS) Model

In the common reference string model [5] the parties have access to a CRS choses from a specified trusted distribution D, which is captured via the following ideal functionality $\mathcal{F}_{CRS}^{D}$ (Figure 7.1) that samples a string r from the distribution D and sets it as a CRS.

Functionality $\mathcal{F}_{CRS}^{D}$

1. Upon activation with an id sid run the sampling algorithm D with uniformly distributed random input $\rho \in \{0,1\}^n$ to obtain a reference string $r = D(\rho)$. Store D, ρ, r and send (sid, r) to the adversary.
2. On input (CRS, sid) from a party P with session id sid', return (sid, r) to that party only if $sid = sid'$, and ignore the message, otherwise.

Figure 4: Common Reference String Functionality

We construct a puzzle in the $\mathcal{F}_{CRS}^{G}$-hybrid, where G is a pseudorandom generator.

Protocol $\langle S, R\rangle$: On input sid, S and R request a common reference string from ideal functionality $\mathcal{F}_{CRS}^{G}$ by sending sid.

Relation: $\mathcal{R} = \{(x,y) | y = G(x)\}$

Figure 5: Common Reference String Puzzle

THEOREM 4. *Assume the existence of a simulatable PKE scheme and the existence of an $O(t)$-round* EQNMCom *scheme. Let G be a pseudorandom generator. Then, for every well-formed ideal functionality $\mathcal{F}$, there exists an $O(t)$-round protocol π that realizes $\hat{\mathcal{F}}$ with adaptive UC security in the $\mathcal{F}_{CRS}^{G}$-hybrid.*

7.2 Adaptive UC in the Uniform Reference String (URS) Model

When the distribution D in the CRS model is fixed as the uniform distribution, we obtain the uniform reference string model [7]. Let the URS-functionality be $\mathcal{F}_{URS} = \mathcal{F}_{CRS}^{I}$, where I is the identity function. Since the $\mathcal{F}_{CRS}^{G}$-functionality implements $\mathcal{F}_{URS}$-functionality when G is pseudo-random generator, any protocol that realizes f in the $\mathcal{F}_{CRS}^{G}$-hybrid also realized the same functionality in the $\mathcal{F}_{URS}$-hybrid.

THEOREM 5. *Assume the existence of a simulatable PKE scheme and the existence of an $O(t)$-round* EQNMCom *scheme. Then, for every well-formed ideal functionality $\mathcal{F}$, there exists an $O(t)$-round protocol π that realizes $\hat{\mathcal{F}}$ with adaptive UC security in the $\mathcal{F}_{URS}^{G}$-hybrid.*

7.3 Adaptive UC in the Key Registration Model

In the key registration model [1] includes a service that allows all parties to obtain a public key derived from a seed, which is kept secret by the service. The service is modeled as an ideal functionality $\mathcal{F}_{KR}^{f}$ parameterized by a function $f : \{0,1\}^* \to \{0,1\}^*$, which is presented in Figure 7.3.

Using the KR-functionality we construct a puzzle as follows:

Functionality $\mathcal{F}^f_{KR}$

Upon activation with input sid and security parameter n initialize a set R of empty strings.

Registration: On input message $(register, sid)$ from party P_1 send to the adversary $\mathcal{A}$ $(register, sid, P_i)$ and receive a value p'. If $p' \in R$, then set $p \leftarrow p'$. Otherwise, choose $r \leftarrow \{0,1\}^n$ and set $p \leftarrow f(r)$ and add p to R. Finally, record (P_i, p) and return (sid, p) to both P_i and $\mathcal{A}$.

Registration by corrupted party: On input message $(register, sid, r)$ from a corrupted party P_i, add $P_i, f(r)$ but does not add $f(r)$ to R.

Retrieval: On input $(retrieve, sid, P_i, P_j)$ from party P_j, send $(retrieve, sid, P_i, P_j)$ to $\mathcal{A}$ and get back a value p. If (P_i, p) is recorder, return (sid, P_i, p) to P_j. Otherwise, return $(sid, P_i, \perp)$ to P_j

Figure 6: Key Registration functionality

Protocol $\langle S, R\rangle$: On input sid, R sends $(retrieve, sid, S, R)$ to the ideal functionality $\mathcal{F}^f_{KR}$ to obtain a public key.

Relation: $\mathcal{R} = \{(x, y) | y = f(x)\}$

Figure 7: Key Registration Model Puzzle

THEOREM 6. *Assume the existence of a simulatable PKE scheme and the existence of an $O(t)$-round* EQNMCom *scheme. Let f be a one-way function. Then, for every well-formed ideal functionality $\mathcal{F}$, there exists an $O(t)$-round protocol π that realizes $\hat{\mathcal{F}}$ with adaptive UC security in the $\mathcal{F}^f_{KR}$-hybrid.*

7.4 Non-Uniform Adaptive UC

In this model, we consider environments that are $\mathcal{PPT}$ machines and ideal-model adversaries that are n.u.$\mathcal{PPT}$ machines. First, we construct an adaptive puzzle in this model and then state our main theorem. To construct an adaptive puzzle in this model, we make the same complexity theoretic assumptions as those made in [23]; namely, we assume the existence of an *evasive set* $\mathcal{L}$ in P.

Recall the definition of an evasive set [23]:

Definition 10. *A set $\mathcal{L}$ is said to be evasive, if for all n, $S \cap \{0,1\}^n \neq \emptyset$ and for any $\mathcal{PPT}$ machine M, there is a negligible function $v(\cdot)$, such that,* $\Pr[M(1^n) \in S \cap \{0,1\}^n] \leq v(n)$

In [23], several other assumptions sufficient for constructing puzzles in this model. We note that in the adaptive case we can also construct puzzles under each of the assumptions used by [23]. However, for simplicity, we focus only on the assumption that there exists an an evasive set in P.

Lemma 6. *Assume the existence of an evasive set $\mathcal{L}$ in* P. *Then there exists an adaptive puzzle in $(\mathcal{PPT}, n.u.\mathcal{PPT})$ with an empty protocol.*

Proof. Let λ denote the empty string. Define the puzzle $\mathsf{P}_{nu} = (\langle \mathsf{S}, \mathsf{R}\rangle, \mathcal{R})$ as follows (see Figure 7.4):

We prove soundness and adaptive, statistical simulatability of the puzzle.

Protocol $\langle \mathsf{S}, \mathsf{R} \rangle$: S and R on input 1^n run the empty protocol.

Relation: $\mathcal{R} = \{(x, \lambda) | x \in \mathcal{L}\}$

Figure 8: Non-uniform Puzzle

Soundness: Since $\mathcal{L}$ is evasive, no cheating $\mathcal{PPT}$ receiver can output x such that $(x, \lambda) \in \mathcal{R}$, i.e. $x \in \mathcal{L}$ with more than negligible probability.
Adaptive Simulatability: Consider an adversary $\mathcal{A}$ that participates in a concurrent adaptive puzzle execution with environment $\mathcal{Z}$. We construct a n.u.$\mathcal{PPT}$ adversary $\mathcal{A}'$ that receives $\delta \in \mathcal{L}$ as non-uniform advice and proceeds as follows: It incorporates $\mathcal{A}$ internally and emulates an execution with A. It forwards all messages from $\mathcal{A}$ to $\mathcal{Z}$, except the messages involved in the puzzle interactions with $\mathcal{A}$. However, since the protocol is empty, there are no messages exchanged in the puzzle interaction. Clearly, dealing with adaptive corruptions is trivial since no messages are exchanged in the puzzle interaction. To outputs a witness, $\mathcal{A}'$ simply outputs δ on its special output tape whenever $\mathcal{A}$ sends $(\mathsf{TRANS} = \lambda, C)$ to $\mathcal{Z}$ for a puzzle interaction. Finally, since the interaction between $\mathcal{A}'$ with $\mathcal{Z}$ is identical to the interaction between $\mathcal{A}$ with $\mathcal{Z}$, the real and ideal executions are perfectly indistinguishable to $\mathcal{Z}$. □

THEOREM 7. *Assume the existence of simulatable PKE secure against n.u.$\mathcal{PPT}$, the existence of an $O(t)$-round* $\mathsf{EQNMCom}$ *scheme secure against n.u.$\mathcal{PPT}$, and the existence of an evasive set $\mathcal{L}$. Then, for every well-formed ideal functionality $\mathcal{F}$, there exists an $O(t)$-round protocol π that realizes $\hat{\mathcal{F}}$ with adaptive Non-Uniform UC security.*

7.5 Quasi-Polynomial Adaptive UC

Recall that the Quasi-Polynomial Simulation model is a relaxation of the standard simulation-based definition of security, allowing for a super polynomial-time or Quasi-polynomial simulation (QPS).

THEOREM 8. *Assume the existence of simulatable PKE secure against $\mathcal{PQT}$, the existence of an $O(t)$-round* $\mathsf{EQNMCom}$ *scheme secure against $\mathcal{PQT}$, and the existence of one-way functions that can be inverted with probability* 1 *in $\mathcal{PQT}$. Then, for every well-formed ideal functionality $\mathcal{F}$, there exists an $O(t)$-round protocol π that realizes $\hat{\mathcal{F}}$ with adaptive QPS-UC security.*

We remark that one-way functions that are invertible by $\mathcal{PQT}$ machines as well as $O(n)$-round $\mathsf{EQNMCom}$ schemes can be constructed based on one-way functions with sub-exponential hardness. Thus, assuming simulatable PKE secure against $\mathcal{PQT}$ and one-way function with subexponential hardness, we obtain as a corollary an $O(n)$-round protocol that securely realizes any functionality with adaptive QPS-UC security.

The notion of security we achieve is analgous to the one in [23] which guarantees that the output of the simulator is indistinguishable also for $\mathcal{PQT}$. This means that anything an attacker can learn "on-line" (in poly-time) can be simulated "off-line" (in qpoly-time) in a way that is indistinguishable also "off-line".

We present the following Adaptive UC-Puzzle in the QPS model (See Figure 7.5). Let f be a one-way function that can be inverted with probability 1 in $\mathcal{PQT}$.
Soundness: This follows directly from the one-wayness of f and the witness-hiding property of the proof given by the sender.
Adaptive Simulatability: The simulator $\mathcal{A}'$ simply plays the part of the honest receiver. Upon adaptive corruption, $\mathcal{A}'$ reveals the randomness of the honest receiver. Clearly, this simulation is identically distributed to a real execution. To output a witness, we require $\mathcal{A}'$ to compute the inverse of $y = f(x)$ for a random x. While emulating $\mathcal{A}$, if $\mathcal{A}$ completes a puzzle-interaction by convincing the receiver in the WHPOK, then $\mathcal{A}'$

Protocol $\langle S, R\rangle$:

$S \to R$: Pick $x \leftarrow \{0,1\}^n$ and send $y = f(x)$ to R.

$S \leftrightarrow R$: a witness-hiding argument of knowledge of the statement that there exists x' such that $y = f(x')$.

Relation: $\mathcal{R} = \{(x, y)|y = f(x)\}$

Figure 9: QPS Puzzle

Functionality $\mathcal{F}_{\mathsf{sun}}$.

1. Upon activation with session id sid proceed as follows. Send the message $(\mathsf{Activated}, sid)$ to the adversary, and wait to receive bad a message (n, sid, D). Run the sampling algorithm d on a uniformly distributed random input ρ from $\{0,1\}^n$ to obtain a reference string $r = D(\rho)$. Store D, ρ, r and send $(\mathsf{CRS}, sid, r, \rho)$ to the adversary.
2. When receiving input (CRS, sid) from some party P with session id sid', send (CRS, sid, r) to that party if $sid = sid'$; otherwise ignore the message.

Figure 10: $\mathcal{F}_{\mathsf{sun}}$

inverts x to obtain a witness y such that $y = f(x)$. If an inverse exists, it finds one since f is invertible by $\mathcal{PQT}$ machines. From the soundness property of the WHPOK, it follows that, if $\mathcal{A}$ convinces the receiver, then except with negligible probability, x has an inverse w.r.t. f.

7.6 Adaptive UC in the Sunspots model

Below we describe the functionality $\mathcal{F}_{\mathsf{sun}}$ (See Figure 7.6).

We construct an adaptive UC-puzzle in the sunspots model which relies on a statistically hiding commitments $\langle C, R\rangle$ with additional algorithms $(\tilde{C}, \mathsf{Adap})$ that have the following properties:

Invertibility: For every (expected) PPT machine R^*, let τ be the transcript of the interaction between R^* and C on input bit β and random tape $r \in \{0,1\}^*$ for C. Then $\mathsf{Adap}(r, \tau)$ produces a random tape r' such that $\langle C^*, R^*\rangle$ yields transcript τ when C^* uses random tape r'.

Strong Oblivious Simulation: For every (expected) PPT machine R^*, it holds that, the following ensembles are statistically indistinguishable over $n \in N$.

- $\{(\mathsf{sta}^{R^*,r_1}_{\langle C^*,R\rangle,r_1}(z), r_1)\}_{n\in N, r_1,r_2\in\{0,1\}^n, z\in\{0,1\}^*, \beta\in\{0,1\}}$
- $\{(\mathsf{sta}^{R^*}_{\langle C,R\rangle,r_2}(\beta, z), \mathsf{Adap}(r_2, \tau))\}_{n\in N, r_1,r_2\in\{0,1\}^n, z\in\{0,1\}^*, \beta\in\{0,1\}}$

where $\mathsf{sta}^{R^*,r_1}_{\langle C^*,R\rangle}(\beta, z)$ denotes the random variable describing the output of R^* after receiving a commitment from C^* using random tape r_1, $\mathsf{sta}^{R^*}_{\langle C,R\rangle,r_2}(\beta, z)$ denotes the random variable describing the output of R^* after receiving a commitment from C to bit β using random tape r_2 and τ denotes the transcript produced by $\langle C, R\rangle$.

We note that the standard construction of statistically-hiding commitment scheme from collision-resistant hash function (CRHF) fulfills the above definition when the CRHF has ”random outputs” (i.e. for randomly chosen input x, the output of the CRHF is statistically indistinguishable from random). Such a CRHF was constructed by [19] from lattice-based assumptions. Additionally, the construction of statistically-hiding commitment from one-way permutation (OWP) of [26] has the desired properties, since C^* can simply choose a random image $y = \pi(x)$ of the OWP π, without knowing the corresponding x and run the interactive hashing protocol oblviously. We note that the construction of [26] relies on a general hardness assumption but requires $\mathrm{poly}(n)$ rounds while the construction of [19] relies on a concrete hardness assumption but is constant-round. For concreteness, we state the theorem below for the case of CRHF with random output. Our proof is written for the general case, assuming any commitment scheme that satisfies the properties above.

THEOREM 9. *Assume the existence of simulatable PKE, collision-resistant hash-functions with random output, and an $O(t)$-round* $\mathsf{EQNMCom}$ *scheme. Then, for every well-formed ideal functionality $\mathcal{F}$, there exists a $O(t)$-round protocol π in the $\mathcal{F}_{\mathsf{sun}}$-hybrid that realizes $\hat{\mathcal{F}}$ with adaptive UC-security w.r.t.* (μ, d, t)*-conforming adversaries where $\mu(n) - d(n) > n^{\epsilon}$ for some $\epsilon > 0$.*

We first consider a $\mathcal{F}^{G}_{\mathsf{sun}}$-hybrid model, where $\mathcal{F}^{G}_{\mathsf{sun}}$ is the ideal functionality identical fo $\mathcal{F}_{\mathsf{sun}}$, with the exception that, instead of running the sampling algorithm D on a uniformly distributed ρ, it runs D on input $G(x)$ for a uniformly random x, where G is a pseudo-random generator. We conclude that the protocol constructed in the $\mathcal{F}^{G}_{\mathsf{sun}}$-hybrid also securely realizes the functionality in the $\mathcal{F}_{\mathsf{sun}}$-hybrid.

We proceed towards constructing a puzzle in the $\mathcal{F}^{G}_{\mathsf{sun}}$-hybrid. Let $G : \{0,1\}^{n^{\delta}} \rightarrow \{0,1\}^*$ be a pseudo-random generator that expands a seed of length n^{δ} (for $\delta > 0$) to a stream of bits such that $d(n) + n^{\delta} + |G| < \mu(n)$. Such a $\delta(n)$ is guaranteed to exist since $\mu(n) - d(n) > n^{\epsilon}$. Such a generator can be constructed from any one-way function, which exists under the assumption of simulatable PKE.

Our construction of the puzzle is similar to the construction used in [23] (which is based on [8]), the only difference is the type of commitment we use in the construction.

Let $(V_1, P_1, V_2, P_2, V_3)$ be the respective verifier and prover algorithms for a public-coin univeral argument for the language

$$\mathcal{L}_{\mathsf{KOL}} = \{r \mid r \in \{0,1\}^n \text{ and } \mathsf{KOL}(r) < n^{\frac{\epsilon+\delta}{2}}\},$$

where $\mathsf{KOL}(x)$ is the *Kolmogorov complexity* of a string x. Such a system can be constructed based on collision-resistant hash functions. We describe a language of transcripts of universal arguments in which the prover's messages are committed instead of sent to the verifier. In order to commit, we use a special statistically hiding commitment scheme $\langle C, R\rangle$, which satisfies the properties listed above. We describe the puzzle construction below (See Figure 7.6).

Soundness: Suppose a $\mathcal{PPT}$ receiver R^* is able to break the soundness by outputting the witness for a puzzle with probability p. We use R^* to construct another efficient algorithm P which breaks the soundness property of the universal argument system with probability $\mathrm{poly}(p)$. The soundness of the universal argument system therefore implies that p must be negligible which implies the soundness of the puzzle. We show that P breaks the soundness of the universal argument w.p. $\mathrm{poly}(p)$ on the statement that the reference string r sampled from $\mathcal{F}^{G}_{\mathsf{sun}}$-functionality has a "short" description. Since G is pseudo-random, if p is non-negligible, then P breaks the soundness with non-negligible probability in the hybrid experiment when r is sampled from the $\mathcal{F}_{\mathsf{sun}}$ functionality. Since, D has min-entropy $\mu(n)$, w.p. at most $2^{-n^{\frac{\epsilon-\delta}{2}}} = 2^{-O(n^{\epsilon})}$, r has a short description and therefore no computationally bounded prover can succeed in the universal argument with non-negligible probability. Thus, p is negligible.

More precisely, P upon receiving the verifier message v_1, feeds v_1 to R and then internally simulates the rest of the puzzle until R outputs the witness. By hypothesis, this succeeds with probability p. Let p_1 be a decommitment to the first message sent by R. P forwards p_1 externally to the verifier and receives the next

Protocol $\langle S, R\rangle$: S and R obtain the reference string r from the $\mathcal{F}^{G}_{\mathsf{sun}}$-functionality.

$S \to R$: Pick $m_1 \leftarrow V_1(r, n)$ and send to R.

$R \leftrightarrow S$: R and S interact using $\langle C^*, R\rangle$, where R plays the role of C^*. We denote by c_1 the resulting transcript.

$S \to R$: Pick $m_2 \leftarrow V_2(r, n)$ and send to R.

$R \leftrightarrow S$: R and S interact using $\langle C^*, R\rangle$, where R plays the role of C^*. We denote by c_2 the resulting transcript.

Relation:

$$\mathcal{R} = \left\{ (\mathsf{TRANS}, w) \mid \begin{array}{l} \mathsf{TRANS} = (r, v_1, c_1, v_2, c_2), w = ((p_1, r_1), (p_2, r_2)) \\ \exists r_1, r_2 c_1 \leftarrow \langle C, R\rangle(p_1, r_1), c_2 \leftarrow \langle C, R\rangle(p_2, r_2) \text{ and} \\ V_3(s, v_1, p_1, v_2, p_2) = 1 \end{array} \right\}$$

Figure 11: Sun Spots Puzzle

message v_2. At this point, P rewinds R and feeds v_2 instead of the second message (simulated before) from the verifier and continues to simulate the rest of the puzzle. If R outputs a witness $((p'_1, r'_1), (p_2, r_2))$ then we argue that the p'_1 outputted must, with all but negligible probability, be the same as p_1 outputted during the first simulation. Otherwise, R breaks the binding of the equivocal commitment and we obtain a witness M to $r \in \mathcal{L}_{\mathsf{KOL}}$. In particular, this means that R distinguishes the output of G from a truly random string. Now, we argue that with probability at least p^2, the transcript (v_1, p_1, v_2, p_2) is an accepting transcript for the universal argument.

Adaptive Simulatability: We achieve statistical simulation by allowing the simulator $\mathcal{A}'$ to set the reference string and obtain the witness, which is the description of D, G and x, whose combined size by construction is $n^{\delta} + O(1) + d(n) < n^{\frac{\epsilon+\delta}{2}}$. Furthermore, while emulating a receiver in a puzzle with adversary $\mathcal{A}$, instead of following the honest receiver's code, $\mathcal{A}'$ runs the protocol $\langle C, R\rangle$ with the sender S in the second and fourth step of the puzzle interaction. The simulator runs the code of an honest prover (P_1, P_2) in the universal argument with witness (D, G, x) obtaining transcript (v_1, p_1, v_2, p_2) and sends commitments to p_1 and p_2 using $\langle C, R\rangle$. Thus, the values committed to by $\mathcal{A}'$ and the randomness used to commit amount to a trapdoor for the puzzle. Upon adaptive corruption, $\mathcal{A}'$ uses Adap to produce randomness r' to show that the transcript "could have" been produced using C^*. Notice that, due to the properties of $\langle C, R\rangle$, even after the randomness r' has been produced, the puzzle sender's view is statistically indistinguishable in the real and simulated interaction.

7.7 Adaptive UC in the Timing model

We prove feasibility of our result in the timing model, which is the same as presented in [23], in the following theorem.

THEOREM 10. *Let $\epsilon > 1$ and $\Delta > 0$ be constants. Assume the existence of simulatable PKE and a $2\epsilon^2\Delta$-delayed $O(t)$-round* $\mathsf{EQNMCom}$ *scheme. Then, for every well-formed ideal functionality $\mathcal{F}$, there exists an $O(t)$-round protocol π that realizes $\hat{\mathcal{F}}$ with $(\Delta, \epsilon, 2\epsilon^2\Delta)$-timed adatpive UC-security.*

For the proof of the above theorem we need to show that Lemma 5 holds the timing model and also adapt the definition of a puzzle to handle entities with clock tapes. To achieve the first task we require that the puzzle environment is δ-delaying and soundness and simulatability hold with respect to ϵ-drift preserving adversaries. Thus we obtain the following claim for the lemma:

Lemma 7 (**Adaptive-Puzzle-Lemma in the Timing Model**). *Let $\epsilon > 1$ and $\Delta > 0$ be constants. Let Π' be a $\epsilon^2\Delta$-delayed protocol in the $\mathcal{F}_{\mathsf{mcom}}$-*hybrid *model. Assume the existence of a $(\mathcal{C}_{\mathsf{env}}, \mathcal{C}_{\mathsf{sim}})$-secure t_P-round adaptive puzzle $\langle \mathsf{S}, \mathsf{R} \rangle$ in a $\mathcal{G}$-hybrid model, $\epsilon^2\Delta$-delayed t_C-round stand-alone EQNMCOM $\langle \mathsf{S}_{\mathsf{com}}, \mathsf{R}_{\mathsf{com}} \rangle$ secure w.r.t $cl(\mathcal{C}_{\mathsf{sim}}, \mathcal{C}_{\mathsf{env}})$ and simulatable PKE scheme secure w.r.t $\mathcal{C}_{\mathsf{sim}}$. Then, there exists a $O(t_P + t_C)$-round protocol Π in the $\mathcal{G}$-hybrid such that, for every uniform $\mathcal{PPT}$ adversary $\mathcal{A}$ that is ϵ-drift preserving, there exists a simulator $\mathcal{A}' \in \mathcal{C}_{\mathsf{sim}}$, such that, for every $\epsilon^2\Delta$-delaying environment $\mathcal{Z} \in \mathcal{C}_{\mathsf{env}}$, the following two ensembles are indistinguishable over N w.r.t $\mathcal{C}_{\mathsf{sim}}$.*

- $\left\{\mathsf{Exec}^{\mathcal{G}}_{\Pi,\mathcal{A},\mathcal{Z}}(\mathsf{n})\right\}_{\mathsf{n}\in\mathbb{N}}$
- $\left\{\mathsf{Exec}^{\mathcal{F}_{\mathsf{mcom}}}_{\Pi',\mathcal{A}',\mathcal{Z}}(\mathsf{n})\right\}_{\mathsf{n}\in\mathbb{N}}$

We adapt the proof of Lemma 5 to the timing model. There we considered a sequence of hybrid experiments starting with the execution of the adversary $\mathcal{A}$ in the real world to the execution with the simulator $\mathcal{A}'$ in the $\mathcal{F}_{\mathsf{com}}$-hybrid world. We constructed non-abusing adversaries in each of the hybrids and showed that the executions in the hybrids are indistinguishable for the environment $\mathcal{Z}$. The first step was to construct an adversary relying on the simulatability of the puzzles. In hybrid H_1 we construct an adversary $\mathcal{A}'$ that incorporates $\mathcal{A}$ and simulates all puzzles interactions. In order to show that hybrid H_0 (the real world) and hybrid H_1 are indistinguishable we constructed an adversary $\mathcal{A}_{puz}$ in a concurrent puzzle execution, which incorporates $\mathcal{A}$ and emulates the interaction of $\mathcal{A}$'s environment. Thus the indistinguishability of H_0 and H_1 is reduced to indistinguishability of $\mathcal{Z}_{puz}$ in concurrent puzzle execution with $\mathcal{A}_{puz}$ and its simulator $\mathcal{A}'_{puz}$. To ensure that this holds in the timing model we require that (1) $\mathcal{Z}_{puz}$ is $\epsilon^2\delta$-delaying environment and (2) the internal emulation of the execution by $\mathcal{A}_{puz}$ is identical to H_0. The first conditions holds since $\mathcal{Z}_{puz}$ incorporates $\mathcal{Z}$ and the honest parties and emulates only the interactions of these parties that are not part of the puzzle-interactions. Thus all messages sent from $\mathcal{Z}$ or the honest parties to the adversary that are forwarded from $\mathcal{Z}_{puz}$ to $\mathcal{A}_{puz}$ are $\epsilon^2\delta$-delayed since $\mathcal{Z}$ is $\epsilon^2\delta$-delaying, and messages from the honest parties which are not part of the puzzles interactions are $\epsilon^2\delta$-delayed.

In order to satisfy the second condition we have to account for the special messages $(time, *, *)$ and $(reset - time, *, *)$ that the adversary $\mathcal{A}$ can send to alter the parties' clock-tapes. We introduce two modifications of $\mathcal{A}_{puz}$ and $\mathcal{Z}_{puz}$ to achieve this. First, we require that $\mathcal{A}_{puz}$ forward all special messages from $\mathcal{A}$ to $\mathcal{Z}_{puz}$ and also adjust appropriately the local clock-tapes of the parties in the internal emulation. Since $\mathcal{A}_{puz}$ forwards to the external receiver the messages between $\mathcal{A}$ and the honest parties where $\mathcal{A}$ acts as a sender, we need to synchronize the clocks of those external receivers for the puzzle interactions. For this we require that $\mathcal{Z}_{puz}$ forward the appropriate message for the clock-tapes to the external receives. The above modifications of $\mathcal{A}_{puz}$ and $\mathcal{Z}_{puz}$ suffice for the proof of the non-abusing property as well (the only difference in the puzzle environment is the final output). The rest of the hybrids in the proof of the lemma are the same as before since they use the simulated puzzles and rely only on the EQNMCOM properties.

We turn towards constructing an adaptive puzzle in the timing model. Define the puzzle $\mathcal{P}_{tim}(\langle S, R\rangle, \mathcal{R})$ as follows (see Figure 7.7).

Soundness: The soundness of the puzzle follows directly from the one-wayness of f and the witness-hiding property of the protocols.

Adaptive Simulatability: To simulate a concurrent puzzle-execution with $\mathcal{A}$ and its environment $\mathcal{Z}$, $\mathcal{A}'$, as before, internally emulates an execution with $\mathcal{A}$ while playing the role of the honest receiver. Upon adaptive corruption, $\mathcal{A}'$ simply reveals the inputs and randomness used while running the code of the honest receiver during puzzle interactions (note that the inputs and randomness used in puzzle interactions are independent of the inputs of the honest receiver to the commitment functionality). To extract the witness in a puzzle

Protocol $\langle S, R\rangle$:

$S \to R$: Pick $x \leftarrow \{0,1\}^n$ and send $y = f(x)$ to R.

$S \leftrightarrow R$: a witness-hiding special-sound argument of knowledge of the statement that there exists x' such that $y = f(x')$. R issues a $\mathsf{time} - \mathsf{out}$ if more than $2\epsilon\Delta$ local time units elapsed since the challenge in the WHPOK was issued and the response was received from S.

Relation: $\mathcal{R} = \{(x, y)|y = f(x)\}$

Figure 12: Timing Model Puzzle

challenged by $\mathcal{A}$, $\mathcal{A}'$ essentially rewinds $\mathcal{A}$ in the witness-hiding proof-of-knowledge sub-protocol to obtain another accepting transcript. Using the special-sound property of the proof-of-knowledge protocol, the adversary $\mathcal{A}'$ can then extract the witness used in the proof and outputs that as the witness for the puzzle transcript.

More formally, whenever $\mathcal{A}$ completes a puzzle-interaction with a receiver, $\mathcal{A}'$ temporarily stalls the emulation and rewinds $\mathcal{A}$ to the state where it receives a challenge in the WHPOK sub-protocol. It feeds a new challenge and continues the emulation to obtain a response. While performing emulation from a given challenge, $\mathcal{A}$ expects to exchange messages with $\mathcal{Z}$ and other receivers. Since, the receivers are internally emulated, messages exchanged between $\mathcal{A}$ and the receivers can be emulated internally. Messages exchanged with $\mathcal{Z}$ are delicate, since we cannot rewind the external $\mathcal{Z}$. Note, however, that in a rewinding, $\mathcal{A}$ receives two kinds of messages from $\mathcal{Z}$: (1) messages that were sent before the new challenge was fed to $\mathcal{A}$ in a rewinding, and (2) messages that were sent after. The former messages were received by $\mathcal{A}$ in the main execution can be replayed by $\mathcal{A}'$ to $\mathcal{A}$. For the latter kind of messages, we claim that $\mathcal{A}'$ does not have to emulate them. As $\mathcal{A}$ is ϵ-drift-preserving, the receivers clock-tape advances at least $2\epsilon\Delta\frac{1}{\epsilon}$ time units before the puzzle-environment's clock-tape advances $2\epsilon\Delta$ time units. Since, the receiver issues a time-out when its clock-tape advances $2\epsilon\Delta$ steps since it sent the challenge, $\mathcal{A}$ needs to respond to the challenge before the message from $\mathcal{Z}$ reaches $\mathcal{A}$. Finally, messages to $\mathcal{Z}$ from $\mathcal{A}$ in a rewinding are ignored by $\mathcal{A}'$. Finally we need to argue that $\mathcal{A}'$ runs in polynomial time. Let $q(n)$ be the expected time that $\mathcal{A}'$ spends to extract the witness. Let p be the probability that the receiver is not corrupted during the rewinding and $\mathcal{A}$ responds to a challenge in the WHPOK of the puzzle before the receiver times out. Then the expected number of times that $\mathcal{A}'$ has to rewind before $\mathcal{A}$ responds to the challenge before the receiver times out (conditioned that the receiver is not corrupted) is $\frac{1}{p}$. Therefore, the total time spent is $p \cdot \frac{1}{p} \cdot q(n)$, which is polynomial.

7.8 Adaptive UC in the Tamper-Proof Hardware Model

The tamper-proof hardware model introduces a physical assumption that enables protocols to be executed in an isolated environment. This assumption is instantiated through the existence of tamper-proof hardware tokens, which allows a party P_i to create a hardware token that implements a functionality F and give this token to any party P_j. Now the party P_j can interact with the token and access the embedded functionality in a black-box manner. The tamper-proof property means that an adversary that has a token can do nothing more than observe the input and output from the interaction with it, i.e. he cannot alter in anyway the functionality that the token implements. The notion of a tamper-proof hardware token in formalized though the ideal functionality $\mathcal{F}_{wrap}$ in Figure 7.8 introduced by Katz [21].

The following theorem states our result in the tamper-proof model.

THEOREM 11. *Assume the existence of simulateable PKE and a $O(t)$-round* $\mathsf{EQNMCom}$ *scheme. Then, for*

Functionality $\mathcal{F}_{wrap}$.

Let p be a polynomial and n be a security parameter for $\mathcal{F}_{wrap}$.

Create: On input $(create, sid, P_i, P_j, M)$ from P_i, where P_j is another user of the system and M is an interactive Turing machine, do:

1. Send $(create, sid, P_i, P_j, M)$ to P_j.
2. If there is no tuple of the form $(P_i, P_j, *, *, *)$ stored, then store $(P_i, P_j, M, 0,)$.

Execute: On input (run, sid, P, msg) from P', find the unique stored tuple $(P, P', M, i, state)$ (if no such tuple exists, then do nothing). Then do:

Case 1 $(i = 0)$: Choose random $w \leftarrow \{0,1\}^{p(k)}$. Run $M(msg; w)$ for at most $p(k)$ steps, and let out be the response (set $out = \perp$ if M does not respond in the allotted time). Send (sid, P, out) to P'. Store $(P, P', M, 1, (msg, w))$ and erase $(P, P', M, i, state)$.

Case 1 $(i = 1)$: Parse state as (msg_1, w). Run $M(msg_1 || msg; w)$ for at most $p(k)$ steps, and let out be the response (set $out = \perp$ if M does not respond in the allotted time). Send (sid, P, out) to P'. Store $(P, P', M, 0,)$ and erase $(P, P', M, i, state)$.

Figure 13: $\mathcal{F}_{wrap}$

every well-formed ideal functionality $\mathcal{F}$, there exists an $O(t)$-round protocol π that realizes $\hat{\mathcal{F}}$ with adaptive UC-security in the $\mathcal{F}_{wrap}$-hybrid model.

In order to prove the theorem it suffices to construct an adaptive UC puzzle in the $\mathcal{F}_{wrap}$-hybrid model. Unlike the other puzzles this will be a ”stateful” puzzle in the sense that a party is required to spawn a subroutine of S at the beginning of the execution and use this subroutine to generate any consecutive puzzle. This routine can keep state across multiple executions and thus the generated puzzle instances are not independent. Figure 7.8 presents the resulting puzzle in the tamper-proof model.

We argue the soundness and simulatability properties of the puzzle in Figure 7.8 as follows:

Soundness: It follows from the one-wayness the function f and the witness-hiding property of the protocol.

Simulation: To simulated concurrent puzzle execution with the adversary $\mathcal{A}$ and the environment $\mathcal{Z}$, $\mathcal{A}'$ emulates internally an execution with $\mathcal{A}$ where it acts as $\mathcal{F}_{wrap}$. $\mathcal{A}'$ obtains the message $(create, sid, P_i, P_j, M^*)$ sent by $\mathcal{A}$. Later in a challenge protocol by $\mathcal{A}$ to P_j, $\mathcal{A}'$ extract the witness to a puzzle y by rewinding M^* in the witness-hiding argument-of-knowledge sub-protocol. Since M^* does not receive messages from any other parties other than P_j during the execution (and the rewinding), the extraction can finish in isolation without intervening the adversary $\mathcal{A}$ and the environment $\mathcal{Z}$. If party P_j is corrupted during rewinding, $\mathcal{A}'$ does not have to execute the simulation.

7.9 Adaptive, Partially Isolated Adversaries Model

In this section, we consider a model that incorporates the physical assumption that protocols can be run in a (partially) isolated environment. In particular, we assume that a player P_j can ensure that another player P_i

Protocol $\langle S, R \rangle$:

S proceeds in two phases:

- When it is first spawned and invoked on inputs the identity of the sender P_i and the session id sid, it uniformly picks a string $x \in \{0,1\}^n$, computes its image y through the one-way function f, and stores (y, P_i, sid) as an internal state.
- Later when S is invoked on inputs the identity of the puzzle receiver P_j to challenge P_j, S checks whether this is the first time interacting with party P_j, if so, it ”creates” and ”gives” P_j a token, which encapsulates the functionality M that gives a witness-hiding argument-of-knowledge of the statement that y is in the image set of f, by sending the message $(create, sid, P_i, P_j, M)$ to $\mathcal{F}_{wrap}$. To actually challenge P_j, S simply sends y as the puzzle to the receiver.

Upon receiving y from the sender, R accesses M via $\mathcal{F}_{wrap}$ as follows: it sends (run, sid, S, ϵ) to $\mathcal{F}_{wrap}$ (ϵ is an empty string), and then receives from M a WHAOK of the statement that y is in the image set of f (forwarded by $\mathcal{F}_{wrap}$).

Relation: $\mathcal{R} = \{(x,y) | y = f(x)\}$

Figure 14: Tamper-Proof Model Puzzle

is *partially isolated* for a short portion of the computation. During this time, P_i can only exchange a limited number of bits with the environment but P_j's communication is unrestricted. More specifically, we assume the existence of some threshold ℓ, such that P_j can prevent P_i from exchanging more than ℓ bits with the environment.

The partially isolated adversaries model was introduced by [13, 14], and formalized as the isolate ideal functionality $\mathcal{F}_{\mathsf{isolate}}$. We recall the formal description of $\mathcal{F}_{\mathsf{isolate}}$ as in [14] in Figure 7.9.

We obtain an analogue of the result of [14], using our puzzle framework:

THEOREM 12. *Assume the existence of simulatable PKE scheme, and the existence of an $O(t)$-round* $\mathsf{EQNMCom}$ *scheme. Then, for every well-formed ideal functionality $\mathcal{F}$, there exists an $O(t)$-round protocol π that realizes $\hat{\mathcal{F}}$ in the Adaptive, Partially Isolated Adversaries model.*

To prove the theorem, it suffices to construct a puzzle in the $\mathcal{F}_{\mathsf{isolate}}$-hybrid model. In all the previous models, the puzzle protocols $\langle \mathsf{S}, \mathsf{R} \rangle$ are executed in a ”stateless” way, that is, whenever a party intends to challenge (acting as the sender of the puzzle) another, it spawns *independently* a new subroutine of S to generate the puzzle. In this model, we consider a ”stateful” puzzle, which requires a party to spawn a subroutine of S at the beginning of its execution, and use this subroutine to generate all the puzzles it needs throughout its lifetime. (Note that the receiver part of the puzzle protocol is still ”stateless”.) It is stateful in the sense that the subroutine can keep states across multiple invocations, and hence the puzzle instances generated are not independent to each other, but correlated. More precisely, we define the puzzle $\mathsf{P}_{\mathsf{isolate}} = (\langle \mathsf{S}, \mathsf{R} \rangle, \mathcal{R})$ for the $\mathcal{F}_{\mathsf{isolate}}$-hybrid model as follows. The interactive Turing machine S, proceeds in two phases:

- When it is first spawned and invoked on inputs the identity of the sender P_i and the session id sid–called the initialization phase–it uniformly picks a string $x \in \{0,1\}^n$, computes its image y through the one-way function f, and stores (y, P_i, sid) as an internal state.

- Let Π be an ℓ-Isolated Proof of Knowledge Protocol as defined by [13], where parties P_i, P_j interact

The $\mathcal{F}_{\mathsf{isolate}}$ ideal functionality is parameterized by an isolation paramter ℓ, a security paramter κ and a polynomial p.

Isolation of P_i: Wait until receiving messages $(\mathsf{isolate}, sid, P_i, P_j)$ from P_j and $(\mathsf{isolate}, sid, P_i, P_j, M)$ from P_i. If there is already a stored tuple of the form $(P_i, P_j, \cdot, \cdot, \cdot, \cdot)$ then ignore the command. Otherwise:

1. Parse the string M as the description of an ITM with four communication tapes; two tapes (”in” and ”out”) for regular protocol communication with P_j and two tapes for secret communication with P_i. Let the value state encode the initial state of M (including the value of a work tape and an initialized random tape). Define new values $\mathsf{inCom} = 0, \mathsf{outCom} = 0$ and store the tuple $(P_i, P_j, M, \mathsf{state}, \mathsf{inCom}, \mathsf{outCom})$.
2. Send $(\mathsf{isolate}, sid, P_i)$ to P_j.

Interaction with P_j: On input $(\mathsf{run}, sid, P_i, P_j, \mathsf{msg})$ from P_j, retrieve the tuple $(P_i, P_j, M, \mathsf{state}, \mathsf{inCom}, \mathsf{outCom})$. If there is no such tuple then ignore the command.

1. Place the string msg on the ”in” tape designated for P_i and run M for $p(\kappa)$ steps.
2. If there is any value msg' on the output tape for P_j then send the message $(\mathsf{reply}, sid, P_i, \mathsf{msg}')$ to P_j.
3. If there is any value msg' on the output tape for P_i and $\mathsf{outCom} + |\mathsf{msg}'| < \ell$ then send the message $(\mathsf{secretCom}, sid, P_j, P_i, \mathsf{msg}')$ to P_i and update $\mathsf{outCom} = \mathsf{outCom} + |\mathsf{msg}'|$.
4. Update the value of state in the stored tuple to encode the updated state of M and the values of its tapes.

Communication: On input $(\mathsf{secretCom}, sid, P_i, P_j, \mathsf{msg})$ from P_i, if there is no tuple of the form $(P_i, P_j, M, \mathsf{state}, \mathsf{inCom}, \mathsf{outCom})$ then ignore. Also if the tuple has $\mathsf{inCom} + |\mathsf{msg}| > \ell$ then ignore the command. Otherwise:

1. Update $\mathsf{inCom} = \mathsf{inCom} + |\mathsf{msg}|$, place msg on the ”in” tape for P_i and run M for $p(\kappa)$ steps.
2. Proceed with steps $2, 3, 4$ of the above command.

Release of P_i: On input $(\mathsf{release}, sid, P_i, P_j)$ from P_j, retrieve the tuple $(P_i, P_j, M, \mathsf{state}, \mathsf{inCom}, \mathsf{outCom})$ and send $(\mathsf{release}, sid, P_i, P_j, \mathsf{state})$ to P_i.

Figure 15: The $\mathcal{F}_{\mathsf{isolate}}$ Ideal Functionality

and P_i proves that it knows a witness w to an NP-statement z. We note that by definition, such a protocol is standalone zero-knowledge and hence, is also *witness-hiding*.

P_j, playing the part of receiver, initializes a puzzle interaction with S by sending the message $(\mathsf{isolate}, sid, P_i, P_j)$ to the Ideal Functionality. S replies with the message $(\mathsf{isolate}, sid, P_i, P_j, M)$, where M is a description of an ITM playing the part of the Prover in protocol Π, interacting via protocol communication with verifier P_j and via secret communication with P_i. The NP-statement being proved is simply that y is in the range of f, and by the end of the protocol, P_j should be convinced that P_i knows x such that $f(x) = y$.

S and P_j interact with M via the Ideal Functionality messages run and $\mathsf{secretCom}$. When the protocol completes, P_j sends a message $(\mathsf{release}, sid, P_i, P_j)$ to the Ideal Functionality.

To actually challenge P_j, S simply sends y as the puzzle to the receiver. The puzzle relation $\mathcal{R}$ is simply $\{(x, y) \mid y = f(x)\}$.

The soundness of the puzzle follows directly from the one-wayness of the function f and the witness-hiding property of the protocol. Furthermore, to adaptively simulate a concurrent puzzle interaction with adversary $\mathcal{A}$ and environment $\mathcal{Z}$, $\mathcal{A}'$ internally emulates an execution with $\mathcal{A}$ and acts as the $\mathcal{F}_{\mathsf{isolate}}$ functionality for $\mathcal{A}$. Whenever $\mathcal{A}$ sends a message $(\mathsf{isolate}, sid, P_i, P_j, M)$ to $\mathcal{F}_{\mathsf{isolate}}$, $\mathcal{A}'$ obtains the message. Later to extract the witness of a puzzle y challenged by $\mathcal{A}$ (controlling P_i) to P_j, $\mathcal{A}'$ simply runs the knowledge extractor of the ℓ-Isolated Proof of Knowledge to extract the witness. Using the [13] construction of ℓ-Isolated Proofs of Knowledge, we have that the simulation of $\mathcal{A}'$ is perfect; addtionally, we note that since the [13] verifier is public-coin, dealing with adaptive corruptions is trivial. Thus, we achieve perfect, adaptive simulation.

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A Constructing Non-Interactive, Language-Based, Equivocal Commitments

Let Com be a non-interactive commitment scheme with a pseudorandom range. Such a commitment scheme can be constructed from OWF.

Let L be an NP-Language and $\mathcal{R}$, the associated NP-relation. Since the language $\mathcal{L} \in \mathsf{NP}$, we can reduce $\mathcal{L}$ to the NP-complete problem Hamiltonian Cycle. Thus, given the public input x (which may or may not be in $\mathcal{L}$), we can use a (deterministic) Karp reduction to a graph G which contains a Hamiltonian cycle. Moreover, finding a Hamiltonian cycle H in the graph G, implies finding a trapdoor w such that $\mathcal{R}(x, w) = 1$. Let Φ denote the deterministic mapping from strings x to a graphs G induced by the Karp reduction.

The protocol is specified in Figures 16, 17 and has appeared before in [7]. For completeness, we present it again here and show that it satisfies the properties of an equivocal commitment scheme as specified in Definition 2

We omit the security analysis of the non-interactive, language-based equivocal commitment scheme $\langle \mathsf{S}_{\mathsf{eq}}, \mathsf{R}_{\mathsf{eq}} \rangle$ presented in Figures 16 and 17, since it is standard.

B Constructing Adaptively-secure WIPOK

The adaptively-secure (without erasures) WIPOK construction given here is similar to the one given in [25]. As in [25], it is based on Blum's Σ-protocol for graph Hamiltonicity [3]. Let Com be any commitment scheme. The Σ-protocol proceeds as follows (see figure 18):

We construct adaptively-secure WIPOK by replacing each commitment Com in the Σ-protocol with a non-interactive equivocal commitment $Com^*(\pi(G')_{i,j})$, as constructed above.

Lemma 8. *When commitments Com are replaced with equivocal commitments Com^* generated by running the protocol $\langle \mathsf{S}_{\mathsf{eq}}, \mathsf{R}_{\mathsf{eq}} \rangle$ presented in Figures 16 and 17 then we have that the protocol in Figure 18 is a WIPOK (with soundness $1/2$) and is secure under adaptive corruptions.*

sketch. The analysis of the soundness of the protocol follows from the analysis of the underlying Σ-protocol, which we omit since it is by now a standard argument.

Next we need to prove the witness-indistinguishability and proof of knowledge properties as well as the fact that the protocol is secure under adaptive corruptions. In fact, we show that the above construction is not only a WIPOK, but is a Zero Knowledge Proof of Knowledge. We now present a simulator which satisfies the zero-knowledge property and can also handle *adaptive* corruptions (for simplicity, we consider here only post-execution corruptions). This implies that the scheme above is zero-knowledge as well as secure under *adaptive corruptions*.

On input graph G', the Simulator does the following:

Simulation of Prover's first message: Use the simulator for $\langle \mathsf{S}_{\mathsf{eq}}, \mathsf{R}_{\mathsf{eq}} \rangle$ to compute a commitment for each position in an $n \times n$ matrix (each position in the matrix can now be opened to either 0 or 1).

$\langle \mathsf{S_{eq}}, \mathsf{R_{eq}} \rangle$ **on common input x and private input β: Commitment phase**

To commit to $\beta = 1$:

1. $\mathsf{S_{eq}}$ chooses an $n \times n$ adjacency matrix H of a random n-node Hamiltonian cycle.
2. $\mathsf{S_{eq}}$ sends a matrix $\overline{\mathsf{Com}}$ of $n \times n$ strings where the following holds:
   - $\overline{\mathsf{Com}}_{i,j}$ contains a random commitment to 1 under Com iff $H_{i,j} = 1$.
   - $\overline{\mathsf{Com}}_{i,j}$ contains a random string iff $H_{i,j} = 0$.

To commit to $\beta = 0$:

1. $\mathsf{S_{eq}}$ chooses an $n \times n$ adjacency matrix I which corresponds to a random isomorphism of $G = \Phi(x)$.
2. $\mathsf{S_{eq}}$ sends a matrix $\overline{\mathsf{Com}}$ of $n \times n$ strings where the following holds:
   - $\overline{\mathsf{Com}}_{i,j}$ contains a random commitment to 1 under Com iff $I_{i,j} = 1$.
   - $\overline{\mathsf{Com}}_{i,j}$ contains a random commitment to 0 under Com iff $I_{i,j} = 0$.

Let $C = \mathsf{EQCom}^x(\beta; r)$ denote the transcript of the commit phase when $\mathsf{S_{eq}}$ uses randomness r.

$\langle \mathsf{S_{eq}}, \mathsf{R_{eq}} \rangle$ **on common input x: Decommitment phase**

To decommit to a 0:

1. $\mathsf{S_{eq}}$ opens the commitments in $\overline{\mathsf{Com}}$ where $\overline{\mathsf{Com}}_{i,j}$ is a commitment to 1 and shows that these correspond to a random Hamiltonian cycle.
2. $\mathsf{S_{eq}}$ produces the randomness it used to sample the remaining random strings in the matrix $\overline{\mathsf{Com}}$.

To decommit to a 1:

1. $\mathsf{S_{eq}}$ opens the commitments in $\overline{\mathsf{Com}}$ to obtain adjacency matrix I and shows an isomorphism from $G = \Phi(x)$ to this graph.

Figure 16: Non-interactive, language-based equivocal commitment scheme $\langle \mathsf{S_{eq}}, \mathsf{R_{eq}} \rangle$

Simulation of Prover's second message

- If $b = 0$, choose a random permutation π and equivocally open the commitments of the $n \times n$ matrix to be consistent with $\pi(G')$.
- If $b = 1$, choose a random cycle C and equivocally open the commitments that correspond to the Hamiltonian cycle to be consistent with the cycle.

Upon post-execution corruption of Prover: Upon corruption, the simulator learns the witness, the cycle H of graph G'.

- If $b = 0$, all the commitments have already been opened, the permutation π has been revealed and there is no additional information revealed to the adversary upon corruption.
- If $b = 1$, find some permutation π' of the vertices of G' such that $\pi'(H) = C$. Note that since

$\langle \tilde{\mathsf{S}}_{\mathsf{eq}}, R_{\mathsf{eq}} \rangle$ **on common input** $x \in L$ **and private input** w **where** $w \in \mathcal{R}(x)$**: Equivocal Commitment**

1. $\tilde{\mathsf{S}}_{\mathsf{eq}}$ chooses an $n \times n$ adjacency matrix I which corresponds to a random isomorphism of $G = \Phi(x)$.
2. $\tilde{\mathsf{S}}_{\mathsf{eq}}$ sends a matrix $\overline{\mathsf{Com}}$ of $n \times n$ strings where the following holds:
   - $\overline{\mathsf{Com}}_{i,j}$ contains a random commitment to 1 under Com iff $I_{i,j} = 1$.
   - $\overline{\mathsf{Com}}_{i,j}$ contains a random commitment to 0 under Com iff $I_{i,j} = 0$.

Let $C = \mathsf{EQCom}^{*x}(r)$ denote the transcript of the commit phase when $\tilde{\mathsf{S}}_{\mathsf{eq}}$ uses randomness r.

$\mathsf{Adap}_{\mathsf{eq}}(x, w, r, \tau, v)$**, where** τ **is the transcript generated by** $\langle \tilde{\mathsf{S}}_{\mathsf{eq}}, R_{\mathsf{eq}} \rangle$ **on common input** $x \in L$**: Equivocal Decommitment**

$\mathsf{Adap}_{\mathsf{eq}}$ **decommits to** $v = 0$ **as follows:**

1. $\mathsf{Adap}_{\mathsf{eq}}$ opens all the commitments in $\overline{\mathsf{Com}}$ to reveal adjacency matrix I and shows an isomorphism from $G = \Phi(x)$ to this graph.

$\mathsf{Adap}_{\mathsf{eq}}$ **decommits to** $v = 1$ **as follows:**

1. $\mathsf{Adap}_{\mathsf{eq}}$ uses w to open the commitments in $\overline{\mathsf{Com}}$ that correspond to the Hamiltonian cycle in $G = \Phi(x)$ and shows that these correspond to a random Hamiltonian cycle.
2. $\mathsf{Adap}_{\mathsf{eq}}$ produces random coins for sampling the remaining strings in $\overline{\mathsf{Com}}$ at random.

Figure 17: Non-interactive, language-based equivocal commitment scheme–Equivocator $(\tilde{\mathsf{S}}_{\mathsf{eq}}, \mathsf{Adap}_{\mathsf{eq}})$

H and C are simply n-node cycles, finding such a π' takes linear time. Equivocally open the commitments of the remaining entries of the $n \times n$ matrix to be consistent with $\pi'(G')$.

We omit the analysis of the above simulator. It is straightforward to check that the simulator simultaneously satisfies the zero-knowledge property and also simulates adaptive corruptions successfully.

We additionally omit the proof that the protocol is a proof of knowledge, which is also straightforward.

□

Σ Protocol

Prover's input: Graph G' (we also use the notation G' to represent the adjacency matrix of G') with Hamiltonian cycle H.

Prover's first message:

- Choose a permutation π of the vertices of G'.
- Commit to the adjacency matrix of $\pi(G')$ by sending $[Com(\pi(G')_{i,j})]_{1\leq i\leq n, 1\leq j\leq n}$ to the Verifier.

Verifier's message: Verifier chooses $b \in \{0,1\}$ at random and sends to Prover.

Prover's second message:

- If $b = 0$, reveal π and open the commitments of the entire adjacency matrix.
- If $b = 1$, reveal only the cycle $\pi(H)$ in $\pi(G')$ by opening the commitments that correspond to the Hamiltonian cycle.

Verifier checks the following:

- If $b = 0$, do the following: Given π, check that the opened adjacency matrix is equal to $\pi(G')$. Check that each of the commitments was opened correctly.
- If $b = 1$, check that the opened commitments correspond to a Hamiltonian cycle. Check that each of the commitments was opened correctly.

Figure 18: Σ Protocol