

# “Oops, I did it again” – Security of One-Time Signatures under Two-Message Attacks

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**Abstract.** One-time signatures (OTS) are called one-time, because the accompanying reductions only guarantee security under single-message attacks. However, this does not imply that efficient attacks are possible under two-message attacks. Especially in the context of hash-based OTS (which are basic building blocks of recent standardization proposals) this leads to the question if accidental reuse of a one-time key pair leads to immediate loss of security or to graceful degradation.

In this work we analyze the security of the most prominent hash-based OTS, Lamport’s scheme, its optimized variant, and WOTS, under different kinds of two-message attacks. Interestingly, it turns out that the schemes are still secure under two message attacks, asymptotically. However, for typical parameters this does not mean anything. Our results show that for Lamport’s scheme, security only slowly degrades in the relevant attack scenarios and typical parameters are still somewhat secure, even in case of a two-message attack. As we move on to optimized Lamport and its generalization WOTS, security degrades faster and faster, and typical parameters do not provide a reasonable level security under two-message attacks.

Besides this destructive context, we also analyze two-message attacks in a constructive context that appears for example when OTS are used to prevent double-spending in bitcoin.

**Keywords:** Hash-based signatures, one-time signatures, few-time signatures, post-quantum cryptography, two-message attacks.

## 1 Introduction

The possible advent of large-scale quantum computers threatens the security of all widely deployed public key cryptography. Shor’s algorithm [20] allows to factor and compute discrete logarithms in polynomial time on a quantum computer with a few thousand logical qubits. While it is not yet known for sure if it will be possible to build such a machine, it is a question of risk assessment to be prepared. The implied disastrous consequences by now also motivated standardization bodies (see e.g. [16]) and security agencies [17] to prepare the transition to post-quantum cryptography – cryptography, secure against attacks using quantum-computers.

The first post-quantum signature schemes considered for standardization are hash-based Merkle Signature Schemes [13,9]. These schemes form the most confidence-inspiring post-quantum solution for digital signatures as their security only relies on some mild assumptions about properties of cryptographic hash-functions [11]. This is in contrast to

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This work was supported by the Commission of the European Communities through the Horizon 2020 program under project number 645622 PQCRYPTO. Permanent ID of this document: 85629c7dc69dad1c4be4fbd7e360086c. Date: November 4, 2016

all other proposals where security in addition to assumptions about the used hash function is based on rather new intractability assumptions like the  $\mathcal{MQ}$ -problem (see e.g. [18]) or the approximate shortest vector problem [6]. Hash-based signature schemes can be split into stateful [15,5,4,3,10,11] and stateless [1] proposals. In this context, statefulness means that the secret key changes after every signature. In case a 'secret key state' is used twice, all security guarantees vanish. In practice it turns out that in many scenarios keeping a state becomes a complicated issue [14]. However, currently stateful schemes are the ones considered for standardization as these schemes are far more efficient in terms of signature size and signing speed than the stateless alternatives.

The reason these schemes are stateful is that the core building block of these hash-based signature schemes is a so-called one-time signature scheme (OTS). A one-time signature scheme allows to use a key pair to sign a single (arbitrary) message. If a key pair is used to sign a second, different message no security guarantees are given. The security reductions only apply as long as just a single message is signed. While this is commonly interpreted as the schemes are entirely broken if a key pair is used to sign twice, this is not necessarily the case. It is known that if an adversary has full control about the messages to be signed, the schemes are fully broken after two signatures, i.e. the secret key can be extracted without any effort. However, in practice the OTS causing statefulness are used to sign the digest of the adversarial chosen message. Moreover, in both recent proposals for standardization [13,9] these message digests are randomized. Hence, the actually signed message (digest) is unpredictable for an adversary.

Taking the message digest into account is one of the crucial steps in the construction of hash-based few-time signature schemes like HORS [19] that allow to use a key pair to sign a small number of messages before security drops below the acceptable limit. This opens up the question if classical hash-based OTS are still one-time when we take the message digest into account or if a similar argument applies as for HORS. For practice, this question translates to the question if reuse of a secret key state leads to a hard fail or if one is "only" facing graceful degradation of security.

**Our contribution.** In this work we analyze the security of hash-based one-time signature schemes under different kinds of two-message-attacks. We carry out the analysis for the most prominent proposals Lamport's scheme [12], the optimized version of Lamport's scheme [15], and the Winternitz OTS (WOTS) [15]. It turns out that actually, all three schemes are still secure under two-message attacks if we take into account that a message digest is signed, at least asymptotically (see Table 1).

The general working of these schemes is as follows. If necessary, a message  $M$  is first compressed using a cryptographic hash function  $H$  to obtain a fixed length message digest  $M^* = H(M)$ . A mapping function  $G$  is used to map  $M^*$  to some index set  $B = (B_1, \dots, B_\ell) = G(M^*)$ . Finally, secret values indicated by the index set  $B$  are published as signature. Generally, the secret values are the preimages of public key values under a cryptographic hash function  $F$ . Verification works by applying  $F$  to the given values and comparing the results to the respective public key values. In case of WOTS secrets are arranged in hash chains. The end nodes of the chains are the public key values. In this case, there exists some dependency, i.e., if a value from a chain is part of the signature, all later values of that chain can be derived applying  $F$ .

After seeing two signatures, there exist two possible ways to forge a signature. First, an adversary can try to find a message that is mapped to an index set which is covered by the union of the index sets of the two seen signatures. In this case, all the required secret values are contained in the two signatures. Second, an adversary can try to compute the missing secret values for a signature from the respective public key values. However, this requires to break one of the security properties of  $F$  and would also allow to forge signatures after seeing just the public key. Parameters in practice are chosen such that this is infeasible. Consequently, we just consider the first approach in this work. The

**Table 1.** Complexity for an existential forgery under a random message attack for the given signature scheme with typical parameters (see text).

Signature scheme	Attack Complexity
Lamport	$\mathcal{O}((1.34)^m)$
Optimized Lamport	$\mathcal{O}((1.14)^{m+\log m})$
Winternitz	$\mathcal{O}((1.09)^{m+\log m})$

possibility and complexity of attacks of this type depends on the properties of hash function  $H$ , the message mapping function  $G$ , and possible dependencies of secret values (as in the case of WOTS). In our analysis we focus on the latter two. For  $H$  we assume that it behaves like a random oracle. This decision follows the same reasoning as above. Vulnerabilities of  $H$  would already allow for forgeries under one-message attacks. For WOTS this implies that the obtained results also apply to the recent variants of WOTS that minimize security assumptions [2,8,11] as the mapping function and the arrangement of secret values for these variants is the same as in the original scheme.

For Lamport’s scheme, we obtain exact complexities for two-message attacks. For the optimized Lamport scheme and WOTS analysis becomes extremely complex when looking at the actual mapping function. This is caused by a checksum which is added to the message. This checksum introduces a lot of dependencies between probabilities, eventually leading to sums with an exponential number of summands. Therefore, we decided to analyze a simplified variant where we assume that the checksums are independent and uniformly distributed. For this simplified message mapping, we obtain exact complexities. We experimentally verified the results obtained for the simplified mapping function.

We analyze security in terms of full break resistance, universal, selective, and existential unforgeability under random and adaptively chosen message attacks. The crucial case for practice is existential unforgeability under random message attacks (EU-RMA). It covers the case of accidental reuse of a OTS key pair when using one of the recent proposals to standardize hash-based signatures. The random message attack setting applies as the signer randomizes the message digest function which we model as random oracle in our analysis. While the all three schemes turn out to be secure under two-message attacks in the asymptotic setting, we get different results for typical parameter choices. For Lamport’s scheme with a message digest size of 256 bits, the complexity to produce existential forgeries under two-random-message attacks is still  $2^{106}$  hash function calls, ignoring the costs for pairwise comparison of all message digests. Hence, in this setting a signer is still on the safe side even after using a one-time key pair twice. For the optimized Lamport OTS with 256 bit message digests, the complexity to produce existential forgeries under two-random-message attacks is already down to  $2^{51}$ . Which means attacks are not for free, but they are possible. For WOTS in the same setting, using the parameters from [9], we are left with an attack complexity of  $2^{34}$  hash function computations. This can be done on a modern computer within few days if not hours. This includes a Winternitz parameter of  $w = 16$ , i.e. hash chains of length 16. For bigger values of  $w$ , the attack complexity goes down even further. These results show that Lamport’s scheme is still somewhat forgiving but especially for WOTS, measures have to be taken that prevent OTS key reuse in any case. However, as soon as we are considering attacks on quantum-computers, complexities go down to their square-roots. In this case even Lamport’s scheme has to be considered broken after two-random-message attacks.

**Constructive applications.** Forgeries under two-message attacks are not necessarily of destructive nature. They can actually be used to enable fast payments using decentralized crypto-currencies like bitcoin. The big problem with fast payments in this

setting is the issue of double-spending attacks. In this case, a buyer transfers the same money to two different entities at the same time. Due to the decentralized nature of the system, this can go unrecognized for some time. However, eventually only one of the two transfers will make it into the public record and one of the recipients will be left empty handed.

In these crypto-currencies, temporary accounts with a assigned balance are public keys for a signature scheme. Ownership consists of knowing the respective secret key. A transaction is basically a signature using the secret key of the current account on a document stating that some amount of the balance goes to an account with the public key of the recipient and the remaining amount (the change) goes to a new account of the owner with a new public key. A possible solution for the double spending issue is to use an OTS as signature scheme [21]. In this case, double-spending, which means signing twice, would allow a third party to change the transaction at their will. If one requires that the change in case of a fast transaction has to be above a certain threshold, this allows to use the change to pay both vendors and to use the remaining value as penalty payment.

This use case opens up a new question. Namely, what if the signer intentionally signs twice? Is it possible for the signer to increase the complexity of a two-message attack? We coin this setting *signer-chosen message attack* and model it as a game between a signer and an adversary where the signers goal is to maximize the costs for the adversary. We also analyze the three hash-based OTS in this setting, showing their applicability in this setting. However, our results also show that malicious signers must not be given the chance to carry out significant pre-computation as in this case, they can increase the costs for a forgery such that an attack becomes infeasible, even for WOTS.

**Organization.** In Section 2 we discuss the models we use as well as required notation. We start our analysis in Section 3 with Lamport’s scheme. We continue in Section 4 with the optimized Lamport scheme and in Section 5 with WOTS. In Section 6, we experimentally verify our results. Finally, we discuss signer chosen message attacks in Section 7.

**Acknowledgement.** This research was motivated in part by suggestions by Burt Kaliski of Verisign that double-signing forgeries against hash-based signatures could be used as a kind of ‘punitive malleability’ to deter double-spending. The authors would also like to thank Aziz Mohaisen for helpful discussions.

## 2 The model

Security of one-time signature schemes (OTS) can be analyzed with regard to all traditional security definitions for general signature schemes. The difference is that the number of adversarial signature queries is limited to  $q = 1$ . Formally, any signature scheme that achieves EU-CMA-security (see definition below) when the adversary may only make a single signature query is a OTS. To understand the security of a OTS under two-message attacks in any of the models, we simply investigate the security for  $q = 2$ . We first discuss the traditional definitions and afterwards we discuss how to analyze security within these models.

### 2.1 Digital signature schemes

First, what exactly are we talking about? From a formal perspective the objects we are talking about are digital signature schemes, defined as follows:

**Definition 1 (Digital Signature Scheme).** Let  $\mathcal{M}$  be the message space. A digital signature scheme  $\text{DSS} = (\text{kg}, \text{sign}, \text{vf})$  is a triple of probabilistic polynomial time algorithms:

- $\text{kg}(1^n)$  on input of a security parameter  $1^n$  outputs a private signing key  $\text{sk}$  and a public verification key  $\text{pk}$ ;
- $\text{sign}(\text{sk}, M)$  outputs a signature  $\sigma$  under  $\text{sk}$  for message  $M$ , if  $M \in \mathcal{M}$ ;
- $\text{vf}(\text{pk}, \sigma, M)$  outputs 1 iff  $\sigma$  is a valid signature on  $M$  under  $\text{pk}$ ;

such that the following correctness condition is fulfilled:

$$\forall(\text{pk}, \text{sk}) \leftarrow \text{kg}(1^n), \forall(M \in \mathcal{M}) : \text{vf}(\text{pk}, \text{sign}(\text{sk}, M), M) = 1.$$

Throughout this work *signature scheme* always refers to a digital signature scheme.

## 2.2 Security of signature schemes

The definition above is only a functional definition of the object at hand that says nothing about security. It leaves the question of how to define security for a signature scheme. In general we can split security notions into the goals an adversary  $\mathcal{A}$  has to achieve (e.g., a valid signature on any new message for existential unforgeability) and the attack capabilities given to  $\mathcal{A}$  (e.g., adaptively learning signatures on messages of its choice after seeing the public key). For the goals, the relevant notions<sup>1</sup> are:

**Full break (FB):**  $\mathcal{A}$  can compute the secret key.

**Universal forgery (UU):**  $\mathcal{A}$  can forge a signature for any given message.  $\mathcal{A}$  can efficiently answer any signing query.

**Selective forgery (SU):**  $\mathcal{A}$  can forge a signature for some message of its choice. In this case  $\mathcal{A}$  commits itself to a message before the attack starts.

**Existential forgery (EU):**  $\mathcal{A}$  can forge a signature for one arbitrary message.  $\mathcal{A}$  might output a forgery for any message for which it did not learn the signature from an oracle during the attack.

On the other hand, for the attacks we got (We omit key-only attacks as these allow for no signature queries at all):

**Random message attack (RMA):**  $\mathcal{A}$  learns the public key and the signatures on a set of random messages.

**Adaptively chosen message attack (CMA):**  $\mathcal{A}$  learns the public key and is allowed to adaptively ask for the signatures on messages of its choice<sup>2</sup>.

These two attacks are parameterized by the number of signature queries  $q$  the adversary is allowed to ask. For one-time schemes we only require that a notion is fulfilled for  $q = 1$ .

Any combination of a goal and an attack from the above sets gives a meaningful notion of security. The strength of the notion increases going down each list. Accordingly, a scheme that is only secure against a full break under a random message attack offers the weakest kind of security while a scheme that offers existential unforgeability under adaptively chosen message attacks offers the strongest security guarantees.

<sup>1</sup> We omit strong unforgeability here as it is irrelevant for this context

<sup>2</sup> We omit the non-adaptive setting as it turns out that there is no difference in the given setting.

### 2.3 Formal definitions

We now give formal definitions for the notions from above. We define EU-CMA as an example. The definitions for the remaining notions can be found in Appendix A.

**EU-CMA.** The standard security notion for digital signature schemes is existential unforgeability under adaptive chosen message attacks (EU-CMA) which is defined using the following experiment. By  $\text{DSS}(1^n)$  we denote a signature scheme with security parameter  $n$ .

**Experiment**  $\text{Exp}_{\text{DSS}(1^n)}^{\text{EU-CMA}}(\mathcal{A})$   
 $(\text{sk}, \text{pk}) \leftarrow \text{kg}(1^n)$   
 $(M^*, \sigma^*) \leftarrow \mathcal{A}^{\text{Sign}(\text{sk}, \cdot)}(\text{pk})$   
 Let  $\{(M_i, \sigma_i)\}_1^q$  be the query-answer pairs of  $\text{sign}(\text{sk}, \cdot)$ .  
 Return 1 iff  $\text{vf}(\text{pk}, M^*, \sigma^*) = 1$  and  $M^* \notin \{M_i\}_1^q$ .

For the success probability of an adversary  $\mathcal{A}$  in the above experiment we write

$$\text{Succ}_{\text{DSS}(1^n)}^{\text{EU-CMA}}(\mathcal{A}) = \Pr \left[ \text{Exp}_{\text{DSS}(1^n)}^{\text{EU-CMA}}(\mathcal{A}) = 1 \right].$$

A signature scheme is called  $(t, \epsilon(t), q)$ -EU-CMA-secure if any adversary running in time at most  $t$ , making no more than  $q$  queries to the signing oracle has at most a success probability of  $\epsilon(t)$  for breaking the scheme:

**Definition 2 (EU-CMA).** Let  $n \in \mathbb{N}$ ,  $\text{DSS}$  a digital signature scheme as defined above. We call  $\text{DSS}$   $(t, \epsilon(t), q)$ -EU-CMA-secure if  $\text{InSec}^{\text{EU-CMA}}(\text{DSS}(1^n); t, q)$ , the maximum success probability of all possibly probabilistic adversaries  $\mathcal{A}$  running in time  $\leq t$ , making at most  $q$  queries to  $\text{Sign}$  in the above experiment, is bounded by  $\epsilon(t)$ :

$$\text{InSec}^{\text{EU-CMA}}(\text{DSS}(1^n); t, q) \stackrel{\text{def}}{=} \max_{\mathcal{A}} \{ \text{Succ}_{\text{DSS}(1^n)}^{\text{EU-CMA}}(\mathcal{A}) \} \leq \epsilon(t).$$

A  $(t, \epsilon(t))$ -EU-CMA-secure one-time signature scheme (OTS) is a DSS that is  $(t, \epsilon(t), 1)$ -EU-CMA secure, i.e. the number of signing oracle queries of the adversary is limited to one.

We can give similar definitions for the remaining notions. The difference between the different notions is described by a modified experiment. The definition of success probability and what it means for a scheme to fulfill the notion can be obtained replacing the experiment in the above definitions (and, of course, tracing the resulting changes through the definition). The experiments of the remaining notions are given in Appendix A.

**Attack complexity.** For a  $(t, \epsilon(t))$ -secure scheme, we define the attack complexity as  $2t^*$  for  $t^* = \min_t \{ \epsilon(t) \geq \frac{1}{2} \}$ . As the most costly operations of all attacks are calls to the message digest function  $\text{H}$ , we measure attack complexity as the number of calls to  $\text{H}$ .

**Further model decisions.** For our analysis we made several decisions on how we are analyzing the security in the above models. We are not interested in attacks that exploit weaknesses of the used hash-functions as these already apply in the one-message attack setting. Therefore, we model all used hash functions as random oracles. Due to this decision, RMA-attacks model the setting where randomized hashing is used for the initial message digest. Hence, we do not do a separate analysis for variants of the schemes that use randomized hashing.

### 3 Lamport’s scheme

We start with analyzing Lamport’s scheme which was the first proposal for a hash-based signature scheme. For  $q = 1$  it achieves the strongest security notion EU-CMA-security when the used function is one-way (actually even the ignored stronger SU-CMA-security if the function is second-preimage resistant). This holds even without hashing the message first. Now let us look at the two-message attack case.

#### 3.1 Scheme description

The first and most intuitive proposal for an OTS is Lamport’s scheme (sometimes called Lamport-Diffie OTS) [12]. The scheme uses a one-way function  $F : \{0, 1\}^n \rightarrow \{0, 1\}^n$ , and signs  $m$  bit strings. The secret key consists of  $2m$  random bit strings

$$\text{sk} = (\text{sk}_{1,0}, \text{sk}_{1,1}, \dots, \text{sk}_{m,0}, \text{sk}_{m,1})$$

of length  $n$ . The public key consists of the  $2m$  outputs of the one-way function

$$\text{pk} = (\text{pk}_{1,0}, \text{pk}_{1,1}, \dots, \text{pk}_{m,0}, \text{pk}_{m,1}) = (F(\text{sk}_{1,0}), F(\text{sk}_{1,1}), \dots, F(\text{sk}_{m,0}), F(\text{sk}_{m,1}))$$

when evaluated on the elements of the secret key. Signing a message (digest)  $M^* \in \{0, 1\}^m$  corresponds to publishing the corresponding elements of the secret key:

$$\sigma = (\sigma_1, \dots, \sigma_m) = (\text{sk}_{1,M_1^*}, \dots, \text{sk}_{m,M_m^*}).$$

To verify a signature the verifier checks whether the elements of the signature are mapped to the right elements of the public key using  $F$ :

$$(F(\sigma_1), \dots, F(\sigma_m)) \stackrel{?}{=} (\text{pk}_{1,M_1^*}, \dots, \text{pk}_{m,M_m^*}).$$

For Lamport’s scheme, the message mapping can be considered the identity.

#### 3.2 Security under two-message attacks

Considering a CMA setting, we cannot achieve any security without an initial message hash. An adversary  $\mathcal{A}$  can choose any pair of messages  $(M_1^*, M_2^*)$  such that  $M_1^* = \neg M_2^*$ , where  $\neg$  denotes bitwise negation, and will learn the full secret key. In the following we assume a message  $M$  is first hashed using a hash function  $H : \{0, 1\}^* \rightarrow \{0, 1\}^m$ , i.e., a  $m$ -bit message digest  $M^*$  is used to select the secret key elements. Our results are summarized in Table 2.

**FB-CMA.** A full break requires  $\mathcal{A}$  to find a pair of messages  $(M_1, M_2)$  such that  $H(M_1) = \neg H(M_2)$ . This task has the same complexity as collision finding for  $H$ . The only difference between the two tasks is that the equality condition is replaced by equality after negation. Sadly, this does not mean that we get a reduction from collision resistance as the counter example of the identity function shows: The identity function is collision resistant as no collisions exist but it is trivial to find a pair such that one message is the negation of the other. However, assuming  $H$  behaves like a random function a birthday bound argument shows that the complexity of finding such a pair is  $\mathcal{O}(2^{m/2})$  which can be carried out as pre-computation as long as  $H$  is known.

**EU-CMA.** To produce a valid forgery in a chosen message setting, an adversary  $\mathcal{A}$  has to find a triple of messages  $M_1, M_2, M_3$  such that

$$\text{break}(M_1, M_2, M_3) = (\forall i \in [0, m - 1]) : H(M_1)_i = H(M_2)_i \vee H(M_1)_i = H(M_3)_i$$

**Table 2.** Overview of the computational complexity for two-message attacks against Lamport’s scheme. If the success probability of an attack is not constant in terms complexity, we give the attack complexity to achieve a success probability of  $1/2$ .

Security Goal	Attack Complexity	Pr[Success]
EU-CMA	$\mathcal{O}((4/3)^{m/3})$	$\frac{1}{2}$
SU-CMA	$\mathcal{O}((4/3)^{m/3})$	$\frac{1}{2}$
UU-CMA	$\mathcal{O}(2^{m/2})$	$\frac{1}{2}$
FB-CMA	$\mathcal{O}(2^{m/2})$	$\frac{1}{2}$
EU-RMA	$\mathcal{O}((4/3)^m)$	$\frac{1}{2}$
SU-RMA	-	$(3/4)^m$
UU-RMA	-	$(3/4)^m$
FB-RMA	-	$(1/2)^{m/2}$

where  $H(\cdot)_i$  denotes the  $i$ -th bit of the message digest. In this case, we say that  $M_2, M_3$  form a cover for  $M_1$ .

For random messages  $M_1, M_2, M_3$ , the probability that  $M_2, M_3$  cover  $M_1$  is the inverse probability of each bit of  $M_1^*$  not being covered by  $M_2^*, M_3^*$ :

$$\Pr_{M_1}[\text{break}(M_1, M_2, M_3) = 1] = (1 - (1/2)^2)^m = (3/4)^m$$

For an existential forgery,  $\mathcal{A}$  can start by hashing  $\tau$  random messages, pick a random set of two hashed message and check if these cover a hashed third message. There are  $\binom{\tau}{2}$  such pairs of hashed messages, and  $\tau - 2$  hashed messages that are potentially covered. This means that the probability for an existential forgery equals:

$$\begin{aligned} \Pr_{\{M_1, \dots, M_\tau\}}[\exists(M_a, M_b, M_c) \in \{M_1, \dots, M_\tau\} : \text{break}(M_a, M_b, M_c) = 1] \\ = 1 - (1 - (3/4)^m)^{\binom{\tau}{2}(\tau-2)} \end{aligned}$$

We want to know for which  $\tau$  this probability is bigger than  $1/2$ , where we will use the relation  $1 - x \leq e^{-x} \forall x \in \mathbb{R}$ :

$$1 - (1 - (3/4)^m)^{\binom{\tau}{2}(\tau-2)} \geq 1 - e^{-(3/4)^m \binom{\tau}{2}(\tau-2)} > 1/2 \quad (1)$$

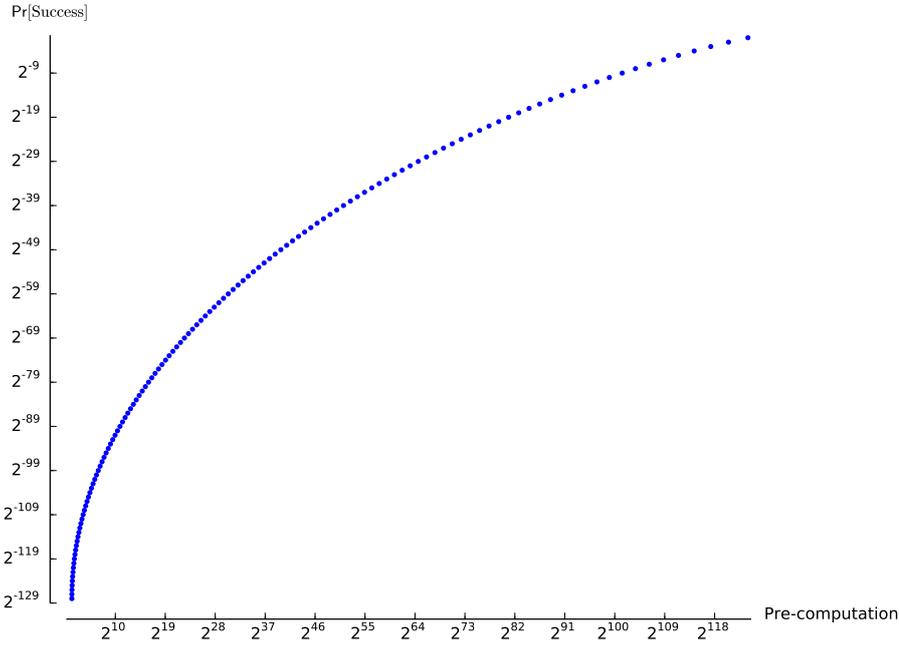
$$(3/4)^m \binom{\tau}{2}(\tau-2) > \ln(2) \quad (2)$$

$$\binom{\tau}{2}(\tau-2) > (4/3)^m \ln(2) \quad (3)$$

This can be roughly estimated as  $\tau > (4/3)^{m/3}$ . As an example, if we consider  $m = 256$  then  $2^{36} > (4/3)^{m/3}$ . Hence, the attack complexity is  $(4/3)^{m/3}$ . It has to be noted that this is all pre-computation, which can be done before choosing a victim: no knowledge of the public key is required.

**SU-CMA.** For selective forgeries,  $\mathcal{A}$  can pick a message  $M$  for which it needs to find a cover before receiving signatures. However, since no knowledge of the public key is needed to start an attack, there is no difference between a selective forgery and an existential forgery.  $\mathcal{A}$  can simply search for three messages  $(M_1, M_2, M_3)$  satisfying the break condition before the attack starts using the correct hash function. It can then commit to  $M_1$  before learning  $\text{pk}$ , and use the signatures of  $M_2, M_3$  to sign  $M_1$ . This means, the complexity of a selective forgery can again be estimated as  $(4/3)^{m/3}$ .

**UU-CMA.** For universal forgeries,  $\mathcal{A}$  can try to find two messages  $M_1, M_2$  such that they have non-overlapping message digests in  $r$  indices. After the experiment,  $\mathcal{A}$  can forge any message with probability  $(1/2)^{m-r}$ , since a messages digest has to overlap with the digests of  $M_1, M_2$  in  $m - r$  indices. The probability to find two messages  $M_1, M_2$  such that they have non-overlapping message digests in  $r$  indices equals  $\binom{m}{r}(1/2)^m$ . Using similar arguments as in the EU-CMA case after  $\tau$  calls to  $H$ , we will find two messages with  $r$  non-overlapping indices with probability bigger than  $1/2$  if  $\binom{\tau}{2} > 2^m \binom{m}{r}^{-1} \ln(2)$ , where we can estimate that  $\tau > 2^{m/2} \binom{m}{r}^{-1/2}$ . It is easy to see that the more pre-computation an attacker is doing, the higher the success probability. Figure 1 shows the success probability as a function of the pre-computation carried out. For  $m = 256$ , a pre-computation of  $2^{136}$  calls to  $H$  is required to achieve a success probability of  $1/2$ .



**Fig. 1.** This plot shows the relation between the amount of pre-computation and the success probability of a universal forgery in a chosen message attack on Lamport’s One-Time Signature Scheme.

**EU-RMA.** In this case, the adversary gets a signature of two random messages ( $M_1, M_2$ ) and has to find a third message  $M_3$  that is covered by  $M_1, M_2$ . The difference to the CMA case is that  $\mathcal{A}$  cannot optimize the choice of  $M_1, M_2$ . This means each index should be covered, which happens with probability  $(3/4)^m$ . In consequence,  $\mathcal{A}$  has to compute about  $\tau > (4/3)^m$  message digests before it has found the forgery, using the same arguments as in the EU-CMA setting. For  $m = 256$ , this means the attacker has to compute about  $2^{106}$  message digests, making this type of forgery computationally infeasible. However, for  $m = 128$  bit message digests, this would mean a computational cost of  $2^{53}$ , which is in reach for strong attackers.

**SU-RMA.** For SU-RMA, the adversary selects a message before it receives two signatures of two random messages. There is no way for  $\mathcal{A}$  to optimize the selection of this message, as  $\mathcal{A}$  does not know (or has influence on) the two random messages for

which it learns the signatures. The probability that  $\mathcal{A}$  can afterwards sign the selected message is  $(3/4)^m$ . This is also the success probability of the attack. Please note that this probability is constant for fixed parameters, i.e., independent of the adversaries efforts.

**UU-RMA.** For random message attacks, there is no difference between universal and selective forgery attacks since the adversary has no power over the signed messages and cannot affect his success probability by choice of a target message. This means also in this case, the probability of a forgery is  $(3/4)^m$ .

**FB-RMA.** The probability of a full break under a random message attack, is simply the probability that two messages are each-others negated version. This happens with probability  $\mathcal{O}((1/2)^{m/2})$  due to the birthday paradox.

## 4 Optimized Lamport

The optimized Lamport scheme is very similar to Lamport's scheme and first appeared in [15]. While it is interesting on its own, it is also of interest as it can be viewed as a special, simplified version of the Winternitz OTS discussed in the next section.

### 4.1 Scheme description

The optimized Lamport scheme uses a one-way function  $F : \{0, 1\}^n \rightarrow \{0, 1\}^n$ , and signs  $m$  bit messages. The secret key consists of  $\ell = m + \log m + 1$  random bit strings

$$\text{sk} = (\text{sk}_1, \dots, \text{sk}_\ell)$$

of length  $n$ . The public key consists of the  $\ell$  outputs of the one-way function

$$\text{pk} = (\text{pk}_1, \dots, \text{pk}_\ell) = (F(\text{sk}_1), \dots, F(\text{sk}_\ell))$$

when evaluated on the elements of the secret key. Signing a message  $M^* \in \{0, 1\}^m$  corresponds to first computing and appending a checksum to  $M^*$  to obtain the message mapping  $G(M^*) = B = M^* \| C$  where  $C = \sum_{i=1}^m \neg M_i^*$ . The signature consists of the secret key element if the corresponding bit in  $B$  is 1, and the public key element otherwise:

$$\sigma = (\sigma_1, \dots, \sigma_m) \text{ with } \sigma_i = \begin{cases} \text{sk}_i & , \text{ if } B_i = 1, \\ \text{pk}_i & , \text{ if } B_i = 0. \end{cases}$$

To verify a signature the verifier checks whether the full public key is obtained by hashing the elements of the signature that correspond to 1 bits in  $B$ :

$$\text{Return } 1, \text{ iff } (\forall i \in [1, \ell]) : \text{pk}_i = \begin{cases} F(\sigma_i) & , \text{ if } B_i = 1, \\ \sigma_i & , \text{ if } B_i = 0. \end{cases}$$

### 4.2 Security under two-message attacks

As with the non-optimized Lamport scheme, we cannot achieve any security without initial message hash. While it is impossible to learn the whole secret key from a two-message attack for typical parameters (this is the case as for  $m$  being a power of two the most significant bit of the checksum is only 1 for the all zero message, and it is impossible to learn the remaining secret key values from the signature of a single message), it is trivial to obtain all secret key elements but the one that corresponds to the most significant bit of the checksum. This allows to sign any message but the all 0 message. An adversary can for example use the all 1 message (to learn the secret key values for

**Table 3.** Overview of the computational complexity for two-message attacks against the optimized Lamport scheme. If the success probability of an attack is not constant in terms complexity, we give the attack complexity to achieve a success probability of  $1/2$  (aside from SU-RMA as the best we can achieve is a success probability of  $\frac{3}{8}$ ).

Security Goal	Attack Complexity	Pr[Success]
EU-CMA	$\mathcal{O}((8/7)^{(m+\log m)/3})$	$\frac{1}{2}$
SU-CMA	$\mathcal{O}((8/7)^{(m+\log m)/3})$	$\frac{1}{2}$
UU-CMA	$\mathcal{O}((3/4)^{(m+\log m)/2})$	$\frac{1}{2}$
FB-CMA	$\mathcal{O}((3/4)^{m+\log m})$	$\frac{1}{2}$
<hr/>		
EU-RMA	$\mathcal{O}((8/7)^{m+\log m})$	$\frac{1}{2}$
SU-RMA	$\mathcal{O}(2^{m+\log m})$	$\frac{3}{8}$
UU-RMA	-	$(7/8)^{m+\log m}$
FB-RMA	-	$(3/4)^{m+\log m}$

the message part of  $B$ ) and any message with a single one (to learn the secret key values of the checksum part of  $B$ , besides the one at the most significant position).

In the following we assume a message  $M$  is first hashed using a hash function  $H : \{0, 1\}^* \rightarrow \{0, 1\}^m$  to obtain a message digest  $M^*$  – making attacks significantly harder. The analysis of the scheme as described above turned out too complex to be carried out exactly due to the dependency between  $C$  and  $M^*$ . The problem is that it would be possible to condition on two checksums to cover a third one in the existential forgery setting. These conditions would give an exact Hamming weight for the message parts. However, there would be exponentially many possibilities, each with a specific probability, rendering an analysis impossible. For that reason, we simplified the analysis assuming that  $C$  is uniformly random and thereby that digest  $M^*$  and checksum  $C$  are independent of each other. Note that the neglected dependency can make the attack both easier and harder, depending on the setting. Our theoretical results are summarized in Table 3. For an experimental verification of our results see Section 6.

**FB-CMA.** As mentioned above for  $m$  being a power of two (which is the typical setting), it is impossible to learn the whole secret key from a two-message attack. For other choices of  $m$ , an adversary  $\mathcal{A}$  has to find two messages  $M_1, M_2$  such that  $(B_1)_i = 1$  or  $(B_2)_i = 1$  for all  $i \in \{0, \dots, \ell - 1\}$ .

As  $H$  is modeled as random oracle and we assume the checksum is uniformly random and independent of the message, every random input message  $M$  leads to a random message mapping  $B$  of length  $\ell$ . For two random input messages  $M_1, M_2$ , the probability that at least one of the two corresponding message mappings  $B_1, B_2$  is 1 at each position is:

$$\Pr[\text{FB}(M_1, M_2)] = (3/4)^\ell.$$

Similar to the strategy in Lamport’s scheme, we can hash  $\tau$  messages and check all pairs for a full break. Again using the relation  $1 - x \leq e^{-x} \forall x \in \mathbb{R}$ , we have a full break with probability bigger than  $1/2$  if  $\binom{\tau}{2} > (4/3)^\ell \ln(2)$ . We can therefore estimate the complexity of a full break with  $(4/3)^{\ell/2}$  calls to  $H$ . For  $m = 256$ , this complexity equals  $2^{54}$ .

**EU-CMA.** We will now explore forgeries for a third message, given the signatures for two messages. We define the condition for a break for three messages  $M_1, M_2, M_3$  with message mappings  $B_1, B_2, B_3$  as:

$$\text{break}(M_1, M_2, M_3) := (\forall i \in [0, \ell - 1]) : (B_1)_i = 1 \Rightarrow (B_2)_i = 1 \vee (B_3)_i = 1 \quad (4)$$

where  $(B_j)_i$  denotes the  $i$ -th bit of the mapping of message  $M_j$ . If the condition is fulfilled, we say that  $M_2, M_3$  form a cover of  $M_1$ .

In other words: we only need the secret values for those bits of the first message mapping that are 1, so the probability for a break is higher for target messages with a low weight message mapping. Recall that we assume that  $M_j^*$  and  $C_j$  are independent, meaning we assume we have three independent random bit strings.

To get the probability that we cover a bit of  $B_1$ , we can condition on the value of that bit  $b \in \{0, 1\}$ :

$$\begin{aligned} & \Pr[(B_1)_i \leq (B_2)_i \vee (B_1)_i \leq (B_3)_i] \\ &= \sum_{b \in \{0,1\}} \Pr[(B_1)_i \leq (B_2)_i \vee (B_1)_i \leq (B_3)_i \mid (B_1)_i = b] \Pr[(B_1)_i = b] \\ &= \frac{1}{2} \cdot \Pr[0 \leq (B_2)_i \vee 0 \leq (B_3)_i \mid (B_1)_i = 0] \\ &\quad + \frac{1}{2} \cdot \Pr[1 \leq (B_2)_i \vee 1 \leq (B_3)_i \mid (B_1)_i = 1] \\ &= \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot \frac{3}{4} = \frac{7}{8} \end{aligned}$$

This means that the probability that the break condition is fulfilled for three random messages is  $(\frac{7}{8})^\ell$ .

As with the original Lamport scheme, we can precompute  $\tau$  message mappings, pick two mappings and check if we cover a third mapping. This gives  $\binom{\tau}{2} \cdot (\tau - 2)$  possibilities, which means the probability of an existential forgery equals:

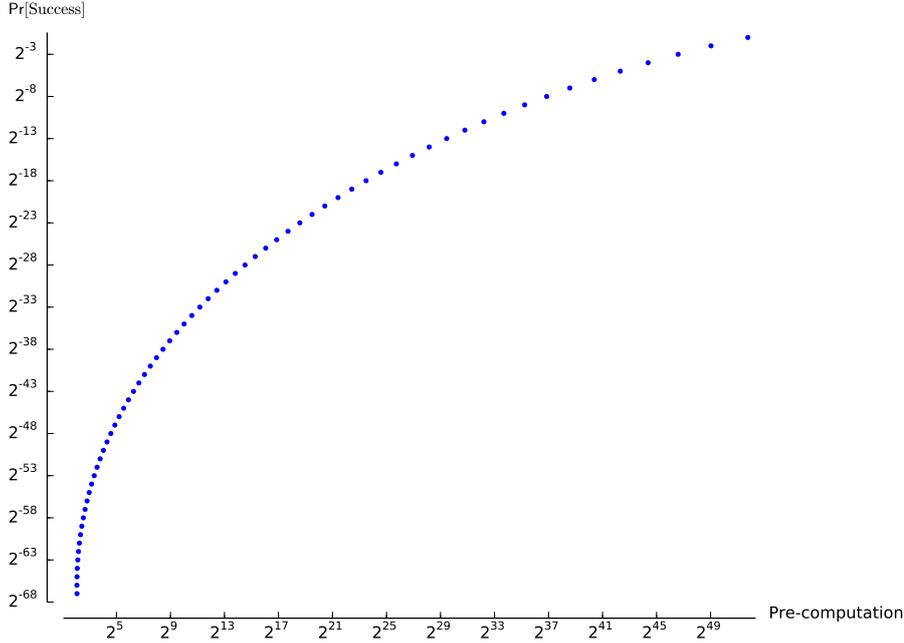
$$\begin{aligned} \Pr_{\{M_1, \dots, M_\tau\}}[\exists(M_a, M_b, M_c) \in \{M_1, \dots, M_\tau\} : \text{break}(M_a, M_b, M_c) = 1] \\ = 1 - (1 - (7/8)^\ell)^{\binom{\tau}{2}(\tau-2)} \end{aligned}$$

This probability is bigger than  $1/2$  for  $\tau > (8/7)^{\ell/3}$ , using similar arguments as in the EU-CMA case for Lamport. For  $m = 256$ , the adversary needs to precompute  $\tau > 2^{17}$  hash digests to find such a cover with probability bigger than  $1/2$ . For  $m = 128$ , this would mean  $\tau > 2^9$  hash digests.

**SU-CMA.** As with the original Lamport scheme, the adversary does not need knowledge of the public key to compute three messages that satisfy the break condition. This means that also for the optimized Lamport scheme, a selective forgery has the same complexity as an existential forgery under chosen message attacks.

**UU-CMA.** The goal of the adversary is to find two messages  $M_1, M_2$  such that their combined mappings have the highest weight possible. The probability to find two messages having weight  $r$  is equal to  $\binom{\ell}{r} (3/4)^r (1/4)^{\ell-r}$ , where we again assume that  $M^*$  and  $C$  are independent. Note that this probability is symmetric around  $\ell \cdot (3/4)$ , which means  $\mathcal{A}$  should not take any  $r$  below  $\ell \cdot (3/4)$ . After  $\tau$  calls to H, we will find two messages  $M_1, M_2$  with a combined weight of  $r$  with probability bigger than  $1/2$  if  $\binom{\tau}{2} > \left( \binom{\ell}{r} (3/4)^r (1/4)^{\ell-r} \right)^{-1} \ln(2)$ . We can estimate the pre-computation complexity as square-root of the right part of this inequality. After the online phase of the attack,  $\mathcal{A}$  can sign a new message with probability  $(1/2)^{\ell-r}$ , since for the positions that are not covered by  $B_1$  or  $B_2$ , the bit of the new message must be 0. The relation between the pre-computation and the success probability is given in Figure 2 for  $m = 256$ .

**EU-RMA.** According to Eqn. 4, two messages  $M_2, M_3$  have a probability of  $(7/8)^\ell$  to cover a random third message  $M_1$ . This means that after receiving the signature of two random messages, the adversary has to search about  $\tau > (8/7)^\ell$  messages to forge a

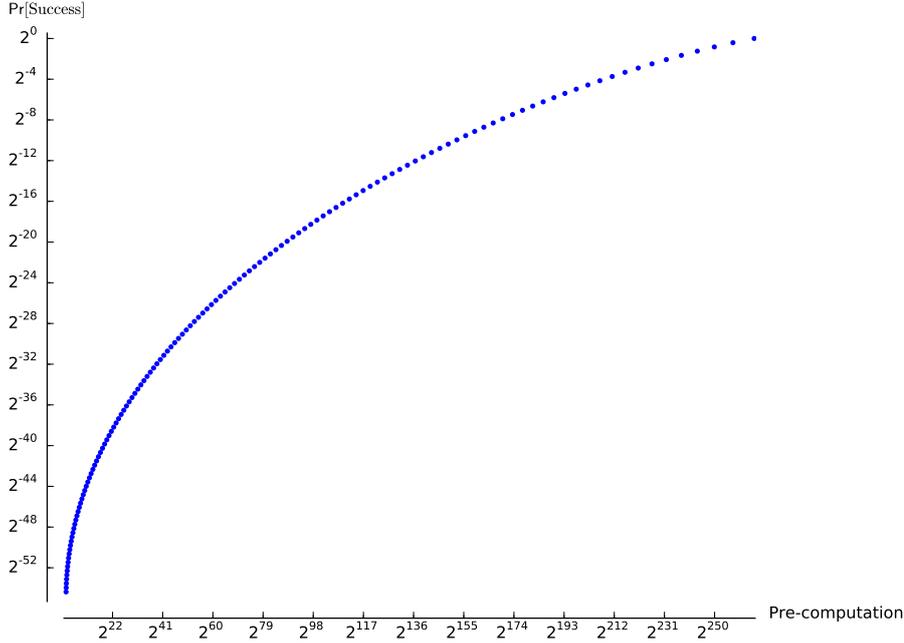


**Fig. 2.** This plot shows the relation between the amount of pre-computation and the success probability of a universal forgery in a chosen message attack on the optimized Lamport scheme.

third signature with probability bigger than  $1/2$  (again using arguments described in earlier analyses), since it only needs the secret values for the bits of  $M_1$  that are 1. For  $m = 256$ , this means a computational cost of about  $2^{51}$ , which is in reach for a strong attacker. For  $m = 128$ , this would mean a computational cost of  $2^{26}$ , which can be done within minutes on today's CPUs.

**SU-RMA.** Unlike with the original Lamport scheme, for the optimized Lamport scheme an adversary can optimize his selection of the target message in a random message attack. Messages that have low-weight message mappings are more likely to be covered by the mappings of two random messages. However, note that we can only select a single target message instead of a whole cover, which makes the pre-computation more costly. The probability to find a message mapping  $B$  with weight  $r$  is equal to  $\binom{\ell}{r}(1/2)^\ell$ , which is again symmetric around  $\ell/2$ . An attacker should therefore always pick a message with weight  $r \leq \ell/2$ . This message can be signed, after receiving the signatures of two random messages, with probability  $(3/4)^r$ , since all positions of  $B$  that are 1 have to be covered by the mappings of the two random messages. If we again estimate the pre-computation as  $\tau > \left(\binom{\ell}{r}(1/2)^\ell\right)^{-1} \ln(2)$  to find a message mapping with weight  $r$  with probability bigger than  $1/2$ , we get the relation with success probability for a selective forgery in Figure 3 for  $m = 256$ . Note that this figure looks similar to Figure 2 but a far more pre-computation is required to achieve the same success probability. Even for strong attackers, it should be infeasible to get a high success probability.

**UU-RMA.** For a universal forgery under a random message attack, the attacker cannot influence anything in the experiment. This means the success probability for this forgery is simply the success probability of the conditional break:  $(7/8)^\ell$ .



**Fig. 3.** This plot shows the relation between the amount of pre-computation and a lower bound for the success probability of a universal forgery in a chosen message attack on the optimized Lamport’s One-Time Signature Scheme.

**FB-RMA.** The probability of a full break under a random message attack, is simply the probability that all bits are covered. This happens with probability  $(3/4)^\ell$ , which is  $2^{-54}$  when  $m = 256$ .

## 5 Winternitz OTS

The Winternitz one-time signature scheme (WOTS) is a further improvement of the optimized Lamport scheme. Instead of using the hash of each secret key value as public key, the public key values are obtained by hashing more than once, i.e.  $w$  times. That way, more than one bit can be encoded per selection of a hash value. The basic idea for the Winternitz OTS (WOTS) was proposed in [15]. What we know as WOTS today is a generalization that was proposed by Even, Goldreich, and Micali [7]. There exist several variants that reduce the assumptions made about the used hash function [2,8,11]. Recent standardization proposals for hash-based signatures [13,9] as well as a recent proposal for stateless hash-based signatures [1] use WOTS as one-time signature scheme.

### 5.1 Scheme description

WOTS uses a length-preserving (cryptographic hash) function  $F : \{0, 1\}^n \rightarrow \{0, 1\}^n$ . It is parameterized by the message length  $m$  and the Winternitz parameter  $w \in \mathbb{N}, w > 1$ , which determines the time-memory trade-off. The two parameters are used to compute

$$\ell_1 = \left\lceil \frac{m}{\log(w)} \right\rceil, \quad \ell_2 = \left\lceil \frac{\log(\ell_1(w-1))}{\log(w)} \right\rceil + 1, \quad \ell = \ell_1 + \ell_2.$$

The scheme uses  $w - 1$  iterations of  $F$  on a random input. We define them as

$$F^a(x) = F(F^{a-1}(x))$$

and  $F^0(x) = x$ .

Now we describe the three algorithms of the scheme:

*Key generation algorithm* ( $\text{kg}(1^n)$ ): On input of security parameter  $1^n$  the key generation algorithm chooses  $\ell$   $n$ -bit strings uniformly at random. The secret key  $\text{sk} = (\text{sk}_1, \dots, \text{sk}_\ell)$  consists of these  $\ell$  random bit strings. The public verification key  $\text{pk}$  is computed as

$$\text{pk} = (\text{pk}_1, \dots, \text{pk}_\ell) = (F^{w-1}(\text{sk}_1), \dots, F^{w-1}(\text{sk}_\ell))$$

*Signature algorithm* ( $\text{sign}(1^n, M^*, \text{sk})$ ): On input of security parameter  $1^n$ , a message (digest)  $M^*$  of length  $m$  and the secret signing key  $\text{sk}$ , the signature algorithm first computes a base  $w$  representation of  $M^*$ :  $M^* = (M_1^* \dots M_{\ell_1}^*)$ ,  $M_i^* \in \{0, \dots, w - 1\}$ . Next it computes the check sum

$$C = \sum_{i=1}^{\ell_1} (w - 1 - M_i^*)$$

and computes its base  $w$  representation  $C = (C_1, \dots, C_{\ell_2})$ . The length of the base- $w$  representation of  $C$  is at most  $\ell_2$  since  $C \leq \ell_1(w-1)$ . We set  $B = (B_1, \dots, B_\ell) = M^* \parallel C$ . The signature is computed as

$$\sigma = (\sigma_1, \dots, \sigma_\ell) = (F^{B_1}(\text{sk}_1), \dots, F^{B_\ell}(\text{sk}_\ell)).$$

*Verification algorithm* ( $\text{vf}(1^n, M^*, \sigma, \text{pk})$ ): On input of security parameter  $1^n$ , a message (digest)  $M^*$  of length  $m$ , a signature  $\sigma$  and the public verification key  $\text{pk}$ , the verification algorithm first computes the  $B_i$ ,  $1 \leq i \leq \ell$  as described above. Then it does the following comparison:

$$\text{pk} = (\text{pk}_1, \dots, \text{pk}_\ell) \stackrel{?}{=} (F^{w-1-B_1}(\sigma_1), \dots, F^{w-1-B_\ell}(\sigma_\ell))$$

If the comparison holds, it returns **true** and **false** otherwise.

*Remark 1.* The difference between the basic WOTS as described above and the variants proposed in [2,8,11] is how  $F$  is iterated. As all the attacks below are independent of this choice, our results apply to all those variants, too.

## 5.2 Two-message attacks

Without hashing the message, the scheme does not offer any security once an attacker can choose two messages to be signed. As always, the adversary simply chooses the all zero and the all one message to be signed, and afterwards knows all secret values (for some parameter choices it will actually be impossible to extract the whole secret key for the same reason as for optimized Lamport. However, in that case, as for the optimized Lamport scheme, it is possible to select two messages that allow learn all but one secret key element).

In the following we assume a message  $M$  is first hashed using a hash function  $H : \{0, 1\}^* \rightarrow \{0, 1\}^m$  to obtain a message digest  $M^*$  – making attacks significantly harder. As for the optimized Lamport scheme, the analysis of the scheme as described above turned out too complex to be carried out exactly due to the dependency between  $C$  and  $M^*$ . We simplified the analysis assuming that  $C$  is uniformly random and thereby

**Table 4.** Overview of the computational complexity for two-message attacks against the Winternitz OTS. If the success probability of an attack is not constant in terms complexity, we give the attack complexity to achieve a success probability of  $1/2$ .

Security Goal	Attack Complexity	Pr[Success]
EU-CMA	$\mathcal{O}\left(\left(\frac{(w+1)(4w+1)}{6w^2}\right)^{-\frac{m+\log m}{3\log w}}\right)$	$\frac{1}{2}$
SU-CMA	$\mathcal{O}\left(\left(\frac{(w+1)(4w+1)}{6w^2}\right)^{-\frac{m+\log m}{3\log w}}\right)$	$\frac{1}{2}$
UU-CMA	$\mathcal{O}\left(\left(1 - \left(\frac{w-1}{w}\right)^2\right)^{-\frac{m+\log m}{2\log w}}\right)$	$\frac{1}{2}$
FB-CMA	$\mathcal{O}\left(\left(1 - \left(\frac{w-1}{w}\right)^2\right)^{-\frac{m+\log m}{\log w}}\right)$	$\frac{1}{2}$
EU-RMA	$\mathcal{O}\left(\left(\frac{(w+1)(4w+1)}{6w^2}\right)^{-\frac{m+\log m}{\log w}}\right)$	$\frac{1}{2}$
SU-RMA	$\mathcal{O}\left(\left(\frac{1}{w}\right)^{-\frac{m+\log m}{\log w}}\right)$	$\frac{1}{2}$
UU-RMA	-	$\left(\frac{(w+1)(4w+1)}{6w^2}\right)^{\frac{m+\log m}{\log w}}$
FB-RMA	-	$\left(1 - \left(\frac{w-1}{w}\right)^2\right)^{\frac{m+\log m}{\log w}}$

that digest  $M^*$  and checksum  $C$  are independent of each other. It applies again that the neglected dependency can make the attack both easier and harder, depending on the setting. Our theoretical results are summarized in Table 4. For an experimental verification of the results see Section 6.

**FB-CMA.** The adversary has to find messages  $M_1, M_2$  with mappings  $B_1, B_2$  such that for all  $0 \leq i \leq \ell$ : either  $(B_1)_i = 0$  or  $(B_2)_i = 0$ . The probability to cover an index of the secret key equals  $(1 - (\frac{w-1}{w})^2)$  for each  $i$ , which means the probability that this is true for all  $i$  equals:  $(1 - (\frac{w-1}{w})^2)^\ell$ . After hashing  $\tau$  messages, the probability to find two messages satisfying the condition of a full break will be larger than  $1/2$  if  $\binom{\tau}{2} > (1 - (\frac{w-1}{w})^2)^{-\ell} \ln(2)$ , which means we can approximately use  $\tau > (1 - (\frac{w-1}{w})^2)^{-\ell/2}$ . As a sanity check, we see that for  $w = 2$  we get  $\tau > (4/3)^{\ell/2}$ , which is the complexity of a full break for the optimized Lamport scheme. Typically for applications  $w = 16$  is used, which would mean  $\tau > 2^{102}$ .

**EU-CMA.** For an existential forgery, we first define the condition for a break for WOTS:

$$\text{break}(M_1, M_2, M_3) := (\forall i \in [0, \ell - 1]) : (B_1)_i \geq (B_2)_i \vee (B_1)_i \geq (B_3)_i \quad (5)$$

where  $(B_j)_i$  denotes the  $i$ -th bit of the base- $w$  values of the message mapping  $B_j$  for message  $M_j$ ;  $j \in \{1, 2, 3\}$ . If the condition is true, we say  $M_2, M_3$  form a cover of  $M_1$ .

We will first see what the probability is to cover one index of  $B_1$ . If we condition on the value of  $(B_1)_i$ , we get:

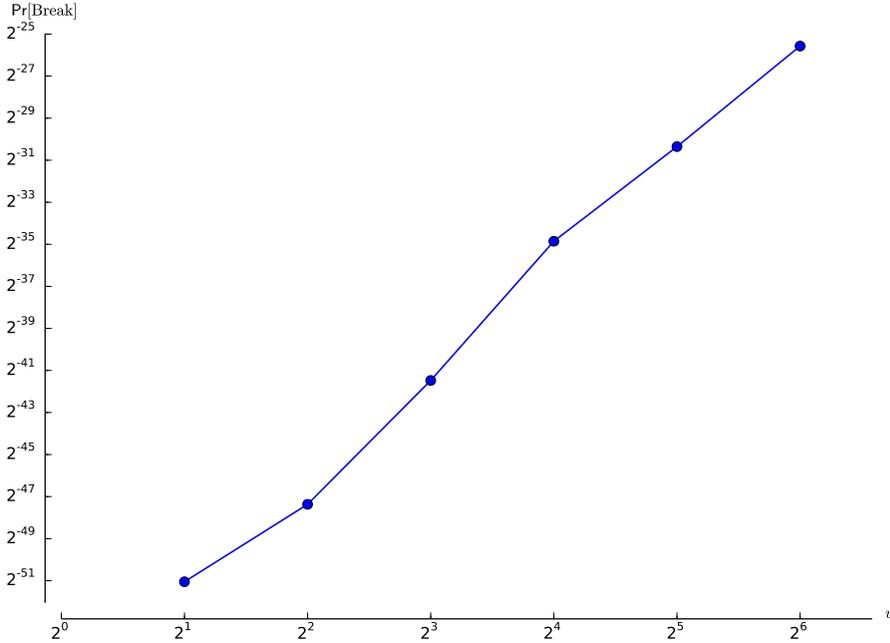
$$\begin{aligned} \Pr[(B_1)_i \geq (B_2)_i \vee (B_1)_i \geq (B_3)_i] &= \\ \sum_{x=0}^{w-1} \Pr[(B_1)_i \geq (B_2)_i \vee (B_1)_i \geq (B_3)_i | (B_1)_i = x] \Pr[(B_1)_i = x] &= \\ \sum_{x=0}^{w-1} \frac{1}{w} \left(1 - \left(\frac{w - (x+1)}{w}\right)^2\right) &= \\ \frac{(w+1)(4w-1)}{6w^2} & \end{aligned}$$

Again as a sanity check, we see that for  $w = 2$ , this probability equals  $(7/8)$ , which we already concluded for the optimized Lamport scheme.

In total we see that the probability for a conditional break is:

$$\Pr[\text{break}(M_1, M_2, M_3) = 1] = \left( \frac{(w+1)(4w-1)}{6w^2} \right)^\ell \approx \left( \frac{(w+1)(4w-1)}{6w^2} \right)^{\frac{m+\log m}{\log w}}$$

We see that for bigger  $w$ , the probability that one of the indices is not covered grows, but the number of indices shrinks. The logarithmic decrease of the exponent is in this case more important, which means the bigger the  $w$ , the bigger the probability of the conditional break (which means less computational power required for forgeries).



**Fig. 4.** This plot shows the logarithmic relation between  $w$  and  $\Pr[\text{break}]$  for  $w \in \{2, 4, 8, 16, 32, 64\}$ . The logarithmic decrease of the exponent in  $\Pr[\text{break}]$  is clearly making the probability grow larger for larger  $w$ .

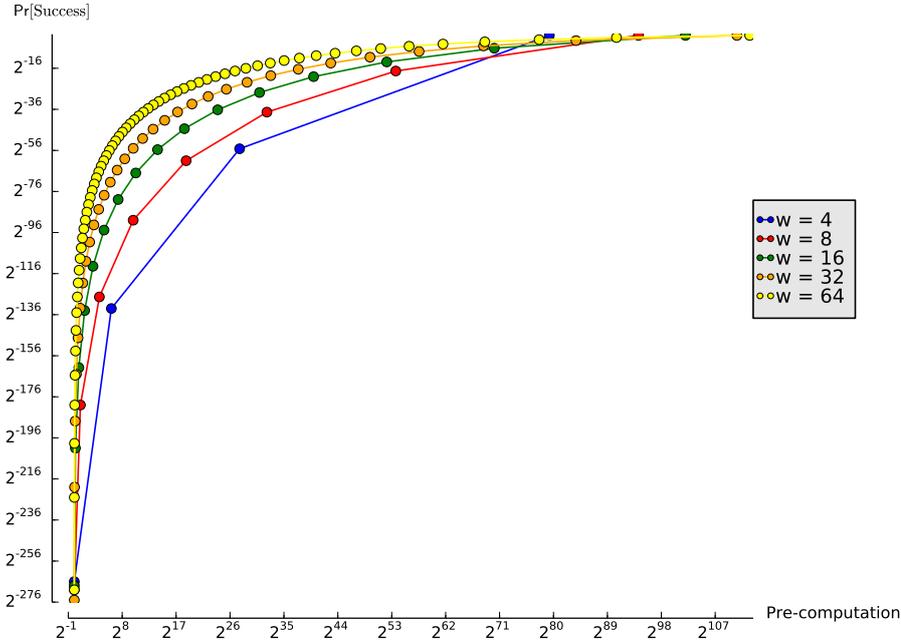
Similar to the arguments for the EU-CMA cases for Lamport and optimized Lamport scheme, an adversary needs to pre-compute about

$$\tau > \left( \left( \frac{(w+1)(4w-1)}{6w^2} \right)^{-\frac{m+\log m}{\log w}} \ln(2) \right)^{1/3}$$

message mappings to find a cover in the list of  $\tau$  message mappings with probability bigger than  $1/2$ . As an example, if we set  $m = 256$  and  $w = 16$ , we have  $\tau > 2^{12}$ . Note that, unlike the FB-CMA setting, it is much easier to forge a third signature for bigger  $w$ : while it becomes harder to get  $B_i = 0$ , the probability for a message cover grows.

**SU-CMA.** As with Lamport’s scheme and the optimized Lamport scheme,  $\mathcal{A}$  does not need knowledge of the public key to start any pre-computation. This means we obtain the same complexity for a selective forgery as for an existential forgery under CMA.

**UU-CMA.** For a universal forgery,  $\mathcal{A}$  can try to compute two message mappings  $B_1, B_2$  such that either  $(B_1)_i \leq r$  or  $(B_2)_i \leq r$  for all  $i \in \{0, \dots, \ell-1\}$ , where  $r \in \{0, \dots, w-1\}$ . The probability to find two messages satisfying these rules equals  $\left(1 - \left(\frac{w-(r+1)}{w}\right)^2\right)^\ell$ , which means we will find such messages in a list of  $\tau$  messages with probability bigger than  $1/2$  if  $\binom{\tau}{2} > \left(1 - \left(\frac{w-(r+1)}{w}\right)^2\right)^{-\ell} \ln(2)$ , using again the same arguments as for Lamport and optimized Lamport. Now  $\mathcal{A}$  obtains a successful forgery for  $M_3$  with probability *at least*  $\left(\frac{w-r}{w}\right)^\ell$ , since we ignored the cases where  $(B_3)_i$  is smaller than  $r$ , but still bigger than  $(B_1)_i$  or  $(B_2)_i$ . To get the exact probability, means summing up over all possible values and would naturally not be feasible to compute. The pre-computation  $\tau$  and corresponding success probability for different values of  $w$  and  $r \in \{0, \dots, w-1\}$  are given in Figure 5.



**Fig. 5.** This plot shows the relation between the amount of pre-computation and the success probability for a universal forgery under a chosen message attack on WOTS for different values of  $w$  and for each  $r \in \{0, \dots, w-1\}$ .

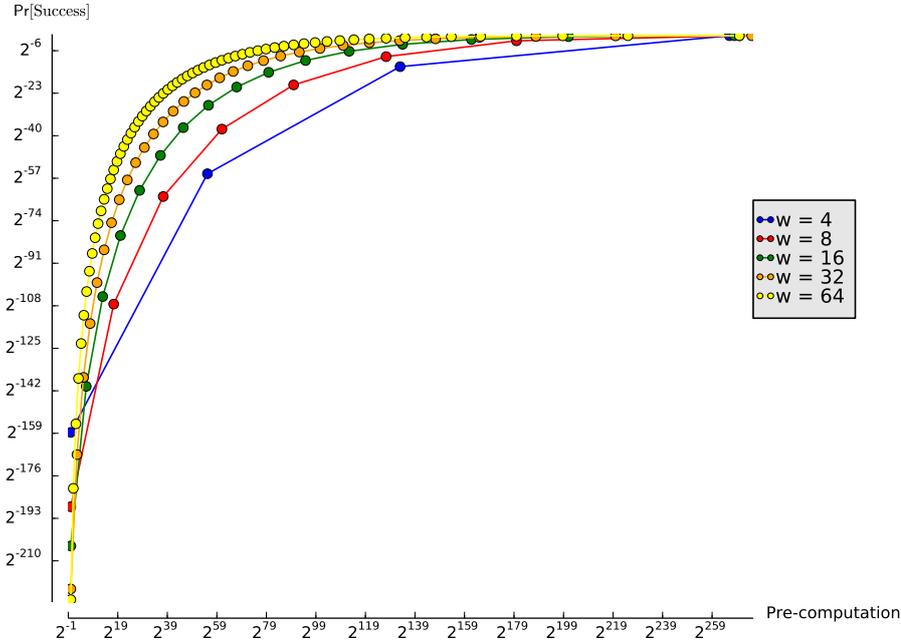
**EU-RMA.** For WOTS, two messages cover a third one with probability:

$$\Pr[\text{break}(M_1, M_2, M_3) = 1] \approx \left(\frac{(w+1)(4w-1)}{6w^2}\right)^{\frac{m+\log m}{\log w}}.$$

This means that when an attacker receives two signatures of two random messages, it has to compute about  $\tau > \left(\frac{(w+1)(4w-1)}{6w^2}\right)^{-\frac{m+\log m}{\log w}} \ln(2)$  messages to find a covered third

message with probability bigger than  $1/2$ . For  $m = 256$  and  $w = 16$ , this equals  $2^{34}$ , which can be done within a few days on today's CPUs.

**SU-RMA.** For the selective forgery, an attacker can select an optimal message with a mapping that contains as high values as possible. For the analysis, we will use the same strategy as for the universal forgery, but in this case we want  $(B_1)_i \geq r$  for all  $i \in \{0, \dots, \ell - 1\}$ , which happens with probability  $\left(\frac{w-r}{w}\right)^\ell$ . We again estimate the pre-computation to be  $\tau > \left(\frac{w-r}{w}\right)^{-\ell} \ln(2)$  to find such a message with probability bigger than  $1/2$  in a list of  $\tau$  messages. The probability that the adversary can sign his selected message after he received two signatures on random messages equals  $\left(1 - \left(\frac{w-(r+1)}{w}\right)^2\right)^\ell$  in this case. A plot of the computational costs with corresponding success probability is given in Figure 6. As for the optimized Lamport scheme, it looks similar to the graph of the universal forgery under chosen message attacks, but with lower success probabilities since  $\mathcal{A}$  only has control over the selected message.



**Fig. 6.** This plot shows the relation between the amount of pre-computation and the success probability of a selective forgery under random message attacks on WOTS for different values of  $w$  and for each  $r \in \{0, \dots, w - 1\}$

**UU-RMA.** The probability of a successful universal forgery under a random message attack equals the probability that three random messages fulfill the break condition:

$$\Pr[\text{break}(M_1, M_2, M_3) = 1] \approx \left(\frac{(w+1)(4w-1)}{6w^2}\right)^{\frac{m+\log m}{\log w}}$$

The attacker has no influence on the process and cannot use any computational power before or after the online phase of the attack to increase his success probability.

**FB-RMA.** Similar to Lamport’s and the optimized Lamport scheme, a full break occurs exactly when all secret values are exposed. For Winternitz with parameter  $w$ , this happens with probability  $(1 - (\frac{w-1}{w})^2)^\ell$ , which is a negligible probability for any  $w$ .

## 6 Experimental verifications

In sections 3, 4, and 5 we discussed the attack complexity of several different attacks. For the optimized Lamport scheme and WOTS, we assumed that the checksum is uniformly random and hence the message digest and its checksum behave as independent bit strings. However, as already mentioned there, the actual situation is that the checksum is dependent of the message digest. To verify the obtained results we carried out experiments for the EU-CMA case for optimized Lamport and WOTS.

We determined a lower bound for the number of calls  $\tau$  to the message digest function  $H$ , such that a list of size  $\tau$  of message digests, allows to find an existential forgery with probability at least  $1/2$ . We performed several experiments for different values of  $\tau$ , to see how realistic our assumption matches the real situation. We checked how many times a list of  $\tau$  message mappings contained a cover for optimized Lamport scheme with digest length of  $m = 128$  bits and for WOTS, with  $m = 256$  and  $w = 16$  (which are the parameters suggested in [9]). We performed 100 experiments per value of  $\tau$ . As can be seen from the results in tables 5 and 6, the experiments closely match the theoretical results using the simplification. The theoretical analysis predicts that  $\tau > 2^9$  is required for existential forgery with probability bigger than  $1/2$  for the optimized Lamport scheme with  $m = 128$ . For WOTS, the analysis suggests  $\tau > 2^{12}$  when  $m = 256$  and  $w = 16$ . From the results of the experiments, we can conclude that the simplifying assumption of independent message digests and checksums is not causing a significant difference to the real setting in the case of EU-CMA.

*Remark 2.* It is important to note that for extreme cases our analysis is not good enough. In the FB-CMA, UU-CMA, SU-RMA and FB-RMA settings for the optimized Lamport and Winternitz schemes, we are trying to push the message mappings to extreme cases to allow for forgeries. However, due to the inverse nature of the checksum, our analysis leads to impossible message mappings. For example, a high weight message part means a low weight checksum part for optimized Lamport, but in our analysis we are trying to push both message and checksum part to high weights. Therefore we expect the complexity to be much higher for these extreme cases, i.e. when  $r$  is very low or very high, with the meaning of  $r$  as described in optimized Lamport and Winternitz.

**Table 5.** Experimental results for the success probability of an EU-CMA adversary, using a list of  $\tau$  message mappings for the optimized Lamport scheme with digest length  $m = 128$

$\tau$	Pr[Success]
$2^8$	0.02
$2^9$	0.13
$2^{10}$	0.77
$2^{11}$	1.0
$2^{12}$	1.0

## 7 Signer Chosen Message Attacks

As discussed in the introduction, in some cases it might actually be intended that the signature scheme allows for existential forgeries whenever a key pair is used twice. In

**Table 6.** Experimental results for the success probability of an EU-CMA adversary, using a list of  $\tau$  message mappings for WOTS with  $w = 16$  and digest length  $m = 256$

$\tau$	$\Pr[\text{Success}]$
$2^{11}$	0.1
$2^{12}$	0.49
$2^{13}$	0.94
$2^{14}$	1.0
$2^{15}$	1.0

these cases, the signer is considered the malicious party that tries to increase the costs for a forgery as much as possible. So, if the signer can control the messages, he can search for two messages that maximize the attack complexity for the forger. In the following we analyze how much a malicious signer can increase the attack complexity for a forger spending some computation time himself. We carry out the analysis for the three schemes discussed in the previous sections. As before, the setting is meaningless without an initial message digest as an adversary can always find a covered message after seeing the signatures on two different messages. Hence, we assume a message  $M$  is first hashed using a hash function  $H : \{0, 1\}^* \rightarrow \{0, 1\}^m$  to obtain a message digest  $M^*$  as in the previous sections. We start with a more formal model of the actual game.

**7.1 Existential unforgeability under signer-chosen message attacks.**

Asymptotically, existential unforgeability under signer-chosen message attacks (EU-SCMA) can be viewed as the worst-case complexity for EU-RMA. However, as we are interested in exact complexities for both – the signer’s attack and the forger’s attack – we describe a game between a signer  $\mathcal{S}$  and a forger  $\mathcal{A}$  to model this setting:

$$\begin{aligned}
 &\text{Game}_{\text{DSS}(1^n)}^{\text{EU-SCMA}}(\mathcal{S}, \mathcal{A}) \\
 &(\text{sk}, \text{pk}) \xleftarrow{\$} \text{kg}(1^n) \\
 &\{(M_i)\}_1^q \xleftarrow{\$} \mathcal{S}(\text{pk}, \text{sk}) \\
 &\{(\sigma_i, M_i)\}_1^q = \{(\text{sign}(\text{sk}, M_i), M_i)\}_1^q \\
 &(\sigma^*, M^*) \leftarrow \mathcal{A}(\text{pk}, \{(\sigma_i, M_i)\}_1^q) \\
 &\text{Return } 1 \text{ iff } \text{vf}(\text{pk}, M^*, \sigma^*) = 1 \wedge M^* \notin \{M_i\}_1^q.
 \end{aligned}$$

We measure the attack complexity of  $\mathcal{S}$  in the above game as his number of calls to  $H$ . In contrast to other games,  $\mathcal{S}$  does not have a success probability. In the game above, the  $\mathcal{S}$ ’s result is a corresponding attack complexity for  $\mathcal{A}$ , i.e. two times the minimum number of calls to  $H$  that  $\mathcal{A}$  has to make such that the game ends with output 1 with probability at least 0.5.

**7.2 Lamport’s scheme**

For Lamport’s scheme, it is important that two message digests  $M_1^*, M_2^*$  share as many bits as possible. That way, the message digest of any third message for which  $\mathcal{A}$  can compute a valid forgery has to comply with these two message digests on those bits.

Each bit of  $M_1^*$  equals the respective bit of  $M_2^*$  with probability 1/2, which means two messages have  $r$  equal bits with probability  $\binom{m}{r}2^{-m}$ . The probability that the digest of a random third message is covered by two such messages is  $(1/2)^r$  in this case, since for the  $m - r$  remaining bits all secret values are known. This naturally means the more pre-computation the signer is doing, the more computation has to be done to sign the third message.

Since the probability of two messages having  $r$  equal bits is symmetric around  $r = m/2$  bits, a malicious signer should always pick two messages such that  $r \geq m/2$ , which happens with probability  $1/2$ . This means even with this simple routine, the probability of a covering a third message is less than  $(1/2)^{m/2}$ , and for  $m$  large enough (for example  $m = 256$ ), this is out of reach even for strong attackers.

For a more general analysis, a pre-computation of hashing  $\tau$  messages will contain a pair of messages with  $r$  equal bits with probability bigger than  $1/2$  if  $\binom{\tau}{2} > \left(\binom{m}{r} 2^{-m}\right)^{-1} \ln(2)$ , which means we can estimate the pre-computation of the signer by  $\tau_S > \left(\binom{m}{r} 2^{-m}\right)^{-1/2}$ . The attack complexity of  $\mathcal{A}$  consists of making  $\tau_{\mathcal{A}} > 2^r$  calls to H in that case, since the adversary will then find a third message that can be covered with probability bigger than  $1/2$ .

### 7.3 Optimized Lamport

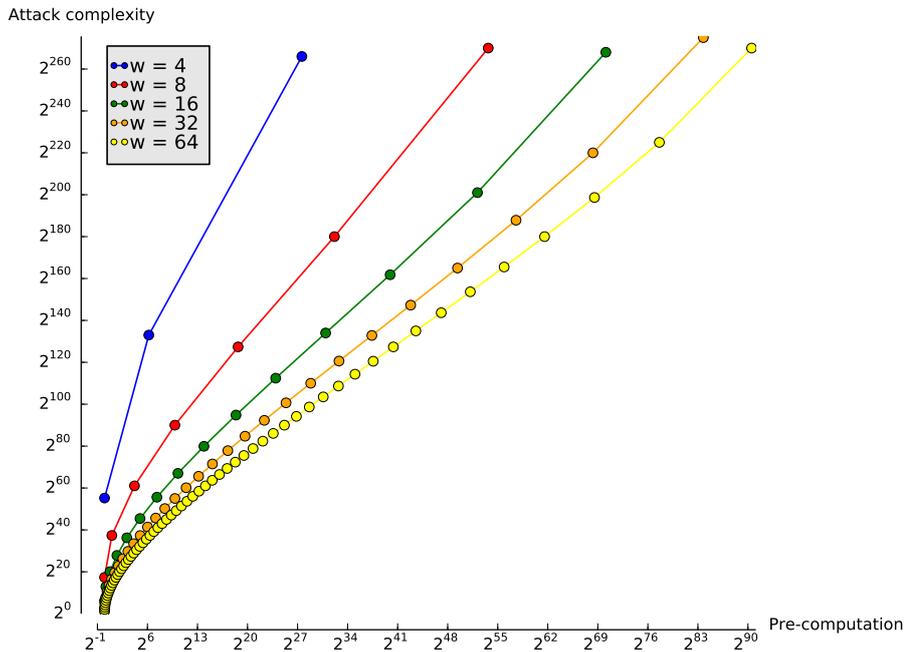
For the optimized Lamport scheme, a signer should pick two messages  $M_1, M_2$  with mappings  $B_1, B_2$  that share as many zero bits as possible, as each one bit leads to leaking the corresponding secret value. A covered message has to have zero's at all positions where both  $B_1, B_2$  were zero.

Unfortunately for the signer, a bit is only zero in both mappings  $B_1, B_2$  with probability  $1/4$ , which means two random message mappings have  $r$  positions where the bit of both  $B_i$  is zero with probability  $\binom{\ell}{r} (1/4)^r (3/4)^{\ell-r}$  where  $\ell = m + \log m + 1$ . The probability that a random third message is covered by two such messages is  $(1/2)^r$ , as in the case of Lamport, since the mapping of the third message has to have a zero bit at all the  $r$  positions where the two message mappings are zero. This time, the probability of two messages having  $r$  equal zero bits is symmetric around  $r = (\ell/4)$ , which means a malicious signer should always pick two messages such that  $r \geq \ell/4$ , again happening with probability  $1/2$ . This means also with this simple procedure, the probability of a cover is less than  $(1/2)^{\ell/4}$ , and for large  $m$ , for example  $m = 256$ , this is still out of reach for a strong attacker. More generally, a signer can precompute  $\tau_S$  messages such that  $\binom{\tau_S}{2} > \left(\binom{\ell}{r} (1/4)^r (3/4)^{\ell-r}\right)^{-1} \ln(2)$  to find two messages with probability bigger than  $1/2$ , where both messages have a zero at  $r$  positions. The attack complexity for the adversary  $\mathcal{A}$  can then be estimated by  $\tau_{\mathcal{A}} > 2^r$  calls to H.

### 7.4 Winternitz OTS

For WOTS, a signer would want to pick two messages  $M_1, M_2$  such that the minimal base- $w$  values of the respective message encodings are above a certain lower bound  $r \in \{1, \dots, w-1\}$ , where  $r$  should be picked as large as possible. Note that we exclude  $r = 0$  here, since it does not make sense for a signer to try to get all zero values as this would leak the entire secret key.  $\mathcal{A}$  can then only create a forgery if all base- $w$  values of the message mapping are above  $r$ . Note that even in those cases it is not necessarily possible as  $r$  was a lower bound. Analogue, but in reversed orientation, to the analysis of the universal forgery under a chosen message attack for WOTS, the success probability that the message mapping of a random third message is covered given below will be an *upper bound*.

The message mappings of two random messages  $M_1, M_2$  have no base- $w$  value less than  $r \in \{1, \dots, w-1\}$  for all  $\ell \approx \frac{m + \log m}{\log w}$  with probability  $\left(1 - \left(\frac{r-1}{w}\right)^2\right)^\ell$ . The probability to cover the message mapping of a random third message in that case is at most  $\left(\frac{w-r}{w}\right)^\ell$ . A pre-computation of  $\tau_S$  messages by the signer, will contain two random messages with base- $w$  values above  $r$  with probability bigger than  $1/2$  if  $\binom{\tau_S}{2} >$



**Fig. 7.** This plot shows the relation between the amount of pre-computation of a malicious signer and the computational cost for an adversary for WOTS. The costs are shown for different values of  $w$  and  $r \in \{1, \dots, w - 1\}$ .

$(1 - (\frac{r-1}{w})^2)^{-\ell} \ln(2)$ . The attack complexity for  $\mathcal{A}$  will be  $\tau_{\mathcal{A}} > (\frac{w-r}{w})^{-\ell} \ln(2)$  calls to  $H$ , since for that  $\tau_{\mathcal{A}}$  the adversary will find a covered message with probability bigger than  $1/2$ . In Figure 7, these values are given for different values for  $w$  and  $r \in \{1, \dots, w - 1\}$ . Note that the bigger the  $w$ , the closer the values move towards the middle, which means for larger  $w$ , it becomes harder for the signer to make forgeries expensive.

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## A Experiments for formal security notions

**SU-CMA.** Selective unforgeability is formally described by the following experiment. In this experiment  $\mathcal{A}$  consists of two independent algorithms  $(\mathcal{A}_1, \mathcal{A}_2)$ . The first of which,  $\mathcal{A}_1$ , outputs the target message and some temporary state  $\mathcal{S}$  that is forwarded to  $\mathcal{A}_2$ .

**Experiment**  $\text{Exp}_{\text{DSS}(1^n)}^{\text{SU-CMA}}(\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2))$

$(M_{\mathcal{A}}, \mathcal{S}) \leftarrow \mathcal{A}_1(1^n)$   
 $(\text{sk}, \text{pk}) \leftarrow \text{kg}(1^n)$   
 $\sigma^* \leftarrow \mathcal{A}^{\text{Sign}(\text{sk}, \cdot)}(\text{pk}, M_{\mathcal{A}}, \mathcal{S})$   
 Let  $\{(M_i, \sigma_i)\}_1^q$  be the query-answer pairs of  $\text{sign}(\text{sk}, \cdot)$ .  
 Return 1 iff  $\text{vf}(\text{pk}, M_{\mathcal{A}}, \sigma^*) = 1$  and  $M_{\mathcal{A}} \notin \{M_i\}_1^q$ .

**UU-CMA.** Universal unforgeability is formally described by the following experiment. The difference to the SU notion is that the target message  $M_{\mathcal{A}}$  is now selected by the experiment.

**Experiment**  $\text{Exp}_{\text{DSS}(1^n)}^{\text{UU-CMA}}(\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2))$

$(\text{sk}, \text{pk}) \leftarrow \text{kg}(1^n)$   
 $\mathcal{S} \leftarrow \mathcal{A}_1^{\text{Sign}(\text{sk}, \cdot)}(\text{pk})$   
 $M_{\mathcal{A}} \xleftarrow{\$} \mathcal{M}$   
 $\sigma^* \leftarrow \mathcal{A}_2(\mathcal{S}, M_{\mathcal{A}})$   
 Return 1 iff  $\text{vf}(\text{pk}, M_{\mathcal{A}}, \sigma^*) = 1$ .

**EU-RMA.** Existential unforgeability under random message attacks (EU-RMA) is defined using the following experiment. Instead of giving the adversary oracle access as in the EU-CMA game, the experiment generates signatures on  $q$  random messages and hands these to the adversary.

**Experiment**  $\text{Exp}_{\text{DSS}(1^n)}^{\text{EU-RMA}}(\mathcal{A})$

$(\text{sk}, \text{pk}) \leftarrow \text{kg}(1^n)$   
 Let  $\{(M_i, \sigma_i)\}_1^q$  be the set of  $q$  message signature pairs, obtained by  
 sampling  $M_i \xleftarrow{\$} \mathcal{M}$  and computing  $\sigma_i = \text{sign}(\text{sk}, M_i)$ .  
 $(M^*, \sigma^*) \leftarrow \mathcal{A}(\text{pk}, \{(M_i, \sigma_i)\}_1^q)$   
 Return 1 iff  $\text{vf}(\text{pk}, M^*, \sigma^*) = 1$  and  $M^* \notin \{M_i\}_1^q$ .

**SU-RMA.** Similarly to the previous notion, SU-RMA is defined by the experiment

**Experiment**  $\text{Exp}_{\text{DSS}(1^n)}^{\text{SU-RMA}}(\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2))$

$(M_{\mathcal{A}}, \mathcal{S}) \leftarrow \mathcal{A}_1(1^n)$   
 $(\text{sk}, \text{pk}) \leftarrow \text{kg}(1^n)$   
 Let  $\{(M_i, \sigma_i)\}_1^q$  be the set of  $q$  message signature pairs, obtained by  
 sampling  $M_i \xleftarrow{\$} \mathcal{M}$  and computing  $\sigma_i = \text{sign}(\text{sk}, M_i)$ .  
 $\sigma^* \leftarrow \mathcal{A}(\text{pk}, \{(M_i, \sigma_i)\}_1^q, M_{\mathcal{A}}, \mathcal{S})$   
 Return 1 iff  $\text{vf}(\text{pk}, M_{\mathcal{A}}, \sigma^*) = 1$ .

**UU-RMA.** Finally, universal unforgeability under random message attacks is formally described by the following experiment.

**Experiment**  $\text{Exp}_{\text{DSS}(1^n)}^{\text{UU-RMA}}(\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2))$

$(\text{sk}, \text{pk}) \leftarrow \text{kg}(1^n)$   
 Let  $\{(M_i, \sigma_i)\}_1^q$  be the set of  $q$  message signature pairs, obtained by  
 sampling  $M_i \xleftarrow{\$} \mathcal{M}$  and computing  $\sigma_i = \text{sign}(\text{sk}, M_i)$ .  
 $\mathcal{S} \leftarrow \mathcal{A}_1(\text{pk}, \{(M_i, \sigma_i)\}_1^q)$   
 $M_{\mathcal{A}} \xleftarrow{\$} \mathcal{M}$   
 $\sigma^* \leftarrow \mathcal{A}_2(\mathcal{S}, M_{\mathcal{A}})$   
 Return 1 iff  $\text{vf}(\text{pk}, M_{\mathcal{A}}, \sigma^*) = 1$ .