New Automatic search method for Truncated-differential characteristics

Application to Midori and SKINNY

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Abstract. In this paper, using Mixed Integer Linear Programming, a new automatic search tool for truncated differential characteristic is presented. While the previous MILP models for truncated differential characteristic has been used just as a facilitator for finding the maximal probability bit-wise differential characteristic, ours treats truncated differential characteristic as an independent distinguisher. our method models the problem of finding a maximal probability truncated differential characteristic, being able to distinguish the cipher from a pseudo random permutation. Our model enjoys a word-wise variable definitions which makes it much simpler and more easily solvable than its bit-wise counterpart.
Using this method, we analyse Midori64 and SKINNY64/64,128 block ciphers, for both of which the existing results are improved. In both cases, the truncated differential characteristic is much more efficient than the upper bound of (bit-wise) differential characteristic proven by the designers, for all number of rounds. More specifically, the highest possible rounds, for which a differential characteristic can exist for Midori64 and SKINNY64/64,128, are 6 and 7 rounds respectively, for which differential characteristics with maximum probabilities of 2^{-60} and 2^{-52} *may* exist. However, we present new truncated differential characteristics for 6-round of Midori64 with probability 2^{-54}. In case of SKINNY64/64,128, the gap is much wider, where for 7 rounds we find a truncated characteristic with probability 2^{-4}, and even a 10-round truncated characteristic can be found with probability 2^{-40}. Moreover, our result outperforms the only truncated differential analysis that exists on Midori64. This method can be used as a new tool for differential analysis of SPN block ciphers.

Keywords: Truncated Differential · MILP · SPN

1 Introduction

Truncated differential attack is a variant of differential attack introduced by Knudsen in 1994 [10]. Despite the basic differential attack, in which the precise bit-wise value of the input/output (and internal) differences are specified, in truncated differential cryptanalysis, one considers the word-wise differences where the word size can be a nibble, byte, etc., usually equal to the Sbox size in the design. Some examples of truncated differential attacks are [16, 11, 12].

Truncated differential characteristics can offer efficient distinguishers for block ciphers, even more efficient than their bit-wise counterpart, in some cases. A well-known instance is KLEIN block cipher, while its security against bit-wise differential attack had been proved in [9], it was broken by differential truncated attacks [11, 16].

However, while the designers prove upper bounds for bit-wise differential characteristics and greedily try to tighten it more and more, there is no provable method to measure

Table 1: Comparison of bit-wise differential and truncated differential characteristics for Midori64

Number of rounds	4	5	6	Ref.
Upper bound for bit-wise differential probability	2^{-32}	2^{-46}	2^{-60}	[4]
Truncated differential probability	$\mathbf{2^{-12}}$	$\mathbf{2^{-24}}$	$\mathbf{2^{-54}}$	**Sec. 4**

Table 2: Comparison of bit-wise differential and truncated differential characteristics for SKINNY64/64

Number of rounds	6	7	8	9	10	Ref.
Upper bound for bit-wise differential probability	2^{-32}	2^{-52}	2^{-72}	2^{-82}	2^{-92}	[5]
Truncated differential probability	$\mathbf{2^{-4}}$	$\mathbf{2^{-4}}$	$\mathbf{2^{-8}}$	$\mathbf{2^{-20}}$	$\mathbf{2^{-40}}$	**Sec. 4**

Table 3: Summary of 4-round truncated differential characteristics for Midori64

Probability	Method	Reference
2^{-44}	Moriai et. al. [13]	[7]
$\mathbf{2^{-20}}$	**MILP-based, Strategy I**	**Sec. 4**
$\mathbf{2^{-12}}$	**MILP-based, Strategy II**	**Sec. 4**

the strength of truncated differential characteristics for block ciphers. Apart from the lack of provable methods, there is not almost any systematic approach for finding efficient truncated differential characteristics. Except a meet in the middle-based method proposed in [12] for finding truncated differential characteristic, applied to CLEFIA and Camellia block ciphers and later to MCrypton and CRYPTON v.1 in [25].

On the other hand, since the variables are defined word-wise in truncated differential attack, the search space is not as large as that of bit-wise differential attacks. So, it is not too much infeasible to intelligently search the whole space to find the best possible truncated differential characteristic by an appropriate search tool. The only work focusing on this problem is a 2 decades old search algorithm proposed by Moriai et al. which was applied to E2 block cipher [13] and later to Midori cipher [7].

Mixed Integer Linear Programming (MILP) has been recently known as an effective automated tool for cryptanalysis of symmetric ciphers such as differential [22], impossible differential [17], zero correlation attacks [6] and integral attack [24]. The scope of MILP is not confined to SPN structure only, as there are innovative results in cryptanalysis of ciphers with ARX structure, mostly linear, differential, impossible differential and zero correlation attacks [8, 2, 3, 15].

In case of bit-wise differential cryptanalysis of SPN ciphers, the MILP-based characteristic search has progressed from the simple problem of counting the minimum number of active Sbox [14] into finding the precise maximal probability characteristic for some lightweight ciphers [21, 1]. MILP modeling of Sboxes which is the most challenging part of the MILP modeling of differential attack is now feasible even for 8×8 Sboxes [1]. However, as far as the truncated differential attack is concerned, the existing MILP models work up to finding a characteristic with minimum number of active Sboxes, supposed to be instantiated by a bit-wise characteristic later [21, 1]. Another trace of MILP modeling of truncated differential characteristic is seen in impossible differential characteristic search [18], where the worst case propagation of truncated differentials is modeled. So, there is not any MILP model concentrated on finding the optimal truncated differential characteristic, as an independent distinguisher with maximum probability.

Our contributions. This paper focuses on the problem of MILP modeling of truncated differential characteristic. In this model the variables are defined byte-wise, so the number of variables does not grow too fast as the number of rounds grows. Moreover, since truncated differential attack is irrelevant to the Sbox specifications, its MILP model is free of modeling Sboxes which was a bottleneck in its bit-wise counterpart. In this model, using an appropriately defined objective function, we can find the optimum truncated differential characteristic distinguishing the cipher from a pseudo random permutation (PRP) covering the most possible number of rounds with highest possible probability.

Having modeled the truncated differential characteristic, we examine our search tool on two remarkable SPN ciphers SKINNY64/64,128 [5] and Midori64 [4]. we observe that for both cipher, for all rounds that bit-wise differential characteristic works, the truncated differential characteristic has a probability higher than the upper bound of the bit-wise differential characteristic proven by the designers or third parties. For more details, see Tabs. 1,2. This shows that, beside the valuable efforts on finding and tightening the upper bound for differential probability, evaluating the strength of the cipher against other kinds of differential attack can be of considerable importance.

SKINNY has not ever been received any internal or external truncated differential analyses. However, for Midori64, some results have been reported on analyzing the cipher against truncated differential attack [13] in which the only other automatic search tool for truncated differential [13] is used. Despite the claim in [7], we found a more probable 4-round truncated characteristic and an efficient 5-round one. For more details see Tab. 3.

Organization of the paper. In Section 2, we bring the preliminaries of the paper, including a brief description of Midori and SKINNY block ciphersand a review of the MILP problem and related work. In Section 3, we present our new method for automatic search for truncated differential attack by MILP. In Section 4, we apply our automatic search to Midori and SKINNY where we bring and compare our results with previous ones. Finally, Section 5 concludes the paper.

2 Preliminaries

2.1 Midori Specifications

Midori is a lightweight SPN block cipher proposed in AISACRYPT 2015 [4]. It has two versions Midori64 and Midori128, with 64-bit and 128-bit block sizes, respectively, however both of which accept 128-bit keys. Both versions have a 128-bit secret key. The state of Midori block cipher is expressed as a 4×4 matrix as below:

$$\begin{pmatrix} s_0 & s_4 & s_8 & s_{12} \\ s_1 & s_5 & s_9 & s_{13} \\ s_2 & s_6 & s_{10} & s_{14} \\ s_3 & s_7 & s_{11} & s_{15} \end{pmatrix} \tag{1}$$

for Midori64 each cell of state matrix, $s_i, i = 0, \ldots, 15$, has 4-bit length (nibble) and for Midori128 each cell is 8-bit (byte). At first, a whitening key is added to the state matrix and then the result goes through round functions. Each round function of Midori consist of the following transformations:

- **AddRoundKey(AK)**: The subkey RK_r is added to the intermediate state.
- **SubCell(SC)**: The intermediate state goes through 16 S-boxes (the size of the S-boxes depends on the cipher variant).

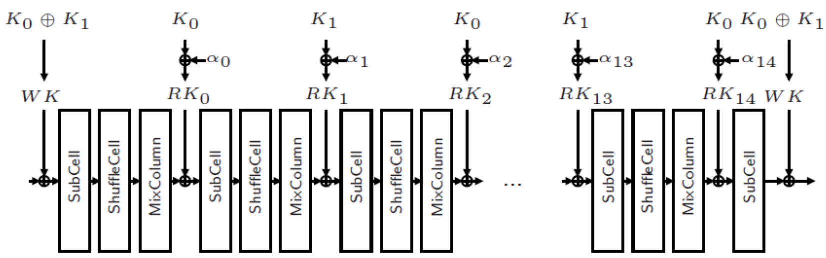

Figure 1: Overview of Midori64 [4]

- **ShuffleCell(ShC)**: The cells in the state are permuted as follows.

$$(s_0, s_1, ..., s_{15}) \longrightarrow (s_0, s_{10}, s_5, s_{15}, s_{14}, s_4, s_{11}, s_1, s_9, s_3, s_{12}, s_6, s_7, s_{13}, s_2, s_8) \quad (2)$$

- **MixColumn(MC)**: an almost MDS binary matrix M is applied to each column of intermediate state matrix

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

The S-box used in Midori64 is a 4-bit S-box, while for Midori128, it is a 8-bit one. Number of rounds for Midori64 is 16 and for Midori128 is 20. In the last round linear layers (SC and MC) are omitted and finally the whitening key is added to state matrix.

Midori128 doesn't employ any key schedule and for Midori64 the key schedule is very simple. For Midori64 the 128 bit secret key is divided into two 64-bit halves K_0, K_1, then the round key is determined by $RK_r = K_{rmod2}$. This keys are XORed with a round constant named α_r before AddRoundKey operation. Whitening key of Midori128 is the same as the secret key while for Midori64 it is $WK = K_0 \oplus K_1$. Fig. 1 shows an overview of Midori64.

2.2 SKINNY Specifications

SKINNY is a lightweight SPN cipher proposed in CRYPTO 2016. It takes Tweakey rather than secret key. SKINNY has many variants depending on the block and tweakey sizes which are detailed in [5]. However, in the following we describe SKINNY64/64 which is the version analyzed in this paper. All variants of SKINNY initialize plaintext in a 4×4 state matrix, then it goes through round functions. Each round function of SKINNY consists of five transformations of SubCell, AddConstant, AddRoundTweakey, ShiftRows and MixColumn. SKINNY does not have any whitening key.

- **SubCell(SC)**: The state matrix goes through 16 S-boxes.

- **AddConstant(AC)**: A round constant is added to the state matrix.

- **AddRoundTweakey(ART)**: The tweakey TK_r is added to the state matrix.

- **ShiftRows(SR)**: The second, third, and fourth cell rows are rotated by 1, 2 and 3 positions to the right, respectively. The first row remains the same.

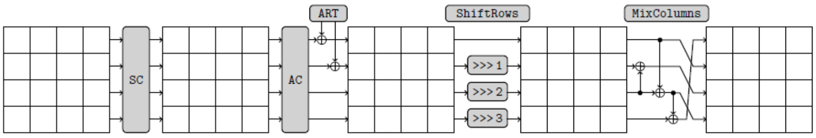

Figure 2: Overview of one round of SKINNY [5]

- **MixColumn(MC)**: A binary matrix M is multiplied by each column of intermediate state matrix

$$M = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \end{pmatrix} \tag{3}$$

Fig. 2 illustrates one round function of SKINNY.

2.3 MILP-based differential cryptanalysis

2.3.1 MILP definition

Linear Programming is a class of optimization problems in which the objective function and all constraints are linear functions in decision variables $\mathbf{x} = [x_1, x_2, \ldots, x_n]$. So, an LP problem is as the following form:

$$\begin{aligned} \text{minimize} \quad & \mathbf{c}^T\mathbf{x} \\ \text{subject to} \quad & A\mathbf{x} \leq \mathbf{b} \\ \text{and} \quad & \mathbf{x} \geq 0 \end{aligned} \tag{4}$$

when all or part of the decision variables are integer-valued, the LP problem is called MILP. An MILP problem is inherently NP-complete, however there are either commercial or open source solvers, which are able to solve some not-too-complicated instances of MILP. A recent trend in symmetric cryptanalysis is using this tool for finding (sub-)optimal (differential, linear, integral, etc.) characteristics in symmetric primitives.

2.4 Related work

In this section we review the developments of MILP-based techniques in differential cryptanalysis of SPN ciphers. The early work in this area belongs to [14, 23] which searches the minimum number of active S-boxes in SPN ciphers with word-oriented diffusion layers. In these models the differential property of the diffusion layer is taken into account up to its branch number while the information of S-boxes differential properties is not included. The next work [21] extends the coverage of this method to the SPN ciphers with bit-wise permutation diffusion layers, though with the same objective function and same limitations. A significant work is done in [20], where the differential properties of the S-box along with its probability are included in the model. This method enables the cryptanalyst to construct a more accurate feasible set for the MILP problem and set the objective function equal to the precise probability of the differential characteristic.

However, the method proposed in [20] for modeling S-box is effective for small S-boxes (at most 4×4). Abdelkhalek et al. developed a method for MILP modeling of large S-boxes (up to 8×8) and applied it to SKINNY block cipher and a AES-based MAC [1]. Besides improvements in differential attack, impossible differential characteristic search is also modeled by MILP in [17].

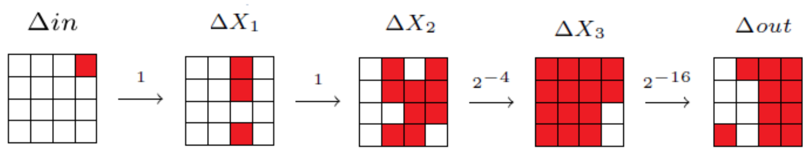

Figure 3: 4-round truncated differential characteristic by strategy I for Midori64.

Therefore there are lots of work in employing MILP technique for finding bit-wise differential characteristic for SPN ciphers, but this is not the case with truncated differential cryptanalysis. MILP modeling for truncated differential characteristic is just paid attention in [1] and [17]. In the former, truncated characteristic is treated just as a facilitator to find the optimal bit-wise differential characteristic. In more details, in [1], finding a minimal active S-box truncated differential characteristic was targeted, in order to be instantiated later by a bit-wise characteristic. In the latter, truncated characteristic is used as a tool for fining the impossible differential characteristic. So, a worst-case scenario for diffusion of the truncated differential characteristic is modeled and utilized for finding impossible differential characteristic.

3 New MILP-based automatic truncated differential search

Despite the bit-wise differential characteristics, there is not any systematic method for proving an upper bound for the probability of the truncated differential characteristic in SPN ciphers. Moreover, the only automatic search algorithm for finding optimal truncated differential characteristic is an exhaustive-type one dating back two decades [13].

In this section, we propose an efficient technique for MILP modeling the problem of searching truncated differential characteristic for SPN structures. Due to the word-wise essence of this attack, we use the word-wise variable definition in the MILP-model, as well, where the word size is equal to the S-box size in the cipher. In this model all the variables are binary, indicating that the associated word is active (1) or inactive (0).

It is clear that in any kind of (single-key) differential characteristic, XORing with constants, round keys and tweakeys are effectless on the characteristic. In addition, word-wise permutations, such as SubCell in Midori and ShiftRows in SKINNY can be modeled by a simple variable change. Moreover, as we adopted the word-wise variable definition, and assuming bijective S-boxes, the S-box layer would be totally bypassed in our model, hence the input and output of the Sbox are indicated by a single binary variable. so, the only layer that plays a decisive and key role in both the propagation pattern of the truncated characteristic and its probability is MixColumn layer.

3.1 MILP model for Diffusion Property of MixColumns

Suppose that the cipher state is a $k \times k$ matrix of m-bit words. So, the MixColumn matrix would be a $k \times k$ matrix M over $GF(2^m)$ and each round contains k parallel MixColumns. For a single MixColumn, the input and output truncated differential variables are denoted by $\mathbf{x} = (x_0, x_1, \ldots, x_k)^T$ and $\mathbf{y} = (y_0, y_1, \ldots, y_k)^T$, where $\mathbf{x}, \mathbf{y} \in (GF(2))^k$.

It is a straightforward task to compute the probability of all 2^{2k} truncated input/output differentials $P(\mathbf{x} \to \mathbf{y})$ and arranging them in a $2^k \times 2^k$ table called *branching property table* of M, whose rows are hexadecimal form of the input difference vector $\mathbf{x}$ and columns are the hexadecimal form of the output differences vector $\mathbf{y}$. [13] proposed a recursive method for computing these diffusion probabilities. Otherwise, they are computable by direct analysis or by means of a simple programming. Since the word size is m bits here, the possible values for the probability $P(\mathbf{x} \to \mathbf{y})$ fall into the range

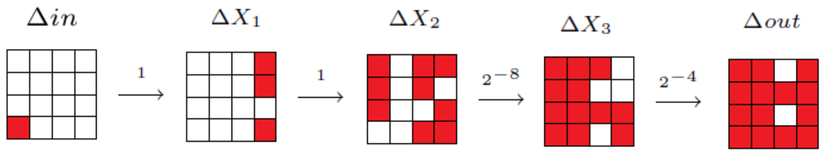

Figure 4: 4-round truncated differential characteristic by strategy II for Midori64.

$$\{0, 1, 2^{-m}, 2^{-2m}, \dots, 2^{-(k-1)m}\}. \tag{5}$$

The branching property tables of Midori and SKINNY MixColumns are shown in **Appendix A**. For more convenience, in the branching property table, the zero probabilities are shown by 0, the one probabilities are shown by 1, and the other probabilities $p \neq 0, 1$ are shown by $-log_2(p)$.

Those differentials with zero probability are called the impossible differentials that should be excluded from the feasible set of our MILP model. All the remaining differentials should be included in the feasible set, while their probabilities are encoded and defined as decision variables of the MILP model. The process is identical to the method of MILP modeling of differential property of (small) S-boxes [21]. We need to define at most $log_2(k)$ decision variables, denoted by $(p_0, p_1, \dots p_{[log_2 k]})$, to encode the probability of all possible differentials of the form $p = 2^{-m\sum_{i=0}^{k-1} p_i 2^i}$ For example for a 4×4 matrix over $GF(2^4)$, which is the case both with Midori and SKINNY, the possible probabilities are $\{0, 1, 2^{-4}, 2^{-8}\}$. The non-zero probabilities are encoded using $[log_2(k)] = 2$ bits (p_0, p_1) as follows.

$$\begin{aligned} 2^0 &\rightarrow (0,0) \\ 2^{-4} &\rightarrow (1,0) \\ 2^{-8} &\rightarrow (0,1) \end{aligned} \tag{6}$$

Finally, each possible differentials can be presented as a $(2k + [log_2(k)])$-tuple of the form

$$(x_0, x_1, \dots, x_k, y_0, y_1, \dots, y_k, p_1, \dots, p_{[log_2(k)]}) \tag{7}$$

Using SAGE computer algebra system [19], one can derive the convex hull of all possible vectors of the form (7). The number of inequalities can be reduced dramatically using the greedy algorithm introduced in [21, 20].

The objective function that should be maximized is the probability of the differential truncated characteristic, which is equal to the product of all MixColumns diffusion probabilities in the characteristic. Equivalently, suppose the cipher under analyze has r rounds, each round contains k MixColumn matrix. The objective function, supposed to be minimized, is defined as

$$P_T = \sum_{j=1}^{rk} \sum_{i=0}^{k-1} p_{j,i} 2^i \tag{8}$$

where $p_{j,i}, i = 0, \dots, k-1$ are the variables encoding the j^{th} MixColumn diffusion probability, $j = 1, \dots, k$.

3.2 Efficient Characteristics

3.2.1 Strategy I

A truncated differential characteristic is effective, not only if its probability is maximized, but also if it is able to distinguish the cipher from a Pseudo Random Permutation (PRP).

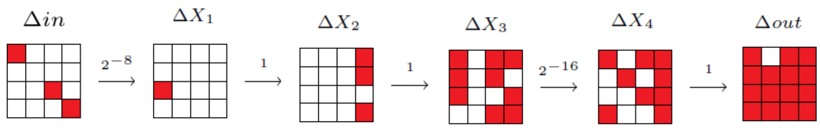

Figure 5: 5-round truncated differential characteristic by strategy II for Midori64.

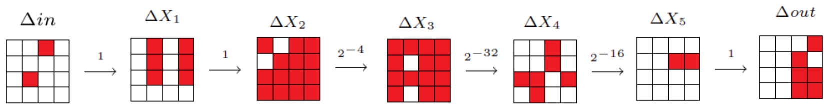

Figure 6: 6-round truncated differential characteristic by strategy II for Midori64.

Therefore, the probability of the truncated characteristic $P(\Delta_{in} \xrightarrow{E} \Delta_{out})$ must be greater than that of a PRP which is equal to $P(\Delta_{in} \xrightarrow{PRP} \Delta_{out}) = \frac{|\Delta_{out}|}{2^n}$, where n is the block size of the cipher in bits, here $n = k^2m$.

A raw way to count $|\Delta_{out}|$ is to count the number of active bytes in the output difference, i.e. $Hw(\Delta_{out})$. If so, $|\Delta_{out}|$ would be $mHw(\Delta_{out})$ as is used in [7]. Therefore the distinguishability condition in our models is translated into a new linear constraint of the following form

$$P_T > k^2 - Hw(\Delta_{out}) \tag{9}$$

However, as we will see in the following, this formula overestimates the $P(\Delta_{in} \xrightarrow{PRP} \Delta_{out})$, which makes the model incorrectly to exclude some efficient truncated characteristics due to constraint (9).

3.2.2 Strategy II

The objection of Strategy I is that this model is not able to take into account the possible *linear dependencies* that may exist between the active bytes of the output difference. Linear dependencies, caused due to the last linear layer, makes $|\Delta_{out}|$ smaller than $mHw(\Delta_{out})$. For precise enumeration of $|\Delta_{out}|$, one should actually counts the number of active bytes of the last non-linear layer output difference. So, in the MILP model, it suffices that the distingushability constraint (9) be modified as follows.

$$P'_T > k^2 - Hw(\Delta_{out}^{N.L.}) \tag{10}$$

where $\Delta_{out}^{N.L.}$ is the output difference of the last round nonlinear layer and P'_T is the probability of the truncated characteristic excluding the last round linear layer. Note that the objective function is defined still equal to P_T.

Although, such an approach finds the maximum probability truncated differential characteristic, this optimum characteristic may activates *all* the output differences. Even in this situation, the linear relations between the words of the output difference, along with (10), ensures that it is still an efficient distinguisher. However, a full state active output becomes challenging in the key recovery phase. Since, it typically demands a full subkey bit guess which definitely is not desirable. To avoid such a situation we add an extra constraint to the model, enforcing that not all the words in Δout are active, i.e. $Hw(\Delta_{out}) < k^2$.

4 Application to Midori64 and SKINNY64/64,128

In this section, we report new truncated differential characteristics we found by applying our proposed search method to two lightweight block ciphers Midori64 and SKINNY64/64,128.

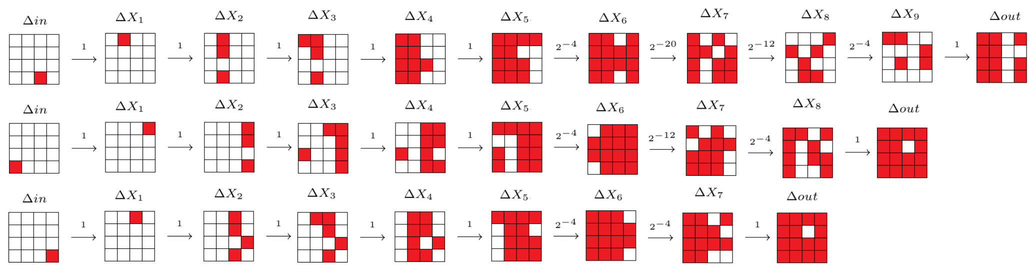

Figure 7: 8,9,10-round truncated differential characteristics by strategy II for SKINNY64/64.

4.1 Midori64 truncated differential characteristics

According the method explained in Sec. 3.2, we first model the truncated differential property of the MixColumn of Midori64. The branching property table of Midori64 MixColumn is shown in **Appendix A** which can be modeled by ?? linear constraints.

4-round characteristic by Strategy I. In order to compare the MILP-based search method with the other automatic search method [13], which was applied on Midori64 in [7], we first examine the less efficient strategy, Strategy I, which is the strategy chosen in [7], too. We found a 4-round characteristic with probability 2^{-20} shown in Fig. 3, while the highest-probability 4-round characteristic introduced in [7] has probability 2^{-44}.

4,5,6-round characteristic by Strategy II. Fig. 4, shows the 4-round characteristic which is found using Strategy II. It has a probability of 2^{-12}. This characteristic demonstrates the large gap that exists between these two strategies. The optimum 5-round characteristic for Midori64 is shown in Fig. 5. This characteristic has a probability of 2^{-24}. The highest possible number of rounds for which truncated distinguishers can be proposed, is 6 rounds for Midori64. This characteristic has probability of 2^{-52} and is shown in Fig. 6. As it can be compared in Tab. 2, in any rounds that an efficient bit-wise differential characteristic may exist, there is a more efficient truncated differential characteristic, with higher probability.

4.2 SKINNY64/64 truncated differential characteristics

The branching property table of SKINNY is shown in **Appendix A** which can be modeled using ?? linear inequalities. The branch number of SKINNY MixColumn is smaller than Midori, expectedly resulting in a more sparse branching property table than Midori. In the following, we report our results on 7,8,10 rounds of SKINNY64/64,128 which is considerably much more effective than midori, covering more number of rounds with larger probabilities.

8,9,10-round characteristic by Strategy II. SKINNY64/64,128 has not ever been analyzed by truncated differential attack. Using Strategy II, we found efficient 8,9,10-round distinguishers for SKINNY64/64,128. These characteristics are shown in Fig. 7.

5 Conclusion

We proposed a new MILP-based automatic method for truncated differential characteristic search for SPN block ciphers. The proposed MILP models maximizes the truncated differential characteristic probability, ensuring its distinguishability from PRP, and avoiding all output words being active. We applied our proposed method to Midori64 and SKINNY64/64,128. Comparing to its bit-wise counterpart, truncated differential attack enjoys a much smaller and less complicated Sbox-free MILP model, which makes it much more faster to be solved. More importantly, our results on Midori64 and SKINNY64/128 shows that the optimal truncated differential characteristics found by this method are much more efficient than the upper bound of bit-wise differential characteristics proven for these two ciphers. This method can be used as a new tool for differential analysis of SPN ciphers.

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Appendix A.

Tabs. 4 and 5 illustrate The branching property tables of Midori64 and SKINNY64/128 MixColumns.

Table 4: Differential property table of Midori64 Mixcolumn

in/out	0x0	0x1	0x2	0x3	0x4	0x5	0x6	0x7	0x8	0x9	0xa	0xb	0xc	0xd	0xe	0xf
0x0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0x1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
0x2	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
0x3	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	1
0x4	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
0x5	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	1
0x6	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	1
0x7	0	0	0	0	0	0	0	4	8	0	0	4	0	4	4	1
0x8	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0x9	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	1
0xa	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	1
0xb	0	0	0	0	8	0	0	4	0	0	0	4	0	4	4	1
0xc	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	1
0xd	0	0	8	0	0	0	0	4	0	0	0	4	0	4	4	1
0xe	0	8	0	0	0	0	0	4	0	0	0	4	0	4	4	1
0xf	0	0	0	8	0	8	8	4	0	8	8	4	8	4	4	1

Table 5: Differential property table of SKINNY64/64 Mixcolumn

in/out	0x0	0x1	0x2	0x3	0x4	0x5	0x6	0x7	0x8	0x9	0xa	0xb	0xc	0xd	0xe	0xf
0x0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0x1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
0x2	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
0x3	0	0	0	4	0	0	0	0	0	0	0	1	0	0	0	0
0x4	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0x5	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
0x6	0	0	0	0	0	0	0	0	0	4	0	1	0	0	0	0
0x7	0	8	0	4	0	0	0	0	0	4	0	1	0	0	0	0
0x8	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
0x9	0	0	0	0	0	4	0	0	0	0	0	0	0	1	0	0
0xa	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	1
0xb	0	0	0	0	0	0	0	4	0	0	0	0	0	0	4	1
0xc	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0xd	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	1
0xe	0	0	0	0	8	0	4	0	0	0	0	0	0	4	0	1
0xf	0	0	0	0	0	8	0	4	0	0	0	0	8	4	4	1