

Machine Learning of Physical Unclonable Functions using Helper Data

Revealing a Pitfall in the Fuzzy Commitment Scheme

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Abstract. Physical Unclonable Functions (PUFs) are used in various key-generation schemes and protocols. Such schemes are deemed to be secure even for PUFs with challenge-response behavior, as long as no responses and no reliability information about the PUF are exposed. This work, however, reveals a pitfall in these constructions: When using state-of-the-art helper data algorithms to correct noisy PUF responses, an attacker can exploit the publicly accessible helper data and challenges. We show that with this public information and the knowledge of the underlying error correcting code, an attacker can break the security of the system: The redundancy in the error correcting code reveals machine learnable features and labels. Learning these features and labels results in a predictive model for the dependencies between different challenge-response pairs (CRPs) without direct access to the actual PUF response. We provide results based on simulated data of a k -SUM PUF model and an Arbiter PUF model. The analysis reveals that especially the frequently used repetition code is vulnerable: Already the observation of 800 challenges and helper data bits suffices to reduce the entropy of the key down to one bit in this case. The analysis also shows that even other linear block codes like the BCH, the Reed-Muller, or the Single Parity Check code are affected by the problem. The code dependent insights we gain from the analysis allow us to suggest mitigation strategies for the identified attack. While the shown vulnerability brings Machine Learning (ML) one step closer to realistic attacks on key-storage systems with PUFs, our analysis also allows for a better understanding and evaluation of existing approaches and protocols with PUFs. Therefore, it brings the community a step further towards a more complete leakage assessment of PUFs.

Keywords: Physical Unclonable Function · PUF · Machine Learning · Supervised Learning · Fuzzy Commitment Scheme · Fuzzy Extractor · Error Correcting Code · Neural Network · Key Storage · Key Distribution

1 Introduction

An ever increasing amount of embedded devices is in need of cryptographic keys to ensure security. Small and low-cost devices in particular struggle with this demand due to cost reasons. Therefore, Physical Unclonable Functions (PUFs) are appealing solutions in this domain. In comparison to traditional secure non-volatile memory (NVM), silicon PUFs [GCVDD02] can store keys at a more affordable price and even at small technology nodes, where no secure on-chip NVM is available. Additionally, PUFs are not susceptible to attacks when powered off. This implies possible power savings when compared to key-storage solutions that require permanent power, e.g., for sensors.

Single-Challenge PUFs (SCPUFs), like SRAM PUFs [GKST07], as well as Multi-Challenge PUFs (MCPUFs), like Arbiter PUFs [GCVDD02], are used to store secret keys (cf. Section 2). In both cases, the PUF measures device-unique manufacturing variations. For MCPUFs, which are of particular interest in this work, the PUF response additionally depends on a challenge determining the specific configuration of the PUF.

Thermal noise, environmental changes, aging, and other effects cause noise in the measured PUF response. Correction of the resulting error is enabled through Helper Data Algorithms (HDAs) which map the PUF response to a codeword of an Error-Correcting Code (ECC). For this process helper data (HD) is generated and stored. Reproduction of the correct secret is only possible when knowing HD and the – possibly noisy – device-specific PUF response. For sufficiently high entropy in the PUF response and an appropriately selected HDA, the secret key is reproduced without any secrecy leakage [PHS17]. However, practically implemented PUFs do not provide full entropy responses. A common practice is to hash the secret from the PUF to a shorter key.

Problem statement: While the secret is deemed secure by applying such mechanism, barely any attention is paid to another problem: *Helper data leak about the PUF response*. We call this phenomenon privacy leakage through HD. It is in particular critical when the secret is stored by means of an MCPUF. The impact of this effect is widely underexplored and not considered in several key-storage scenarios and key-exchange schemes with PUFs. Privacy leakage allows us to model MCPUFs through ML based solely on publicly known challenges and HD. Different from previous ML attacks on PUFs, neither reliability information nor PUF responses are required. Our work highlights the need for using (i) appropriate PUFs and (ii) secure ECCs as well as the need of (iii) limiting the amount of CRPs also in key-storage scenarios with MCPUFs.

Contribution: This work provides insights to the security implications of privacy leakage through HD. While our observations might impact other scenarios, we consider the case of key-storage from MCPUFs when applying Fuzzy Extractors (FEs), and in particular the Fuzzy Commitment Scheme (FCS) as HDA and linear codes as ECCs. For this purpose, we propose a new concept to apply ML to such a system and successfully demonstrate an attack by selecting appropriate ML algorithms. The attack is inspired by the attack on key-storage systems that use pointer based approaches as HDA in combination with MCPUFs [BWG15]. But we take a completely different approach to any previous work. As a result, we do not need to know any PUF responses or any kind of reliability information to predict the secret key. Instead we can model the PUF given the public challenges and HD alone. This is possible by exploiting the inherent dependency of different bits within any codeword of an ECC in combination with the inherent dependency of CRPs of all known MCPUFs.

Our results refute the state of the art, which assumes that for key-storage systems with MCPUFs a high level of modeling resistance can be expected [Del17]. Our analysis rather proves, that the modeling resistance strongly depends not only on the PUF’s quality but also on the used ECC and the amount of published challenges and HD. This aspect has been left out of consideration in previous works. Overall, the contribution of this work is:

- We provide a detailed and formalized presentation of a new ML attack on MCPUFs in key-storage scenarios. The approach is very generic and applicable to various PUF settings.
- We prove the feasibility of the attack for two different scenarios. (i) We demonstrate that this attack is especially devastating for repetition codes. Hence most schemes based on concatenated codes are affected. (ii) To further show the general impact, we apply the attack to a BCH code, a Single-Parity-Check code, and a Reed-Muller

code. This proves that even more complex linear block codes ultimately can be targeted. For the experiments, we use simulated – i.e. ideal – SUM and Arbiter PUFs. More complex PUFs are not in use. While the attack indeed will become more difficult for such PUFs, we consider the increase in learning effort to be similar to the one observed in existing ML attacks on challenge-response protocols.

- Thus, we (i) falsify the assumption in the state of the art, that key-storage systems with MCPUFs have an inherent high modeling resistance. In fact, we (ii) show, that the learnability strongly depends on the used ECCs and (3) motivate, which properties of a code make ML harder.
- Finally, we discuss the findings of our analysis and sketch appropriate means to counter the analyzed ML attack.

Outline The remainder of this work is structured as follows: Section 2 provides required background regarding PUFs and discusses the state of the art regarding ML attacks on PUFs. It is followed by a description of the attack scenario in Section 3. The attack is formally defined in Section 4; Results for the practical application of the attack are provided in Section 5. Section 6 discusses the theoretical and practical findings. We conclude our work in Section 7.

2 State of the Art

This section is split into two parts. First we present the general concepts of PUFs. In particular, we focus on previous works which utilize MCPUFs for key-storage, because in the light of our analysis their security has to be re-evaluated. Second, we give an overview of ML attacks on PUFs in general and in particular on related ML attacks targeting key-storage. This shows, that our novel attack takes a completely different approach in comparison to existing work and advances state of the art.

2.1 Physical Unclonable Functions

The basic PUF Concept A silicon PUF [GCVDD02] exploits manufacturing variations of embedded devices to extract device-specific randomness in the form of a PUF response. This randomness is hard to predict and, thus, provides a unique fingerprint for each device. The fingerprint is mainly used for authentication or to derive a device-specific secret key.

A wide range of different PUF primitives currently exists [MBW⁺19] with more to come based on ongoing research. A fundamental difference between suggested circuits used as PUF primitives is that some only take advantage from manufacturing variations while others additionally allow a challenge to configure the circuit. Consequently, we classify PUFs in accordance with the notation in [WGP18] as either SCPUFs or MCPUFs:

- **Single-Challenge PUF:** This kind of PUF uses a measurement circuit, that cannot be configured by a challenge. Commonly known examples are SRAM [GKST07], ring-oscillator (RO) [MCMS10], and Startup-Value-Based DRAM [TKYC17] PUFs. The use of such PUFs results in an area per bit that scales linearly with the amount of response bits. Therefore, such PUFs are suited to derive few device-specific secret bits. SCPUFs are not suitable for most PUF based challenge-response protocols or to derive a large number of keys, as it is needed in several key-exchange protocols, because their limited amount of responses per device makes them susceptible to replay attacks.
- **Multi-Challenge PUF:** This kind of PUF accepts multiple different challenges, which configure the underlying PUF primitive. This results in multiple different

but probably correlated PUF responses for the same PUF primitive. Yet by careful design choices, storage of multiple keys or performing multiple challenge-response authentications is possible with the same PUF primitive. Examples for such MCPUFs are the Arbiter PUF [LLG⁺05], the k -SUM PUF [YD10a], the Bistable Ring PUF (BR PUF) [CCL⁺11], the Twisted Bistable Ring PUF (TBR PUF) [SH14] or more advanced designs like the interpose PUF (iPUF) [NSJ⁺18]. In comparison to SCPUFs, MCPUFs offer a broader range of suitable applications due to their large challenge-response space.

The definitions of MCPUFs we use are not equivalent to the concept of Weak and Strong PUFs: While it is no consistently used term, Strong PUFs are according to [RH14] PUFs that remain unpredictable, even if a large set of CRPs of the PUF is known. No currently known MCPUF is a Strong PUF in this sense given an arbitrary large number of CRPs. Weak PUFs, in contrast, accept according to [RH14] only few challenges, which does also not apply to many existing MCPUFs.

In this work, we target – similar to previous ML attacks discussed in Section 2.2 – MCPUFs, i.e., PUFs that accept multiple challenges but can potentially be machine-learned. We now introduce the k -SUM PUF and the Arbiter PUF, because we substantiate our analysis in Section 5 with these PUFs.

k -SUM PUF [YD10a]: The basic building blocks for a k -SUM PUF are k pairs of ROs. Each RO consists of an even number of inverters as well as an NAND-gate for the enable signal. When two ROs are enabled simultaneously, two counters measure the RO frequencies. The counter values are then subtracted. Hence, each pair of ROs results in a difference of counter values. To achieve MCPUF behaviour, multiple differences are added where for each summand the sign is flipped or not depending on the challenge. The sign of the overall sum for a specific challenge corresponds to one PUF response bit.

Arbiter PUF [LLG⁺05]: The Arbiter PUF derives a bit based on a race condition between two paths which are designed to be equally long. Due to manufacturing variations, one of the signals has a larger delay than the other. An arbiter at the end of the paths outputs a logical 0 or 1 depending on which path is faster. Challenges are applied to the Arbiter PUF through switch boxes in the paths. Each switch box decides based on a challenge bit if an incoming signal is either forwarded from input to output or crossed over.

For both k -SUM PUF and Arbiter PUF – and equivalently for most other MCPUFs – one challenge results in one PUF response bit. To generate multiple response bits, multiple challenges ch_i are required. These challenges are typically deterministically derived from a challenge seed ch , e.g., using a Linear Feedback Shift Register (LFSR). This allows to produce multiple bits while reducing the overhead of transmitting or storing challenges. In a key-storage scenario, challenge seed and challenge derivation function are considered to be publicly known. In the visualization of the key-storage scenario in Figure 1, we therefore simplify the challenge generation process to a *Challenge Storage*.

Background on Helper Data Algorithms PUF primitives output responses that are derived from measurement results. Consequently, due to effects like thermal noise, changing environmental conditions or aging, a PUF response at two instances of time can differ, i.e., the response is noisy. In the context of authentication, noisy PUF responses can be resolved on a protocol level by tolerating a certain amount of errors [YMVD14, MRK⁺12]. However, if the PUF would be used as a key directly, any error in the response would render the key useless. Therefore, post-processing is needed to derive a key from the PUF response. The most common concept for key-storage with MCPUF and FCS is depicted in Figure 1; The respective modules of this figure are explained in the following. Overall, during *roll out* of a device, a secret s is selected. This secret is encoded to a codeword c . A set of challenges is generated to derive a sufficiently long PUF response r . By XORing PUF response and codeword, the codeword gets masked. The resulting masked codeword

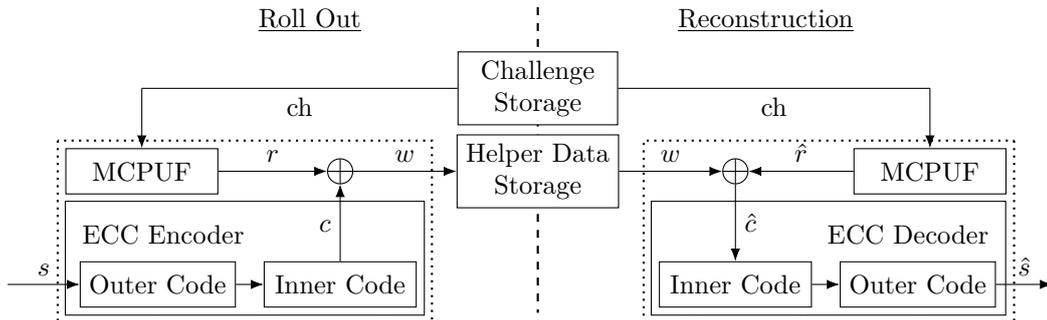


Figure 1: Visualization of the key-storage scenarios with PUFs: At some point in time – during *Roll Out* – a key is stipulated for the device. Later – during *Reconstruction* – the key is derived from the noisy PUF, given public helper data and challenges.

is stored as public HD. Whenever the secret is reproduced later in a *reconstruction* phase, a noisy version of the PUF response \hat{r} is reproduced by the same challenges as during roll out. XORing the response with the corresponding HD un.masks the codeword. Due to noise in the PUF response, the unmasking is not perfect, i.e., errors remain in the codeword. Nevertheless, given an appropriate ECC design, the codeword is decoded to the correct secret $\hat{s} = s$ with high probability.

More generally, the HD in this process are generated by a so called HDA. It generates the HD during *roll out* from a reference PUF response. During the *reconstruction* phase it maps PUF responses and HD to a noisy codeword that can be corrected. This work focuses on two state-of-the-art approaches for HDAs: the already mentioned Fuzzy Commitment Scheme [JW99] and the Code-Offset Fuzzy Extractor [DRS04]. But our concept is applicable to other approaches like the Syndrome Construction [DRS04] or Systematic Low Leakage Coding (SLLC) [HYP15]. Pointer based HDAs like Index Based Syndrome Coding (IBS) [YD10b], Complementary IBS (C-IBS) [HMSS12], and Differential Sequence Coding (DSC) [HYS16] are not considered, as previous work has indicated security vulnerabilities for our scenario, like we discuss in Section 2.2.

Background on Error Correcting Codes For key-storage with PUFs, ECCs are required. As a common design criterion, the error correction capability has to ensure that despite a high bit error probability in the PUF response, a sufficiently long key can be derived with negligible error probability. Previous works like [MSSS11] assume, e.g., error probabilities of 15% for each bit in the PUF response and derive a 128 bit key with an error probability of below 10^{-6} . To ensure a reasonable trade-off between coderate and implementation cost, a widespread approach is to use concatenated codes. Bösch et al. have introduced this concept to the PUF context in [BGS⁺08]. Instead of using one large code, which has high implementation costs, the task of error correction is managed by two concatenated smaller codes. In simplified terms, the inner code has a low rate (i.e. lots of redundant information) to correct the incoming high bit error probability to a moderate one. The outer code processes the output of the inner code to achieve the desired probability to correctly derive a key. A straight forward form of this concatenation method is also sketched in the ECC Encoder and ECC Decoder in Figure 1.

Except for polar codes as in [CIW⁺17], the predominant choice for linear block codes in a PUF context is a code concatenation which uses a repetition code [MTV09, BGS⁺08, PFFS19, MVHV12]. The repetition code has two main advantages, which justify its popularity: (i) It is easy to implement and (ii) it can deal well with the relatively high bit error rate of the PUF response [DMV02]. Reed Muller codes have also been used as

inner codes [HKS⁺15, MTV09]. While there are more possibilities for an outer code, BCH Codes are often used, as they can be implemented well in hardware. We will see in the results of our analysis that the selection of the code strongly influences the learnability of the PUF in our new approach.

State of the Art in Key-Storage based on Multi-Challenge PUFs The concepts from the previous subsections form a basis for several state-of-the-art applications of MCPUFs such as [PFFS19, Del17, TŠ07]. They all derive multiple keys from a noisy PUF by means of HD, e.g., for a PUF-based protocol. Many existing approaches are analyzed in [Del17].

In a very basic sense, such schemes can be represented by Figure 1, while the roll out phase is now interpreted differently: We consider two entities, one called server and the other one is the device with the PUF. In an enrollment phase, the server collects multiple CRPs from the PUF. With the knowledge about these responses, the server can generate HD that result in a specific key on the device. Whenever the server and the PUF device want to derive a new key, e.g., used for one round of authentication, the server provides the device with a challenge seed and the corresponding HD. The server knows the reference PUF response as a result of the enrollment. The device can correct its noisy version of the original PUF response by utilizing the HD. When both entities now have the same secret key, they can authenticate each other or send encrypted messages. In some protocols also fresh CRPs are transferred over this encrypted channel. While this application is allegedly secure, our analysis shows the potential vulnerability of this scheme.

2.2 Machine Learning Attacks on PUFs

We discussed above that MCPUFs have been used in several applications for authentication and key-exchange. In particular, several ML attacks on PUFs used in authentication protocols have been presented in the last years. These attacks are possible, since until today no strong PUF according to the definition in [RBK10] is known. Especially the property, that for all current MCPUFs different sufficiently large sets of CRPs carry mutual information about each other, is exploited by ML attacks on PUFs. Thus, the response sequence collected from an MCPUF does not have full entropy if the challenges are not selected carefully. This is discussed in [RSGD16] for the LOOP PUF; a result that is transferable also to other PUFs, for which a linear model can be found.

Linear models, however, exist for all popular basic MCPUFs types: The linear model for the k -SUM PUF follows directly from its definition; linear models that result from transformations or from the interpretation of physical peculiarities were presented for Arbiter PUF [LLG⁺05], BR PUF [SH14], and TBR PUF [XRHB15]. One approach to prevent ML attacks, and – imputing the learnability of all current MCPUFs – the only really secure approach in the light of our analyses, is limiting the number of challenges used with the MCPUF like suggested in the Hadamard code based challenge selection in [RSGD16] or the lockdown protocol in [YHD⁺16]. But this severely limits the amount of secret bits that can be derived from MCPUFs.

Since the limitation of CRPs strongly reduces the efficiency of MCPUFs and the practical applicability of several protocols, one can try to improve the statistical properties of MCPUFs in order to make ML more difficult. The efforts in this direction started with the recombination of multiple PUF responses like suggested in [SD07] and resulted in today's sophisticated constructions like the iPUF [NSJ⁺18].

However, for every MCPUF and for every approach to prevent ML attacks in the context of challenge-response protocols, sooner or later a successful attack has been demonstrated. The attackability of, among others, not only standard Arbiter PUFs but also its improvements, namely of Feed-Forward Arbiter PUFs and XOR-Arbiter PUFs, was shown in [RSS⁺10] and further substantiated in [RSS⁺13]. Even worse, the learnability of the Arbiter PUF with polynomial number of CRPs was shown under mild assumptions

in [GTS16]; the proof was extended to XOR-Arbiters PUFs in [GTS15]. In [GTFS16], the author of [GTS16, GTS15] also notes that not all challenges have the same influence on the response so that an ML algorithm only has to learn the important characteristics of the PUF.

If reliability information of the XOR Arbiters PUF's responses is available, [Bec15] showed that the required amount of CRPs increases only linearly with the number of XORs. But not only ML attacks on such early constructions have been presented. Also for more complicated structures, the feasibility of ML was demonstrated [Del19], e.g., an approach to learn the IPUF was provided in [WMP⁺19].

To overcome the learnability issue of MCPUFs, several protocols like Slender PUF protocol [MRK⁺12] or the noise bifurcation protocol [YMVD14] have been proposed. But similar to the efforts to strengthen MCPUFs on design level, for every new protocol an appropriate ML attack has been presented after some time. A summary of the strength of several protocols is provided in [Del17]. However, as a conclusion of our work, we disagree with the statement in [Del17, Table 5.1] regarding modeling resistance of approaches that use challenges together with HD. Under the suggested use of an MCPUF, this affects in particular *Reference protocol II-A* and *Reference protocol II-B* in [Del17] as well as the protocols [GCVDD02] and [SVW10] in the form they are provided in [Del17]. According to our findings, these and other protocols and key-storage mechanisms using MCPUFs and HDAs are only model-resistant if the CRP number is limited to an amount, which ensures that the underlying PUF cannot be learned.

Since we focus on MCPUFs in a key-storage scenario, modeling attacks are of particular interest. Only few attacks in this domain are known. Most interestingly, [BWG15] provides an attack on IBS [YD10b], that is also applicable to C-IBS [HMSS12]. The attack shows, that the reliability information implicitly stored when using pointer based HDAs, can be used to derive the secret key from a system that uses an MCPUF for key-storage. The only prerequisite in this case is the knowledge about the challenges and the pointer, i.e., the HD. We expect that this attack can also be applied to other pointer based approaches like DSC [HYS16].

Therefore, we analyze the learnability of MCPUFs in a setting where some sort of FE is used. In this context, [RJA11] discusses a theoretical attack on a key-exchange protocol by P. Tuyls and B. Skoric [TŠ07]. However, there are two notable assumptions in this paper: First, a Strong PUF in the narrow sense of a non-learnable PUF is assumed. Second, it is assumed that the attacker has direct access to the PUF. Due to the first assumption the authors of [RJA11] do not discuss ML. However, since they assume direct access to a Strong PUF, they have direct access to CRPs, making ML possible as soon as the PUF becomes in fact a practically existing MCPUF, i.e., learnable. Due to the assumption of direct access to CRPs, ML becomes straight forward in this case. By applying our approach, we can further reduce the required access to only the HD and challenges like we discuss below. This also turns the useability of the erasable PUF as the suggested countermeasure in [RJA11] into question if such a PUF can be learned.

2.3 Conclusions from the State of the Art

The state of the art shows that MCPUFs are used in several protocols, key-exchange, and key-storage scenarios. Today, such PUFs can be expected to be learnable if an attacker has access to reliability information of MCPUF responses or the responses themselves. This has been proven by different theoretical works and practical ML attacks. However, hardly any risk has been identified regarding ML, when such PUFs are used in settings where no such information is revealed to an attacker. In the following we show, that even then ML of PUFs is an issue, and propose a strategy on how a potential attacker might exploit the information leakage through helper data to enable ML. We will see, that in the generalized case for some state-of-the-art FEs and some linear codes, the problem reduces

for an attacker to the task of learning a PUF with XORed response bit.

3 Attacker Model and Attack Scenario

In this work we focus on scenarios where an MCPUF is used to derive a secret key. Typical cases are the storage of secret keys, where MCPUFs are used for efficiency reasons, or key-exchange protocols, which take advantage of the challenge-response behavior of MCPUFs to derive multiple keys per device. The scenarios have in common that a PUF-specific set of challenges and HD is used to reproduce a key on a certain device. Challenges – or a challenge seed ch from which all challenges ch_i are deterministically derived – as well as HD w are either loaded from memory or transmitted to the device on demand.

3.1 Attacker Knowledge

Since a PUF is involved, we consider HD and challenges as publicly known data. This assumption is well motivated: The amount of data required for w and ch is always larger than the key. Therefore, the use of a PUF implies that no sufficiently secure memory for key-storage, namely storage that provides protection against external read-out, is available on the device; Otherwise it is more efficient to store the key directly instead of using a PUF. To prevent access to w and ch , data can be transmitted in an encrypted way if a master-key is stored on the device, e.g., by means of an SCPUF or protected memory. But in this case an encrypted session key might be transmitted directly so that no MCPUF is needed. Storing the master key with the MCPUF boils down to the previously discussed case of storing any key with an MCPUF.

In addition to the public w and ch we assume in accordance with Kerckhoffs’s principle, that the PUF architecture is publicly known. I.e., the attacker knows the used PUF type, a possible function to derive all used challenges from a challenge seed, the HDA, and the ECC. According to Section 2.2, we assume learnability of the PUF and restrict ourselves to the case of linear ECCs.

With the previous assumptions, our attacker is a *passive eavesdropper* who reads HD and challenges from public memory or a public channel. The attacker cannot read PUF responses directly. Hence, the attacker in this work has fewer prerequisites than in related ML attacks on PUFs. We assume that in cases where direct access to the PUF is necessary, e.g., for a server-side model of the MCPUF, the access can be removed with reasonable effort and in a sufficiently secure way, e.g., by blowing a fuse. If the attacker has direct access to PUF responses, either the task of ML the MCPUF is equivalent to attacks on authentication protocols or no ML is required at all.

The attacker in our setting also does not need to manipulate HD or challenges. I.e., prevention of HD and challenge manipulation does not help against the identified pitfall. The capability to trigger re-enrollment of the key with different challenge and HD is a useful feature for an attacker, since it provides her with an arbitrary number of appropriate challenges and HD, but it is no strict requirement.

Although it is beneficial for an attacker, we exclude Side-Channel Analysis (SCA) capabilities to gain information about the PUF like in [KCG⁺20, MSSS11, TPS17, TPI19] as well as Fault-Injection Analysis (FIA) from our research. Even without such attacks, the attacker can be successful.

3.2 Attack Sketch

The passive eavesdropper Eve (E) in our scenario observes HD and challenges used to store a secret key with an MCPUF. Each known challenge ch_i belongs to an unknown

response bit r_i of the PUF under attack and, therefore, to at least one known HD bit w_j . The HD bit depends on the PUF response as well as on some random bit. Given the PUF response, it is mapped to a codeword by the HDA. This codeword now implies the existence of redundancy. I.e., the HD bits reflect an inherent, unavoidable dependency between specific codeword bits, cf. Section 4.

E knows the mapping during role out to derive HD w from PUF responses r_i and random bits s_j . She transforms the system to eliminate s_j . In the new representation, she replaces each r_i by an unknown function $puf(ch_i)$ where ch_i is known. The result is a mapping from challenges to transformed HD and only puf is unknown.

E provides her knowledge to an ML algorithm for learning. She trains a model that does not learn the PUF responses directly but rather the dependency between the responses of the PUF for several challenges. E now guesses the PUF response \tilde{r} for one challenge \tilde{ch} used in the key-storage scenario. With the tuple (\tilde{ch}, \tilde{r}) she queries the model for the most likely sequence of response bits under this hypothesis and given the set of challenges used to reconstruct the still unknown key. After using HD and HDA with the resulting sequence of PUF responses, she ideally finds the correct codeword if \tilde{r} was correct or the inverted correct codeword if she guesses \tilde{r} wrong. Knowing the ECC, she finally decodes the codeword to the key. The remaining key-entropy from a successful analysis, which E can mount by only passively observing HD and challenges and processing her observations, is therefore one bit. The only requirement is that she can observe enough challenges and HD; It will turn out that for simple PUF primitives in combination with typical code-concatenations only few hundred challenges suffice to break the system. But the situation can be improved not only trivially with a stronger PUF but also by using codes that enforce a large and varying number of XORs in the virtual XOR-PUF.

Remarks: (i) In the described process, it is not necessary to guess the codeword with high accuracy. Rather it suffices to make sufficiently few errors in the guess, so that the ECC corrects to the right key. (ii) The entropy can actually be further reduced if one of the codewords derived after setting $\tilde{r} = 0$ and $\tilde{r} = 1$ is more likely. This happens if the inverse of the correct codeword is not part of the code. But we assume that it is easier for E to try out the two possible solutions than to reveal the bit from statistical properties.

4 Exploitation of Helper Data Leakage

The basic idea of the attack is to exploit dependencies between codeword bits. We use the FCS and systematic codes in the following to ease the explanation. However, the transfer to other HDAs and non-systematic codes is straight forward. First we illustrate the full attack using a repetition code as an example. This is followed by a general formal representation of the attack similar to the notation in [PHS17, HPKS16]. Two subsequent examples demonstrate how to grasp the structure of the underlying ECC to train an ML algorithm and eventually reveal the secret.

4.1 Eve's Attack

To begin with, Figure 2 explains the whole attack using the example of a repetition code of length 3. It summarizes the knowledge of E: She has no direct access to the PUF. But she knows HD w , challenges ch_i , and that ch_i belongs to response r_i . Furthermore, E is aware of the ECC and thus how HD and PUF bits are connected. Through the HD, she can guess possible PUF responses r_{guess} . This is easy for a repetition code, because the codeword c is either $[1, 1, 1]$ or $[0, 0, 0]$. Based on this knowledge E derives so called *XOR-equations* by virtually relating responses to each other and replacing each response with the same unknown function puf that maps a challenge to the PUF response. The pairs of challenges in the XOR-equation constitute the features and the result of

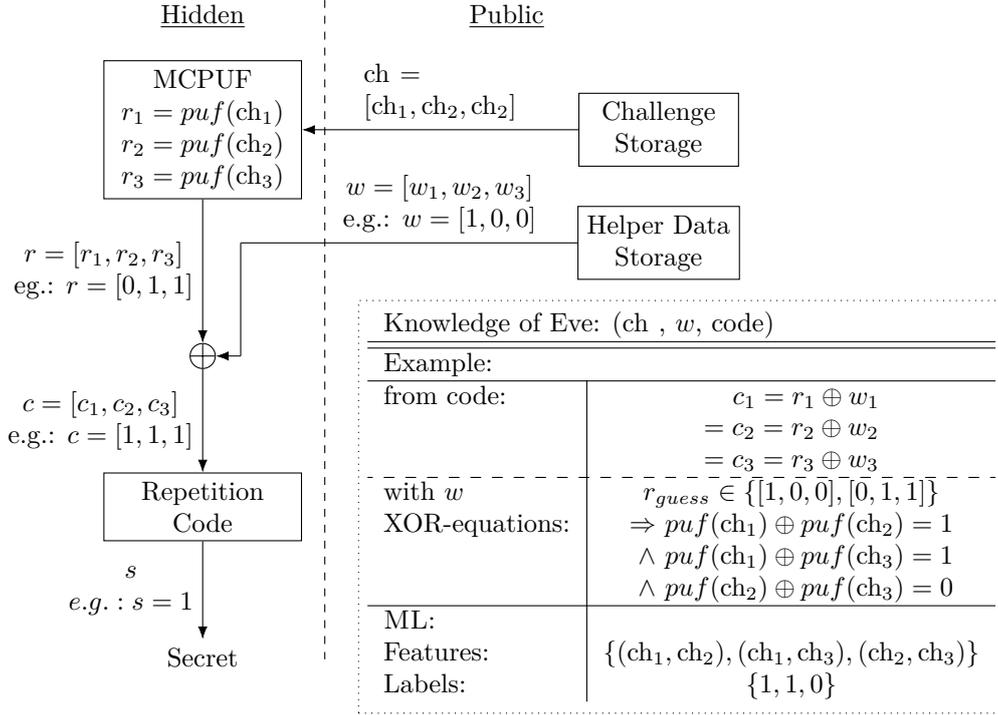


Figure 2: Visualization of the attacker knowledge using a (3,1,3)-repetition code. The left side is hidden from E; Public information and its exploitation is given on the right.

the equation the label for the training of a ML model. The XOR-equations can only be derived using bits of the same codeword. In the training phase these features and labels are used to learn the dependency of different PUF bits with respect to their challenges.

After the training phase, E applies the trained model. She chooses one of the challenges as reference challenge without the knowledge of the respective output. All challenges used for the key-storage are paired with this reference challenge and E queries the model for the responses. By using a single reference challenge the ML algorithm outputs a PUF response for every applied challenge without considering the used code; i.e. the PUF itself is modeled. Since E has to guess the output of the reference challenge one bit of entropy remains. All the other response bits are categorized by the ML model as having the same or the inverse value. E finally uses the same HDA and ECC as the actual PUF construction to correct possible errors in the response string output by the ML algorithm.

4.2 Formalization of the Attack Idea

We introduce an algebraic notation for Eve’s attack and the derivation of the XOR-equations. The inputs to the HDA during roll out are the PUF response r and a random number x . The outputs are a derived secret s and public HD w both determined using the generator matrix G of the underlying ECC:

$$\begin{bmatrix} x^k & r^n \end{bmatrix} \cdot \begin{bmatrix} I^{k \times k} & G^{k \times n} \\ 0^{n \times k} & I^{n \times n} \end{bmatrix} = \begin{bmatrix} s^k & w^n \end{bmatrix}$$

Superscripts denote the length of the respective binary row vectors x, r, s, w and the size of the matrices $0, I, G$. 0 is the all-zero matrix and I the identity matrix. In the FCS, attacker E has no information about $x = s$. We now focus on the computation of the HD

w based on $x = s$ and r . E wants to predict r given w to eventually reveal the unknown secret s . Statements about r are derived from the ECC in the FCS. Hence, we zoom in on the matrices, especially on G with its columns $g_{*,i}$.

$$\begin{bmatrix} x^k & r^n \end{bmatrix} \cdot \begin{bmatrix} g_{*,0} & g_{*,1} & \cdots & g_{*,n-1} \\ 1 & 0 & & 0 \\ 0 & 1 & & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & & 1 \end{bmatrix} = \begin{bmatrix} w^n \end{bmatrix}$$

As an example, $w_i = x^k \cdot g_{*,i} \oplus r_i$. I.e., the i -th HD bit is determined by multiplying the secret random number with the respective column of the generator matrix and XORing the i -th PUF bit. E applies the XOR-sum to two HD bits. This results in:

$$\begin{aligned} w_i \oplus w_j &= (x^k \cdot g_{*,i} \oplus r_i) \oplus (x^k \cdot g_{*,j} \oplus r_j) \\ &= x^k \cdot (g_{*,i} + g_{*,j}) \oplus r_i \oplus r_j \end{aligned} \quad (1)$$

$g_{*,i} + g_{*,j}$ denotes the component-wise XOR of the two columns. The recombination of an arbitrary number of HD and, in parallel, of the corresponding matrix columns and PUF bits is possible in this way.

E now uses this recombination to eliminate the influence of the secret x . In other words, E chooses indices i, j, \dots such that the sum of the columns $g_{*,i} + g_{*,j} + \dots = 0$. Then she ends up with a direct link between a sum of HD bits and a sum of PUF bits.

The selection of such columns is in fact possible and core of the attack: The discussed selection corresponds to choosing linearly dependent columns of G . The generator matrix is, however, constructed such that the codeword of the corresponding code contains redundant information. Therefore, $k < n$ and consequently G contains linearly dependent columns. For each set where according to Eq. 1 the secret part cancels out, E can derive a relation between PUF bits given the HD. We call every resulting equation of the form

$$w_i \oplus w_j \oplus \dots = \{0, 1\} = r_i \oplus r_j \oplus \dots$$

an *XOR-equation*. The bit-value derived from XORing public HD is element of the set $\{0, 1\}$ and is publicly accessible like the HD itself. While this notation has been presented for the FCS, it holds equivalently for the Code-Offset Fuzzy Extractor, since the HD are computed in the same way.

For now, we have shown that by picking suitable indices according to linearly dependent columns, E can set up linear XOR-equations with the only unknowns being the PUF bits. In particular, HD is known, and the secret has been cancelled out. With this knowledge, E starts ML as described in Section 4.1.

4.3 Analysis for Selected Error Correcting Codes and XOR-Equations

To demonstrate the concept, we focus on two different codes in this section: a repetition code and a BCH Code. However, each generator matrix of every linear block code has linearly dependent columns and is, thus, subject to our new type of attack.

Repetition Code: There are mainly three aspects of presenting the repetition code as a first code in this work. (i) It is a simple ECC, which hence allows for an intuitive understanding. (ii) Like discussed in Section 2.1, repetition codes are a very popular choice for error correction based on code concatenation throughout the PUF context. (iii) While their simplicity and error correction capability justify their frequent use, they turn out to be the most vulnerable codes to our attack.

Hereby, it does not matter if n is odd or even. An odd n is the default choice in various previous contributions [BGS⁺08, MVHV12, PFFS19], but the attack works equally well for even n . Hence, [MTV09] is also affected. Also, we neglect possible concatenated outer codes for the moment; Such codes would make the attack easier but confuse during explanation.

Returning to the formal notation, we do not consider an arbitrary generator matrix G , but the one for a repetition code. We use w.l.o.g. a code of length $n = 3$. The HD is then generated by

$$[x^1 \quad r^3] \cdot \begin{bmatrix} \frac{1}{x} & \frac{1}{x^2} & \frac{1}{x^3} \\ \frac{1}{x} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [w^3] .$$

Because $k = 1$, any two columns of the generator matrix are linearly dependent. We are interested in finding as many sets of linearly dependent columns (or their indices) as possible, while at the same time we want the cardinality of these sets to be minimal: The less PUF bits are part of an XOR-equation, the better for the ML as discussed below.

An exemplary XOR-equation of the repetition code is:

$$\begin{aligned} w_0 \oplus w_1 &= x^1 \cdot (g_{*,0} + g_{*,1}) \oplus r_0 \oplus r_1 \\ &= r_0 \oplus r_1 \end{aligned} \quad (2)$$

$w_0 \oplus w_1$, reveals whether or not r_0 and r_1 have the same value. Since for a repetition code the bits of an error free codeword are equal, it holds that $c_1 = c_2 = \dots = c_n$. Consequently it is possible to derive $(n^2 - n)/2$ XOR-equations by comparing the n codeword bits pairwise. The pairs can be permuted so that in total $(n^2 - n)$ XOR-equations are at hand. These XOR-equations contain the information for the ML attack.

BCH Code: BCH Codes are frequently used in the PUF context. Here we introduce a BCH Code or Hamming Code as an example because its structure is more complex than the one of a repetition code. This is reflected in more complex XOR-equations. Thus, we can illustrate more aspects which are relevant from a ML point of view.

Again, we start by representing the HD as result of a vector-matrix-multiplication. The underlying exemplary code is an $(n = 7, k = 4, d_{min} = 3)$ BCH-Code for which the generator matrix can be brought into systematic form. This means, that the codeword is split into a first part containing the redundancy bits and a second part containing the information bits.

$$[x^4 \quad r^7] \cdot \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ \hline & & & I^{7 \times 7} & & & \end{bmatrix} = [w^7]$$

Given this generator matrix one exemplary XOR-equation for this BCH code is:

$$\begin{aligned} w_0 \oplus w_3 \oplus w_5 \oplus w_6 &= x^4 \cdot (g_{*,0} + g_{*,3} + g_{*,5} + g_{*,6}) \oplus r_0 \oplus r_3 \oplus r_5 \oplus r_6 \\ &= r_0 \oplus r_3 \oplus r_5 \oplus r_6 \end{aligned} \quad (3)$$

It can be found by picking a column of the redundancy part and the other columns from the information part with the ones at the correct positions. Due to the structure of the BCH Code, two PUF bits cannot be compared directly. Rather, this example involves four PUF bits in the XOR-equation. This still provides a clue for an attacker. But in

comparison to the repetition code it gets harder to exploit the information. We now discuss the code-independent generation of XOR equations.

The overall complexity of finding XOR-equations and related problems: The goal is to find sets of columns with the smallest size such that the columns are linearly dependent. The smallest amount of columns is also called spark in the context of compressed sensing. Unfortunately, determining the spark is known to be an NP-hard problem [TP13]. Another related problem is computing the minimum Hamming distance for a given parity check matrix of an ECC. This has also been shown to be NP-hard. [Var97].

A practical approach to XOR-equations: While it is beneficial for the attack to find the smallest sets of linearly dependent columns, it is no strict requirement. Any amount of different PUF responses in an XOR-equation gives the attacker insights on how to model the PUF. One practical approach is, thus, to select well suited columns by inspection. Also, any selection of $\text{rank}(G) + 1$ columns gives a set of linearly dependent equations by definition, which results in an XOR-equation.

For our experiments, we follow a brute-force approach: We iterate the amount of columns that shall be combined κ from small to larger, but feasible, values. A brute force search for all combinations of κ linearly dependent columns in the generator matrix is performed in each step. Deriving a large number of equations this way is computationally expensive. But XOR-equations for a specific code have to be found only once to enable the attack on this code for the future.

With the sets of linearly independent equations at hand, the XOR-equation and the results of recombining the HD accordingly – the labels – are easily derived. In addition, the challenges, which belong to an XOR-equation in a specific key-storage scenario – the features – can be looked up. For the ML algorithm below, the ordering of the applied challenges is of relevance. Therefore, to fully exploit the identified independent columns for an ECC, we apply the challenges with the corresponding label in all possible permutations to the ML algorithm.

5 Results of Machine Learning

This section proves the learnability of PUFs by exploiting redundancy information stored in HD. We use a Neural Network (NN) in the attack representing an attacker with very little technical knowledge about the PUF and its underlying structure. We conduct our analysis on simulated data: The k -SUM PUF is built from normalized uncorrelated RO frequencies sampled from a uniform distribution in the range -1 to 1 ; For the Arbiter PUF we use the model implemented by Ruehrmair et al. [RBK10]. The experiment simulates the worst case of an ideal PUF.

For comparability, all challenges were created by the same 128 bit Galois LFSR with primitive feedback polynomial $x^{128} + x^{127} + x^{126} + x^{122} + 1$. The analyses are implemented in Python 3 with Tensorflow 2.0 as ML framework and are executed on a commodity computer with Intel i7 CPU and 16 GB of memory.

The attacker cannot separate a response or key from its inverse, because she has to guess the response bit for the reference challenge as described in Section 4. Therefore, we consider an attack successful if the attacker guesses an n bit PUF response, a key, or their respective inverses correctly. A PUF *success rate* of 80% means: 80% of the bits in the predicted PUF response coincide with either the correct PUF response or its inverse. The *key success rate* is defined equivalently and always refers to a key of same length.

5.1 Selection of a Neural Network

The XOR-equations introduced in Section 4 are interpreted as comparisons of the PUF responses for different challenges. We use a Siamese Neural Network (SNN) to account for this interpretation. SNNs consist of multiple NNs, which have exactly the same structure and share their trainable weights. These NNs are connected by a layer which measures the distance between the NNs' outputs. We use Manhattan distance as the distance metric.

The distance layer is followed by the prediction layer. The loss between predictions and labels while training is calculated using binary cross entropy; an optimizer Adam [KB14] with default parameters is used. The activation functions in our implementation are the ReLU function for the hidden layers and the sigmoid function for the output layer.

The data for learning consists of a set of tuples with challenges (features) and correspondingly XORed HD (labels), derived from XOR-equations. We split this set into training and validation data set with a ratio of 90% to 10%. More precisely, the ratio was aligned with the codeword length, since challenges used for the same codeword must not be part of training and validation set at the same time.

The final selection of hyperparameters like the number of layers, neurons per layer, dropout layers and the dropout rate can be evaluated and optimized by tracking the validation accuracy of each tested network architecture while using the same training and validation set. We manually identified two variants of the SNN used throughout the analysis (cf. Appendix, Section A). Please note, that this selection process could also be automated and the found SNNs still be optimized.

5.2 Exploiting Repetition Codes

The repetition code is commonly used for code concatenation in the PUF context. Therefore, our first evaluation targets a repetition code of length 7 like suggested in [MSS11]. The code is used to correct the response of a 128-SUM PUF. An SNN (cf. Appendix, Figure 10) was trained with a maximal number of 200 epochs. An early stopping condition tracked the validation loss value and stopped training when the validation loss did not further improve within a period of 25 training iterations. If learning was stopped early, the weights were reverted to the iteration with the best validation loss value.

Figures 3(a) and 3(b) depict the accuracy and the validation accuracy of the trained model with respect to the size of the training and validation set together. The results are presented in terms of actually used CRPs of the PUF to allow for better comparison with other code classes. In case of the (7, 1, 7) repetition code, e.g., the number of XOR-equations used for training is per codeword $7^2 - 7 = 42$. The number of codewords is adjusted to fit best into the amount of CRPs in each Figure, e.g., for the repetition code from overall 400 CRPs we use $57 \cdot 7 = 399$ CRPs. Consequently, $57 \cdot 42 = 2394$ XOR-equations are generated for learning; nevertheless, the attacker has observed only 399 challenges and corresponding HD bits.

To account for randomness used in the ML algorithm, the experiment was repeated 10 times for every considered amount of CRPs. While the accuracy of the model is quite high already from the beginning, the validation accuracy increases much slower. However, if the validation accuracy is too low, it is not possible to predict results for not yet seen XOR-equations. Stepwise increase of the used CRPs leads to a roughly logarithmic growth of the validation accuracy. From 800 CRPs on, the validation accuracy reaches a level of above 80%. As it is shown next, this value suffices to derive codewords with sufficiently low error probability to decode them to the correct secret.

Figure 3(c) and Figure 3(d) present the results after training: Figure 3(c) indicates the PUF success rate, that is a measure for the correctness of the prediction. The model was trained to detect the difference between multiple – in case of the repetition code two – responses. Hence, to retrieve specific CRPs from the model, each respective target

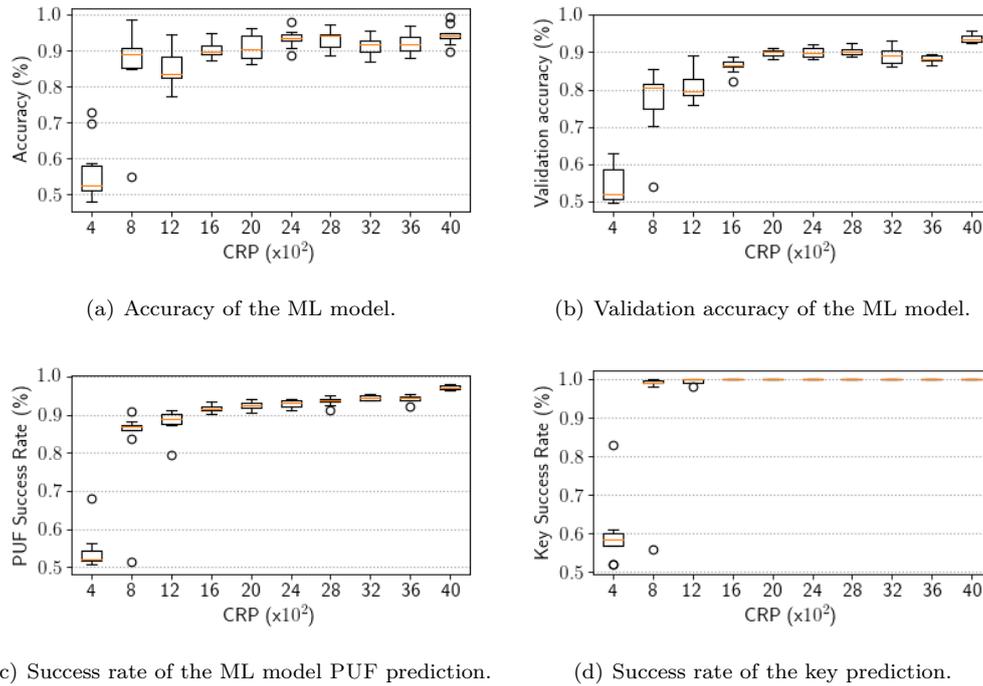


Figure 3: Evaluation of the attack idea using a repetition code with $n = 7$, 128-SUM PUF and a Siamese neural network. For every number of CRPs ten neural network were trained and evaluated.

challenge is applied together with always the same reference challenge. In Figure 3(c) the first challenge used in the key-storage system served as reference challenge, a decision we discuss below. Remarkably, the median of the success rate for the PUF response is always higher than the validation accuracy during training.

With the predicted PUF response at hand, the PUF response together with the HD are now mapped to codewords and decoded with the ECC. Figure 3(d) depicts the resulting key success rate after this process. Due to the error correction capability of the repetition code, we find the correct key with only 800 CRPs. Since the result depends on the guessed response for the reference challenge, the decoding returns either the correct key or its inverse. We did not utilize an outer code connected with the repetition code.

This first experiment shows the feasibility and power of the attack: In typical PUF scenarios using the repetition code, significantly more than 800 CRPs are needed, e.g., 1778 to store a 128 bit key in [MSSS11]. Therefore, scenarios storing a key with only a moderately secure MCPUF are shown to be potentially vulnerable. Especially codes with low coderate are a pitfall in combination with FCSs or FEs and an MCPUF. We now substantiate the analysis to show further aspects of the identified weakness.

Arbiter PUF We repeat the exactly same experiment from above only replacing the k -SUM PUF by the Arbiter PUF model implemented by Ruehrmaier et al. [RBK10]. Figure 4 depicts the corresponding results. The probability that the found key has one bit of entropy left while attacking the Arbiter PUF model is $\approx 100\%$ with 2000 CRPs for training. Again, the validation accuracy of the model grows logarithmically with the number of CRPs. The variance of the probability to find the correct key given 400 CRPs is much larger than the corresponding variance for the experiment with the k -SUM PUF.

Interestingly few outliers reach $\approx 90\%$ which indicates strongly reduced key entropy. The result validates, that – while of course the quality of the MCPUF influences the learnability – the successful attack on the k -SUM PUF was not due a poor implementation of the PUF.

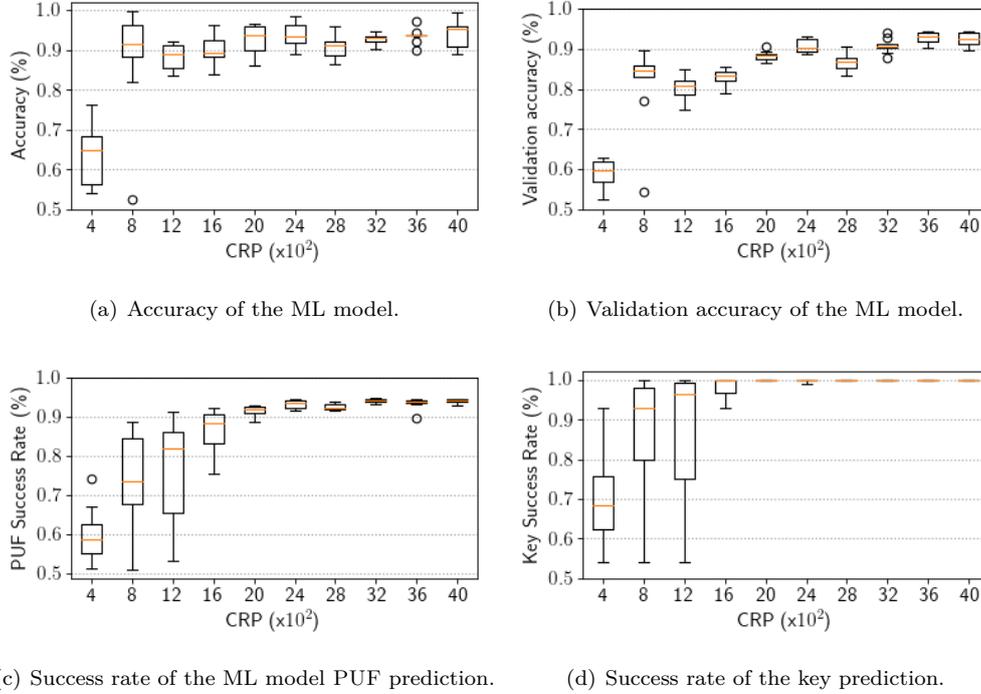
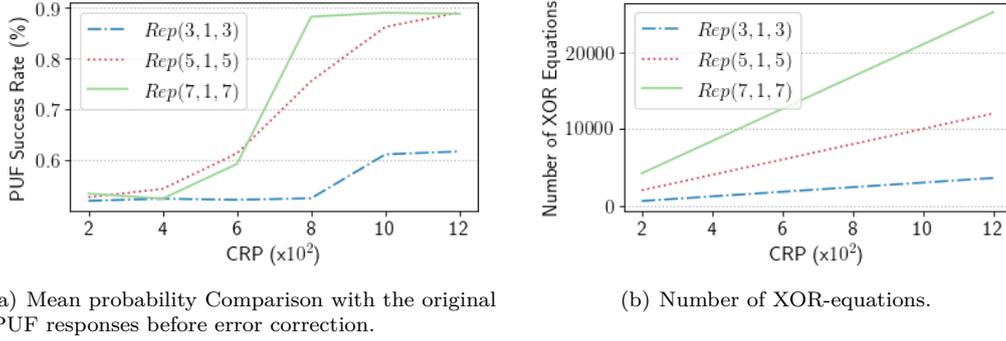


Figure 4: Evaluation of the attack idea using a repetition code with $n = 7$, Arbiter PUF model with 128 bit challenge, and a Siamese neural network. For every number of CRPs ten neural network were trained and evaluated.

5.3 Influence of Error Correction Capabilities and Reference Challenge

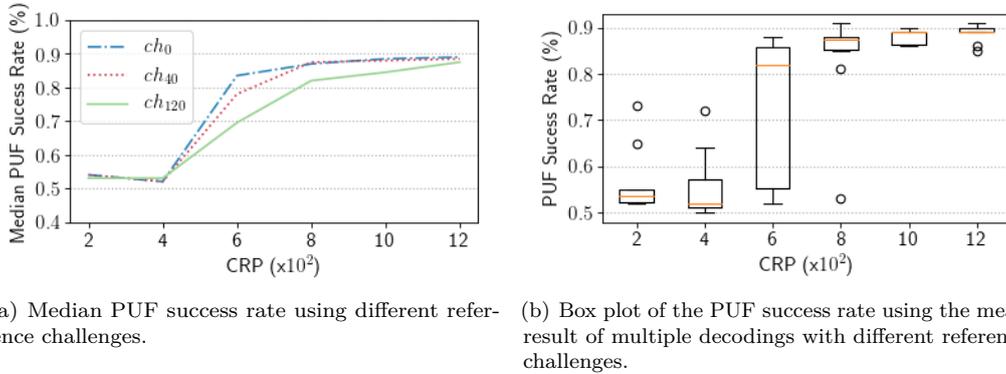
From the previous observations, the code dependency of the number of existing XOR-equations, the influence of the error correction capability itself and the influence of the choice of the reference challenge remained open for further analysis in this section.

Number of XOR-equations: Most important for the learnability is the number of XOR-equations that can be generated. The more XOR-equations are found, the more training samples are at hand to train the model. This is now demonstrated repeating the same experiment as before with the same k -SUM PUF but with codes with different levels of redundancy. Since the repetition code adds redundancy by the size of its codewords, we use always the same number of CRPs as before but create XOR-equations with respect to repetition codes of lengths 3, 5 and 7. Figure 5(a) compares the results of the analysis before decoding. The results indicate that the learnability through these codes is sorted by their codeword length n . This relation is explained by the number of XOR-equations, which is for the repetition code $(n^2 - n)$ (cf. Section 4). Figure 5(b) presents the growth rate of the number of XOR-equations given a CRP number for these three codes. The more XOR-equations a ML algorithm receives, the faster it can learn and generalize.



(a) Mean probability Comparison with the original PUF responses before error correction.

(b) Number of XOR-equations.

Figure 5: Comparison of three repetition codes dependent on the number of XOR-equations which can be created.

(a) Median PUF success rate using different reference challenges.

(b) Box plot of the PUF success rate using the mean result of multiple decodings with different reference challenges.

Figure 6: Comparison between decoding with a single reference challenge and decoding with multiple challenges and creating the mean value.

Error correction rate: Imperfections in the ML model are unavoidable due to the intrinsic limitation of the number of CRPs. The imperfect ML model behaves, however, like a noisy PUF and is corrected in our approach – as already shown – by the ECC for the actual PUF. Recall, that the more redundancy in the code the easier to attack the PUF. Also, the high redundancy can correct many errors. A concatenation of a code with a high error correction capability as inner code with a code with high information rate and low error correction capability as outer code is common practice. Additional XOR-equations, which can be established through dependencies in the outer code, can further advance our attack. In addition, the correction capabilities of the outer code further reduce the required model accuracy for a successful attack. Overall, a high bit error rate in the PUF, which requires a stronger ECC, supports the attack in our analysis.

Choice of the reference challenge: When the ML model has been trained, the PUF is predicted according to a reference challenge. Figure 3(b) shows the median validation accuracy of the ML model while attacking the k -SUM PUF by a repetition code. It lies at $\approx 80\%$ for 800 CRPs. However, the median PUF success rate shown in Figure 3(c) for 800 CRPs is at $\approx 87\%$. This raises the question for a systematic explanation of this observation. We suggest a dependency on the chosen reference challenge as the reason. To substantiate this assumption, we repeated the PUF prediction with different reference challenges. Figure 6(a) depicts the median PUF success rate using three different reference

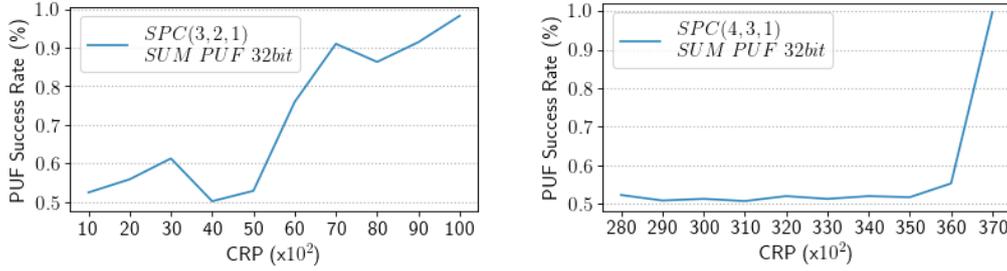


Figure 7: PUF success rate of two attacks on a 32-SUM PUF while utilizing XOR-equations with $\kappa = 3$ of the SPC(3, 2, 1) and $\kappa = 4$ of the SPC(4, 3, 1) code.

challenges. The success rate grows over CRPs similarly for all reference challenges. Also, all reference challenges saturate at $\approx 89\%$. However, the slope of the success rate using ch_{120} as reference challenge is significantly smaller than for the other references. We leave the explanation why some reference challenges perform better than others open for future research. In this work, we consider the task to select the best reference challenge an unsolved problem. To demonstrate the influence of the effect anyway, Figure 6(b) shows the mean value of the prediction with 11 different reference challenges.

5.4 Analyses for Different Codes and PUF Sizes.

We now extend the analysis to three more codes. In these experiments we use k -SUM PUFs with $k = 32$ and $k = 64$.

Single Parity Check Code: We start this part of the analysis with an Single Parity Check (SPC). The consideration of an SPC is motivated by its potential as an inner code generating erasures as well as by its specific structure. This structure enforces that for SPCs the number of equations to be combined κ to get some XOR equation is the code length $\kappa = n$. The longer the codeword, the more XOR operation included in the XOR-equations used to derive features and labels for training. I.e., more PUF responses are XORed. To compare the influence of additional XOR operations we use SPCs with $n = 3$ and $n = 4$. To reduce the influence of the ML model we used an SNN (cf. Appendix, Figure 9) which is able to generalize and learn the attacked PUF by XOR-equations derived for both SPCs. Only one XOR-equation exists per SPC, which can be permuted $3! = 6$ times for SPC(3, 2, 1) and $4! = 24$ times for SPC(4, 3, 1). Figure 7 shows the results for SPC(3, 2, 1) and SPC(4, 3, 1). The results indicate that, although the number of XOR-equations derived for SPC(4, 3, 1) is four times higher than for the SPC(3, 2, 1), also approximately four times more CRPs are needed to generalize and to predict the PUF. Hence, the number of XOR operations in an XOR-equation has a significant influence on the number of CRPs needed to predict a PUF. We conclude that codes which enforce a higher number of XORs within the XOR-equations are harder to exploit.

Analysis of BCH and Reed-Muller Codes. Besides of the repetition codes, two frequently used code classes in the PUF context are BCH and Reed-Muller (RM) codes. Therefore, we now analyze the exploitability of codes from these classes given k -SUM PUFs with $k = 32$ and $k = 64$. More precisely a BCH(7, 4, 3) as well as an RM(16, 5, 8) like in [HKS⁺15] were under consideration. For both codes, XOR-equations with κ equals 4 exist and are used. RM(16, 5, 8) has 140 such XOR-equations per codeword. Each equation is

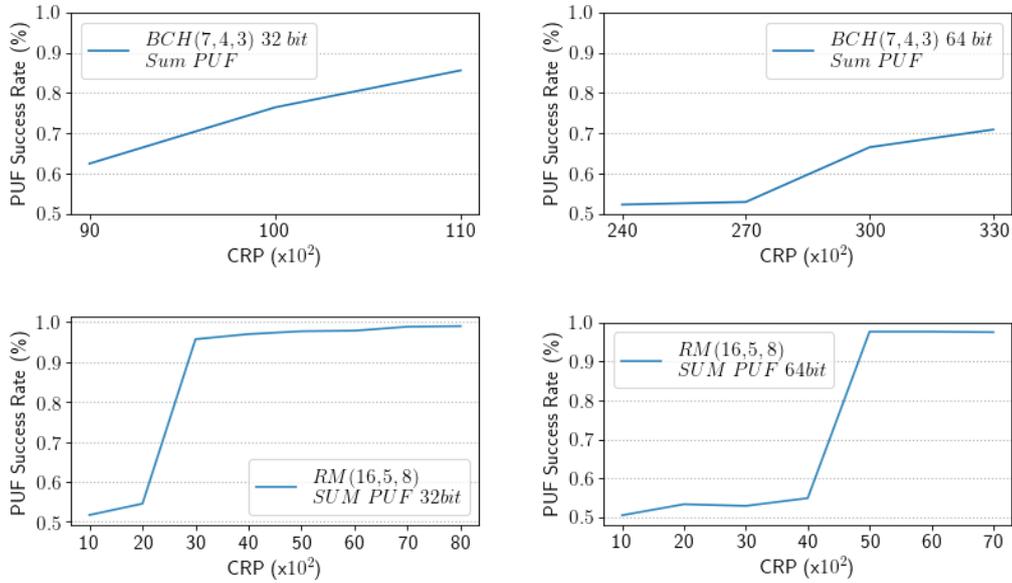


Figure 8: PUF success rate of attacks on a 32-SUM PUF and a 64-SUM PUF while utilizing XOR-equations with $\kappa = 4$ of the RM(16, 4, 8) code and the BCH(7, 4, 3) code.

permuted $4! = 24$ times which allows to derive $140 \times 24 = 3360$ label-feature pairs for one 16 bit codeword. Hence, for each CRP on average we can use $\frac{3360}{16} = 210$ XOR-equations.

In comparison a BCH(7, 4, 3) has only 7 XOR-equations for $\kappa = 4$. Again, each equation can be permuted 24 times which resulting in 168 label-feature pairs per codeword. This is a rate of $\frac{168}{7} = 24$ XOR-equations per CRP. This factor of ≈ 9 times more XOR-equations per CRP impacts also the results.

We applied now the same SNN as in the previous experiment (cf. Appendix, Figure 9) and trained the networks until they reached a PUF success rate of above at least 70%. Figure 8 presents results for both k -SUM PUFs and block codes. For RM(16, 5, 8) both k -SUM PUFs are learned with less than 3000 and, respectively, 5000 CRPs to a PUF success rate of above 97%. In contrast, the BCH(7, 4, 3) is learned with the 32-SUM PUF and using 4.5 times the amount of CRPs with a success rate of only $\approx 80\%$. For the 64-SUM PUF, the ML algorithm needs 7.3 times more CRPs to reach only 70%. Consequently, for RM the key can be derived correctly with high probability, while for the BCH code, due to its higher coderate, the number of considered CRPs does not suffice.

Although there is no obvious direct relation between the number of XOR-equations per CRP and needed CRPs to reach a certain PUF success rate and therefore to mount a successful attack, the results show that the more XOR-equations per CRP in the training phase the easier the ML model can generalize. Further, the findings show that it is possible to adapt the attack to different and more complex codes.

6 Discussion of the Analysis Results

The concept of the attack and the respective findings in Section 5 have shown that key-storage with MCPUFs is much less secure than previously assumed. The knowledge of the public HD and the public challenges alone is sufficient for an attacker to mount an ML attack. This is especially significant for codes with low information rate, like repetition codes, that are frequently used in code concatenations in the PUF context.

This has to be kept in mind when proposing new PUF protocols or other applications in future. The analysis also shows a potential vulnerability in previous protocols such as [PFFS19, Del17]. The security of these protocols is therefore deemed broken if CRPs are not limited. Notably, our analysis has proven that a repetition code as an inner code is an entry point for an attacker, while more complex inner codes make the attack harder.

Therefore, the analysis also gives insights to establish countermeasures against this type of attack. For example, the number of CRPs can be limited like in [YHD⁺16, RSGD16]. However, this reduces the benefit of using an MCPUF. A trade-off is to develop more secure PUF primitives like the iPUF to benefit from the properties of MCPUFs while making ML hard so that the number of usable CRPs remains high.

A more innovative step to make ML harder is to use codes that result in multiple XORs of PUF responses in the XOR-equations. From an ML perspective the resulting problem is related to the well explored recombination of PUF responses through XORs but it causes no additional overhead for the PUF. Consequently, more HD bits and, thus, more CRPs are potentially usable before a ML model can be trained.

The strength of the frequently used code concatenation lies in the usage of simple codes. At the same time, our analysis shows that this is a security weakness. Together with other research [WFP19, TPS17] our analysis highlights the need for a paradigm shift regarding error correction in the PUF context. Not mainly the efficiency but also the security of used codes must be in the focus. Based on our analysis, we claim that the most secure choice for an ECC regarding the new attack is not code concatenation, but rather a standalone code such as a polar code [CIW⁺17, GiSK19]. Not only does a code of large code length (and comparable rate) most likely result in longer XOR-equations, but especially a polar code results also in less HD. Less HD is also provided by schemes like SLLC [HYP15], vector quantization [GiSK19], or syndrome construction [DRS04], but they do not strongly improve secure w.r.t. ML.

While there are countermeasures, our work has also opened the door for more sophisticated attacks. We have focused on FCS and FE, but for other schemes it might be possible to relate HD and PUF bits without the influence of a random secret. Furthermore, more codes of different code classes, rates, or lengths have to be analyzed to see which ones are secure in view of our findings.

Besides, optimized ML algorithms have to be developed to fully understand the risk caused by our findings. Pre-trained models or augmentation of input data, e.g., can further enhance the results such that fewer or more complicated XOR-equations pave the way for a successful attack. Additionally, feeding XOR-equations of different lengths into a ML algorithm remains a challenge we faced during our analysis but which we leave open for future work.

7 Conclusion

This work has exposed a flaw in various MCPUF schemes which use HD to derive a key. ML attacks are possible in such scenarios requiring neither reliability information of the PUF response nor knowledge about the responses themselves. The public HD and challenges alone suffice for an attacker to successfully model the PUF. Our analysis proposes and formalizes the attack thereby confuting state-of-the-art security assumptions. Hence, a multitude of PUF protocols and key-storage schemes have to be re-considered with regard to their security. The vulnerability is especially apparent when the frequently used repetition code is implemented: Our analysis shows for this particular case that attacks with as few as 800 HD bits and challenges are feasible and can result in a complete loss of security. Besides repetition codes, we evaluate the impact on other error correcting codes. This indicates, that future error correction for MCPUF has to be considered from a new perspective. Consequently, we discuss ideas for countermeasures that can

at least reduce the strength of the attack. Most notably, abandoning the popular code concatenation and instead selecting dedicated codes as per our findings would improve the situation.

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A Machine Learning Models

This section documents the machine learning models used throughout this work. Both neural networks are adapted Siamese Neural Networks (SNN) which allow the comparison of parts of the input features. The basic structure of both SNNs is the same. After the input layer a split layer separates the input features in equally sized feature vectors. Each separated feature vector is connected to a copy of the inner sequential neural network marked with dashed lines. All copies of the inner neural network share the weights and bias values. The outputs of the inner neural networks are compared by the L1 Distance Layer. The number of inner networks is dependent on the size of the used XOR-equations (e.g. $\kappa = 4$). Please note, that the networks could further be tweaked which is out of scope of this work.

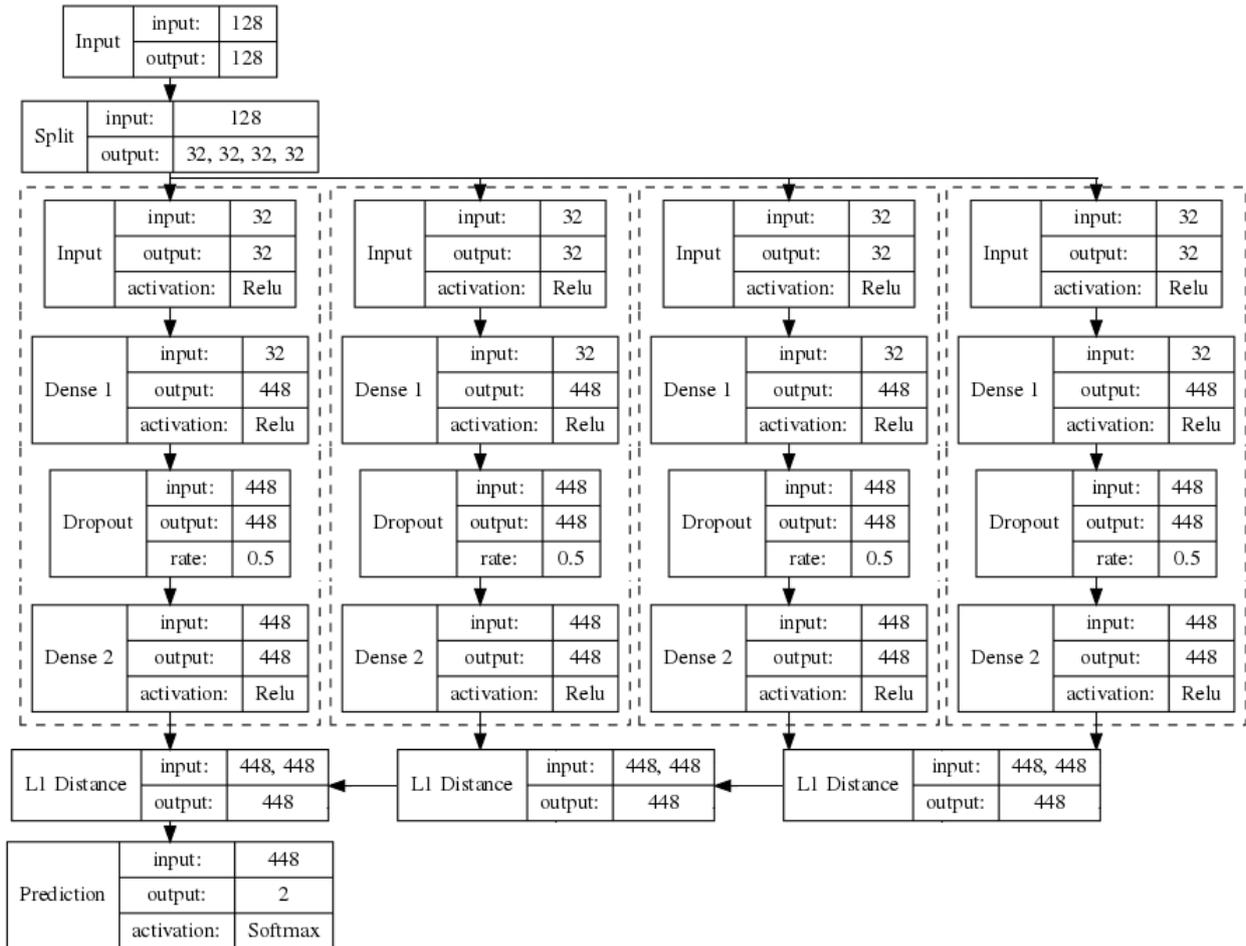


Figure 9: Siamese Neural Network used to learn PUFs using XOR equations derived by SPC, BCH and RM codes. During training the loss is calculated using sparse categorical cross entropy. The chosen optimizer is Adam with default parameters and a learning rate of 0.001.

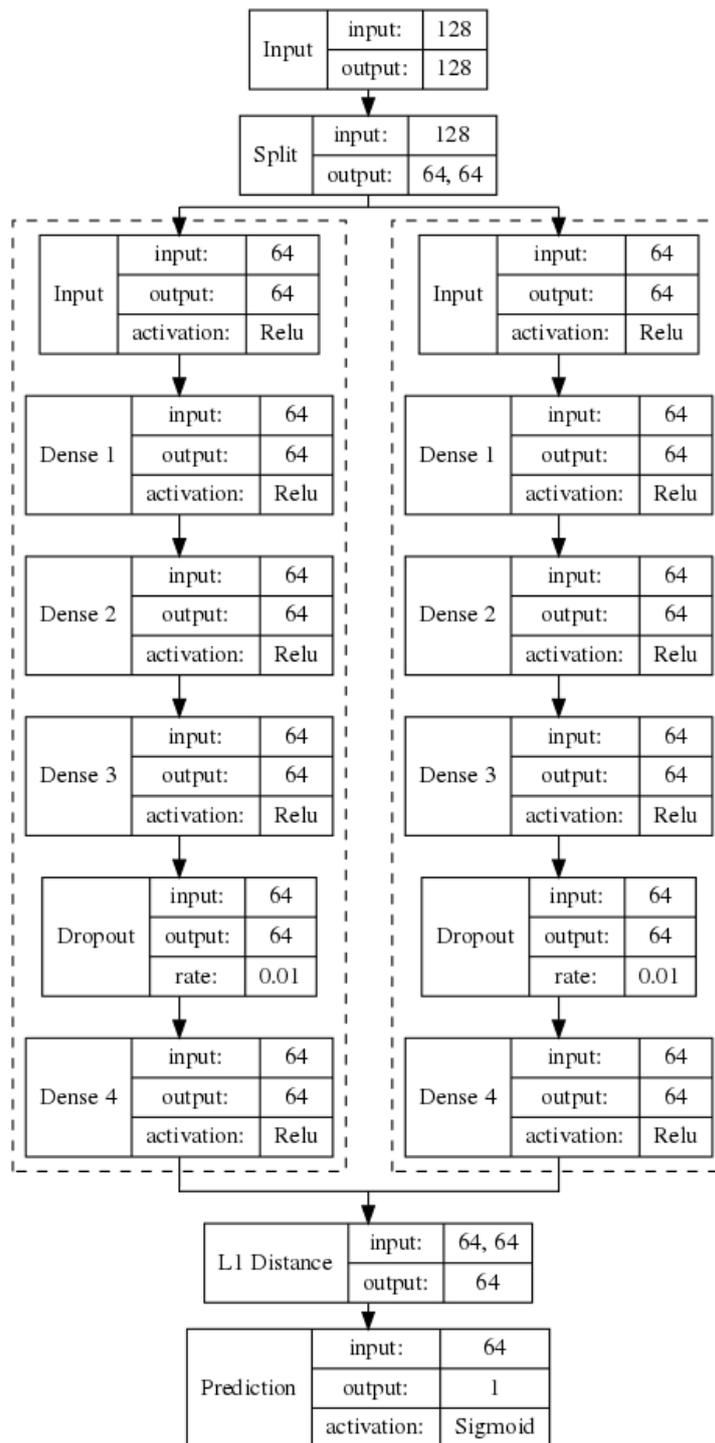


Figure 10: Siamese Neural Network used to learn PUFs using XOR equations derived by a repetition code. During training the loss is calculated using binary cross entropy. The chosen optimizer is Adam with default parameters and a learning rate of 0.01.