

Grover on Caesar and Vigenère Ciphers

Gyeongju Song¹, Kyungbae Jang¹, Hyunji Kim¹, Wai-Kong Lee², and
Hwajeong Seo¹[0000-0003-0069-9061]

¹IT Department, Hansung University, Seoul (02876), South Korea,
{thdrudwn98, starj1023, khj1594012, hwajeong84}@gmail.com

²Department of Computer Engineering,
Gachon University, Seongnam, Incheon (13120), Korea,
waikonglee@gachon.ac.kr

Abstract. Quantum computers can solve or accelerate specific problems that were not possible with classical computers. Grover’s search algorithm, a representative quantum algorithm, finds a specific solution from N unsorted data with $O(\sqrt{N})$ queries. This quantum algorithm can be used to recover the key of symmetric cryptography. In this paper, we present a practical quantum attack using Grover’s search to recover the key of ciphers (Caesar and Vigenère). The proposed quantum attack is simulated with quantum programming tools (ProjectQ and Qiskit) provided by IBM. Finally, we minimize the use of quantum resources and recover the key with a high probability.

Keywords: Quantum computer · Grover’s search algorithm · Caesar · Vigenère

1 Introduction

Quantum computers can solve specific problems much faster than classical computers. International companies, such as IBM and Google, are investing heavily to develop quantum computers. If large-scale quantum computers are developed, cryptography algorithms can be broken through quantum algorithms. The Shor algorithm can break the security of public key cryptography, RSA and ECC (Elliptic Curve Cryptography) [1]. In preparation for the collapse of the security of public key cryptography, NIST (National Institute of Standards and Technology) is currently conducting a PQC (Post-Quantum Cryptography) competition¹.

The Grover search algorithm [2] can be used to break the symmetric key cryptography. By using Grover’s search algorithm, n -bit security symmetric key ciphers can be reduced to $\frac{n}{2}$ -bit security. Recently, researches on estimating quantum resources for applying the Grover search algorithm to symmetric key cryptography have been actively conducted [3–10]. However, it is limited to resource estimation, not recovering the key by applying the actual Grover search algorithm. Large-scale quantum computers have not yet been developed and it is impossible to operate them with the current level of quantum simulators.

¹ <https://csrc.nist.gov/projects/post-quantum-cryptography>

In this paper, we present a practical key search for **Caesar** and **Vigenère** using the Grover search algorithm. We simulate this quantum attack using ProjectQ [11] and Qiskit, quantum programming tools provided by IBM. With proposed compact quantum circuits, it is possible to simulate a key recovery attack on **Caesar** and **Vigenère** ciphers. We describe in detail the implementation of quantum attack circuits for **Caesar** and **Vigenère** ciphers and finally recover the key with a high probability.

2 Related Work

2.1 Caesar Cipher

The **Caesar** cipher is a simple substitution cipher that rotates the plaintext by the value of the key K . Figure 1 shows how the plaintext is encrypted with the **Caesar** cipher when the key is B (i.e. 2). It can be seen that the plaintext is rotated to the right by 2 slots. A changes to C and B changes to D.

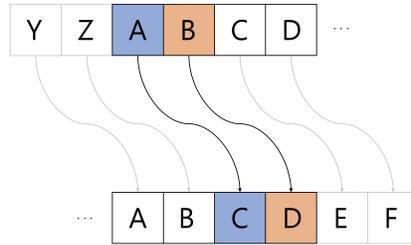


Fig. 1: Caesar encryption with key 2.

2.2 Vigenère Cipher

The **Vigenère** cipher is a multiple substitution cipher. General multi-substitution ciphers, such as Alberti and Trithemius, had a problem that if the algorithm was leaked, all encrypted data in the same way can be decrypted. However, the **Vigenère** cipher uses different key chains for each encryption. If attackers know the algorithm, they cannot recover the plaintext without the valid key. In the **Vigenère** cipher, Table 1 is used. If the plaintext **ABC** is encrypted with the key **DEF**, A is changes to D in the D row. B is changes to F in the E row, and C is changes to H in the F row.

2.3 Grover's Search Algorithm

The Grover search algorithm finds the specific solution for N unsorted data. In a classical brute force attack, $O(N)$ queries are required. However, this can be

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

Table 1: Vigenère table.

found within $O(\sqrt{N})$ queries with the Grover search algorithm. The Grover search algorithm consists of an oracle function and a diffusion operator. The oracle function returns the solution by inverting the sign. The diffusion operator amplifies the amplitude of the solution by calculating (average amplitude) – (each amplitude – average amplitude). These processes are shown in Figure 2. Grover’s search increases the probability of measuring a solution by iterating oracle and diffusion operators. In Figure 2, it is a process of Grover’s search for 2 qubits. A solution can be found with 100 % probability without the repetition.

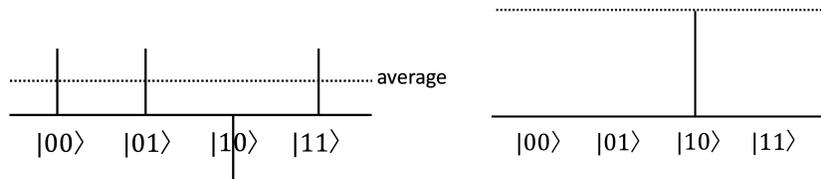


Fig. 2: Oracle (left) and diffusion operators (right).

2.4 Quantum Ripple-Carry Addition Circuit

In [12], a quantum addition circuit called ripple-carry addition that reduces the number of ancillary qubits is presented. We utilize this ripple-carry addition circuit to implement the proposed quantum attack circuit. In the quantum ripple-carry addition circuit, the addition of A and B uses two additional qubits. One is the qubit to store the carry value, and the other is the initial carry qubit. Since the modular addition is required in the circuit, we do not use qubits to store carry values. Detailed descriptions on the ripple-carry addition are described in [12].

3 Proposed Method

We perform a quantum attack using the Grover search algorithm on **Caesar** and **Vigenère** operating in hexadecimal(0 ~ 15) not alphabets (0 ~ 25). This is due to the limitation of the number of qubits in the simulator. The quantum simulator does not use real qubits, but it directly calculates the vast amount of calculation of qubits in a classical way. The ProjectQ simulator utilized in this research can use about 30 qubits. We performed an attack on hexadecimal formats, but proposed method can be applied equally to the alphabet by increasing the number of qubits.

3.1 Grover on Caesar Cipher

In order to perform a key search using Grover's algorithm for the **Caesar** cipher, an encryption quantum circuit must be implemented in Oracle. The **Caesar** encryption can be expressed by the following Equation 1. We utilized the ripple-carry addition. Each element (P_i) of plaintext (P) is encrypted with a single key (K).

$$C_i \leftarrow Enc(P_i, K) = (P_i + K) \bmod 16 \quad (1)$$

In the ripple-carry addition $Add(P_i, K)$, the result of the addition is stored in plaintext qubits (P_i) as follows.

$$Add(P_i, K) = \begin{cases} P_i = (P_i + K) \bmod 16, \\ K = K \end{cases} \quad (2)$$

In the ripple-carry addition, a carry value can be generated. The qubit c_0 for the carry calculation and the qubit z for storing the highest carry value are allocated. Since the modular addition is used for the **Caesar** encryption, the qubit z for the highest carry value can be ignored. Therefore, only a single qubit c_0 is additionally allocated for addition. Since the c_0 is initialized to 0 after addition, it can be reused in the next addition. Figure 3 shows the design of the Grover search circuit that recovers the key for a known plaintext-ciphertext pair $(P, C)=(0xF2, 0x14)$. Because the **Caesar** encryption uses the same key

to each element of the plaintext, there is no need to do a key search for all elements. We recover the key by selecting only two elements $(P_0, C_0)=(2, 4)$ and $(P_1, C_1)=(F, 1)$.

Input Setting For a key search using Grover’s algorithm, we select two elements (P_0 and P_1), and then it encrypts them with the superposition key K using the ripple-carry addition. First, we set the key and elements of plaintext to be entered in the oracle. The Hadamard gate is applied to all qubits of the key K . All values ($0 \sim 15$) exist as probability at once (i.e. superposition state). In Figure 3, the X gate is performed before the H gate to optimize the diffusion operator used later. We will describe this in detail later.

Second, known elements of plaintext (P_0 and P_1) are set by applying X gates. Since the initial values of the qubits P_0 and P_1 are 0, the X gate is applied to the qubit where the plaintext value is 1. Because $P_0 = 0x2(0b0010)$ and $P_1 = 0xF(0b1111)$, the X gate is applied to the second least significant qubit $P_0(p_1)$ for P_0 . All qubits of P_1 must be 1. The X gate is applied to all qubits for $P_1(p_3, p_2, p_1, p_0)$.

Oracle After completing the input setting, we design the oracle. First, the key K in the superposition state is added to the elements (P_0 and P_1), and qubits (P_0 and P_1) become ciphertext.

Second, we check whether the generated ciphertext is compared with the known ciphertext. In Figure 3, known elements of ciphertext (C_0 and C_1) are $0x4(0b0100)$ and $0x1(0b0001)$. If they match, all P qubits are 1 due to the X gates and the sign of the state of key K is reversed by the Controlled- Z gate. Since the Grover’s search finds the key through iterative work of the oracle and diffusion operator. Therefore, the generated ciphertext are returned to plaintext by performing a reverse operation for the next search.

Diffusion Operator The oracle finds the key for a known plaintext-ciphertext pair by inverting the sign of the answer state. Due to the superposition state of qubits, all values still exist with the same probability. In the diffusion operator, the amplitude of the state found in the oracle is amplified and the amplitude of the non-answer states is reduced. The diffusion operator is applied to the key (K). We select an optimized diffusion operator that uses a constant number of X gates utilized in [13]. The basic diffusion operator quantum circuit is shown on the left side of Figure 4. In [13], authors say that in the input setting, if X gate is applied before the Hadamard gate, the same result can be obtained by using the quantum circuit as shown in the right side of Figure 4. Therefore, a constant number of X gates is used regardless of the number of Grover iterations.

Grover Iteration The oracle returns the correct key and the diffusion operator amplifies the amplitude of the returned key. The Grover’s search algorithm iterates the oracle and the diffusion operator to sufficiently increase the probability

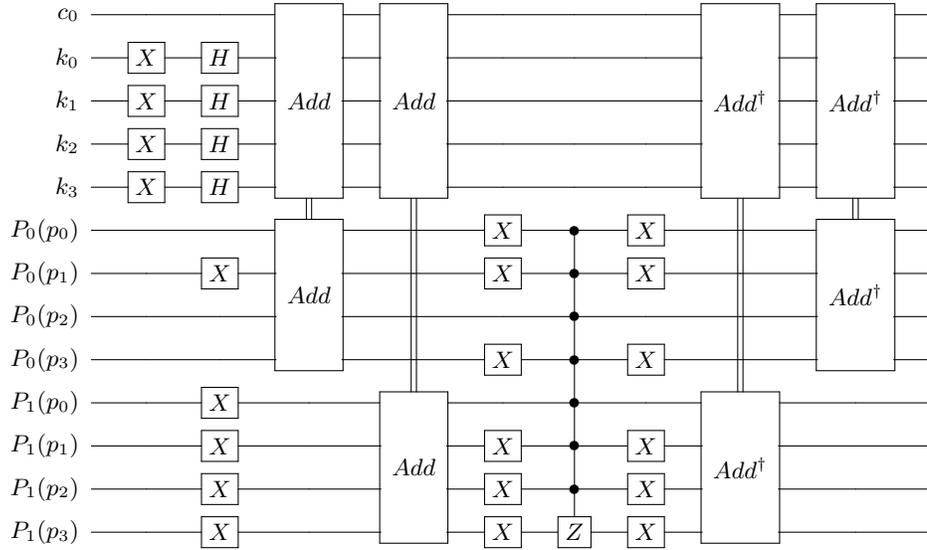


Fig. 3: Input setting and oracle design.

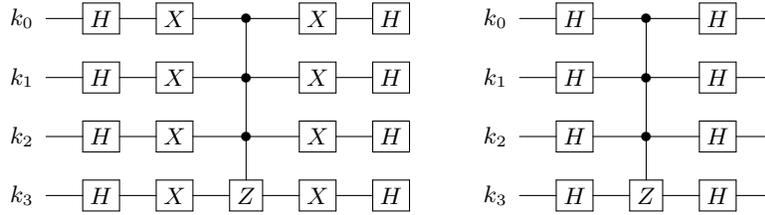


Fig. 4: Basic (left) and constant (right) diffusion operators.

of measuring the correct key and finally measures key qubits. According to [14], the optimal number of Grover iterations to find a single solution in n qubits ($N=2^n$) is $\lfloor \frac{\pi}{4} \sqrt{N} \rfloor$. Since the key is 4 qubits (i.e. $n = 4$) in proposed designs, the oracle and diffusion operators iterate 3 times and find the key ($K=0x2$) for the plaintext-ciphertext pair $((P, C)=(0xF2, 0x14))$ with a high probability.

3.2 Grover on Vigenère Cipher

Unlike the **Caesar** cipher, the **Vigenère** cipher uses a variety of keys and frequency analysis is not feasible. The **Vigenère** encryption can be expressed by the following Equation 3. We implement it using the ripple-carry addition. Each element P_i of plaintext P is encrypted with the key $(K_{i \bmod j})$. j is the length of the key (K).

$$C_i \leftarrow Enc(P_i, K_{i \bmod j}) = (P_i + K_{i \bmod j}) \bmod 16 \quad (3)$$

In the ripple-carry addition ($Add(P_i, K_{i \bmod j})$), the result of the addition is stored in plaintext qubits (P_i) as follows.

$$Add(P_i, K_{i \bmod j}) = \begin{cases} P_i = (P_i + K_{i \bmod j}) \bmod 16, \\ K_{i \bmod j} = K_{i \bmod j} \end{cases} \quad (4)$$

Figure 5 shows the design of the Grover search circuit that recovers the key for a known plaintext-ciphertext pair ($(P, C)=(0xF42, 0x2E5)$). Assuming that attackers don't know the key length (j), attackers search K_i for all P_i and C_i . Due to the limitation of the number of qubits in the simulator, a small length plaintext-ciphertext pair was targeted. The initial key can be recovered due to the nature of the repeated key.

Input Setting In the case of Vigenère cipher (unlike in Caesar cipher), we don't know the key length. For this reason, we have to do a key search for all plaintext elements. we set three elements of plaintext (P_0, P_1 and P_2), and we then encrypt them with the superposition key elements (K_0, K_1 and K_2), respectively using the ripple-carry addition. First, we set the key and plaintext to be entered in the oracle. The Hadamard gate is applied to all qubits of the key (K) to make it the superposition state.

Second, known elements of plaintext ($P_0 = 0x2(0b0010)$, $P_1 = 0x4(0b0100)$ and $P_2 = 0xF(0b1111)$) are set by applying X gates.

Oracle In the oracle, the encryption is performed by adding a key element corresponding to each plaintext element (i.e. $Enc(P_0, K_0), Enc(P_1, K_1), Enc(P_2, K_2)$). If the generated ciphertext is the same as the known ciphertext, all P qubits are 1 due to X gates. There are two options for inverting the sign of the answer using the Controlled- Z gate are shown in Figure 5.

In the Caesar cipher, the sign of the answer 4-qubit key K is reversed when the plaintexts P_0 and P_1 is matched with an 8-qubit Controlled- Z gate. If the **option1** is used in the Vigenere cipher, all key K (i.e. K_0, K_1 , and K_2) qubits are entangled, and iterations are performed to amplify the amplitude of the solution in 12 qubits (i.e. $N = 2^{12}$). According to Equation $\lfloor \frac{\pi}{4} \sqrt{N} \rfloor$ to find the optimal number of Grover iterations, 50 Grover iterations are required. However, the **option2** is applied to a 4-qubit Controlled- Z gate for each plaintext element. K_0, K_1 and K_2 qubits are not entangled to each other and the Grover iteration is performed on each of the key elements. Finally, signs of solutions ($K_0=0x3$, $K_1=0xA$, $K_2=0x3$) are inverted. Since it amplifies the amplitude of the solution in 4 qubits, it is more efficient than **option1** because it only needs to perform 3 times of Grover iterations. Lastly, a reverse operation is performed for the next search.

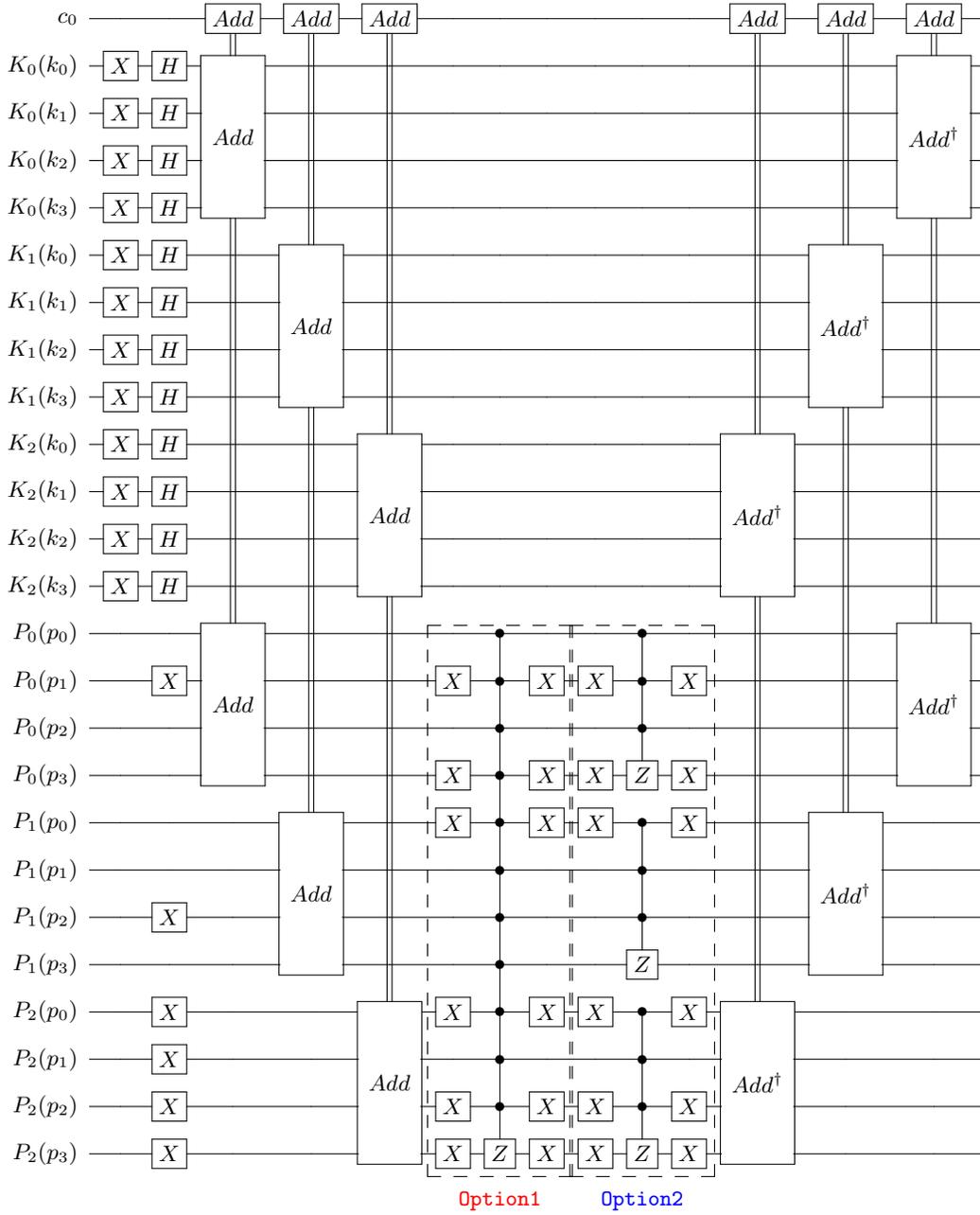


Fig. 5: Input setting and oracle design.

Diffusion Operator Depending on the option, the diffusion operator must be implemented differently. As shown in Figure 6, for `option1` it is working with 12 qubits, and for `option2` it is working with 4 qubits.

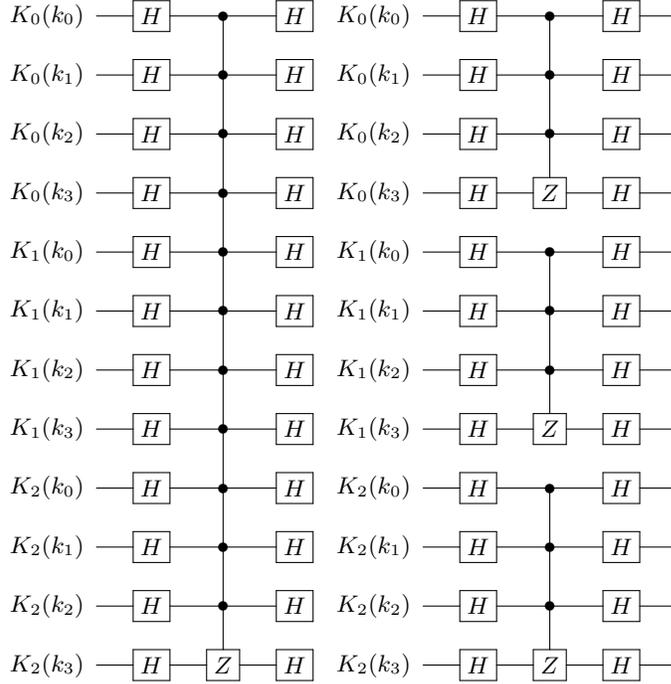


Fig. 6: Option1 (left) and Option2 (right) diffusion operators.

4 Evaluation

We utilized IBM ProjectQ and IBM Qiskit to simulate the quantum key recovery attack using the Grover’s search for **Caesar** and **Vigenère** ciphers. We optimized required quantum resources and recovered a key with a high probability. Figure 7 shows the probability of recovering a key (e.g. 0b0010) according to the number of Grover iterations (r). In the case of **Caesar** cipher, it is the recovery probability of the key (K). In the case of **Vigenère** cipher, it is the recovery probability of each key element (K_i) for `option2`. When the iteration is $r = 3$, the probability of key recovery is the highest, and when the iteration is $r = 4$, the probability is significantly reduced due to overfitting.

Table 2 shows the quantum resources required for a key recovery attack. By utilizing ripple-carry addition, one additional qubit (e_0) is allocated. In the case of **Vigenère** cipher, an individual key search for K_i is performed by selecting

option2. The key is recovered with the same number of iterations as **Caesar** cipher ($r = 3$). In the case of **Caesar** cipher, all plaintext elements are encrypted using the same key. No matter what length plaintext is encrypted, we simply select two plaintext elements and perform a key search. In the case of **Vigenère** cipher, we performed a key search on the plaintext-ciphertext pair limited to length 3 due to the qubit limit of the simulator. In **Vigenère**, since all plaintext elements are encrypted with other key elements, a key search for the entire plaintext must be performed.

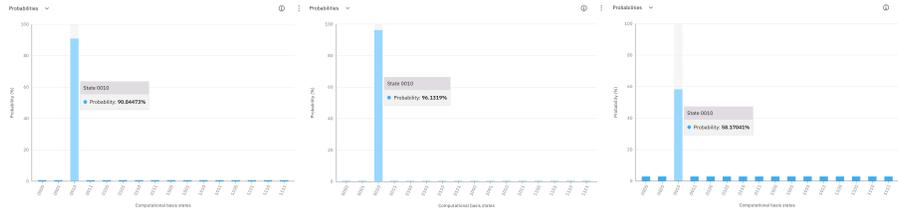


Fig. 7: Key recovery probability for **Caesar** and key element recovery probability for **Vigenère** according to the number of iterations ($r = 2, 3, 4$ from top to bottom).

Table 2: Quantum resources required for a key recovery attack.

Cipher	Qubits	H gates	CX gates	CCCZ gates	X gates	Depth
			CCX gates	CCCCCZ gates		
Caesar	13	28	168	3	39	219
			72	3		
Vigenère	25	84	252	18	48	314
			108	.		

5 Conclusion

The goal of this paper is to explore how the Grover search algorithm is applied to the symmetric key cryptography. We present a practical quantum attack on **Caesar** and **Vigenère** ciphers using the Grover's search algorithm. We perform the actual attack in the simulator. The quantum circuit implementation and the process of performing the attack are described in detail. The result shows that the key is recovered with a high probability by quantum attack using a simulator.

References

1. P. W. Shor, “Polynomial-time algorithms for prime factorization and discrete logarithms on a quantum computer,” *SIAM review*, vol. 41, no. 2, pp. 303–332, 1999.
2. L. K. Grover, “A fast quantum mechanical algorithm for database search,” in *Proceedings of the twenty-eighth annual ACM symposium on Theory of computing*, pp. 212–219, 1996.
3. M. Grassl, B. Langenberg, M. Roetteler, and R. Steinwandt, “Applying Grover’s algorithm to AES: quantum resource estimates,” in *Post-Quantum Cryptography*, pp. 29–43, Springer, 2016.
4. B. Langenberg, H. Pham, and R. Steinwandt, “Reducing the cost of implementing AES as a quantum circuit,” tech. rep., Cryptology ePrint Archive, Report 2019/854, 2019.
5. S. Jaques, M. Naehrig, M. Roetteler, and F. Virdia, “Implementing Grover oracles for quantum key search on AES and LowMC,” in *Annual International Conference on the Theory and Applications of Cryptographic Techniques*, pp. 280–310, Springer, 2020.
6. R. Anand, A. Maitra, and S. Mukhopadhyay, “Grover on SIMON,” *Quantum Information Processing*, vol. 19, no. 9, pp. 1–17, 2020.
7. K. Jang, H. Kim, S. Eum, and H. Seo, “Grover on GIF’T.” Cryptology ePrint Archive, Report 2020/1405, 2020. <https://eprint.iacr.org/2020/1405>.
8. K. Jang, S. Choi, H. Kwon, and H. Seo, “Grover on SPECK: Quantum resource estimates.” Cryptology ePrint Archive, Report 2020/640, 2020. <https://eprint.iacr.org/2020/640>.
9. L. Schlieper, “In-place implementation of quantum-Gimli,” *arXiv preprint arXiv:2007.06319*, 2020.
10. K. Jang, S. Choi, H. Kwon, H. Kim, J. Park, and H. Seo, “Grover on Korean block ciphers,” *Applied Sciences*, vol. 10, no. 18, p. 6407, 2020.
11. D. S. Steiger, T. Häner, and M. Troyer, “ProjectQ: an open source software framework for quantum computing,” *Quantum*, vol. 2, p. 49, 2018.
12. S. Cuccaro, T. Draper, S. Kutin, and D. Moulton, “A new quantum ripple-carry addition circuit,” 11 2004.
13. A. Zulehner and R. Wille, *Simulation and Design of Quantum Circuits*, pp. 60–82. Cham: Springer International Publishing, 2020.
14. M. Boyer, G. Brassard, P. Høyer, and A. Tapp, “Tight bounds on quantum searching,” *Fortschritte der Physik*, vol. 46, p. 493–505, Jun 1998.