Hyperproofs: Aggregating and Maintaining Proofs in Vector Commitments

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ABSTRACT

We present Hyperproofs, the first vector commitment (VC) scheme that is efficiently *maintainable* and *aggregatable*. Similar to Merkle proofs, our proofs form a tree that can be efficiently maintained: updating all n proofs in the tree after a single leaf change only requires $O(\log n)$ time. Importantly, unlike Merkle proofs, Hyperproofs are *efficiently* aggregatable, anywhere from 10× to 100× faster than SNARK-based aggregation of Merkle proofs. At the same time, an individual Hyperproof consists of only $\log n$ algebraic hashes (e.g., 32-byte elliptic curve points) and an aggregation of b such proofs is only $O(\log(b \log n))$-sized. Hyperproofs are also reasonably fast to update when compared to Merkle trees with SNARK-friendly hash functions.

As another added benefit over Merkle trees, Hyperproofs are *homomorphic*: digests (and proofs) for two vectors can be homomorphically combined into a digest (and proofs) for their sum. Homomorphism is very useful in emerging applications such as stateless cryptocurrencies. First, it enables *unstealability*, a novel property that incentivizes proof computation. Second, it makes digests and proofs much more convenient to update.

Finally, Hyperproofs have certain limitations: they are not transparent, have linear-sized public parameters, are slower to verify, and have larger aggregated proofs than SNARK-based approaches. Nonetheless, end-to-end, aggregation and verification in Hyperproofs is 10× to 100× faster than SNARK-based Merkle trees.

1 INTRODUCTION

Vector commitment (VC) schemes [17, 29] such as Merkle trees [30] are fundamental building blocks in many protocols. In a VC scheme, a *prover* computes a succinct *digest* d of a vector $\mathbf{a} = [a_1, \ldots, a_n]$ and proofs $\pi_1, \ldots, \pi_n$ for each position. A *verifier* who has the digest d can later verify a proof π_i that a_i is the correct value at position i. Some VCs, such as Merkle trees, are *maintainable*: when the vector changes *all* proofs can be *efficiently* updated in sublinear time, rather than recomputed from scratch in linear time. Other VCs, such as Pointproofs [21], are *aggregatable*: the prover can take several proofs π_i for $i \in I$ and *efficiently* aggregate them into a single, succinct proof π_I.

Unfortunately, no current VC scheme is both maintainable and aggregatable; at least not efficiently. Yet emerging applications such as *stateless cryptocurrencies* [11, 18, 21, 31, 41, 46, 49] rely on dedicated nodes to efficiently maintain all proofs and also on miners to efficiently aggregate proofs. While generic argument systems (e.g., SNARKs [23, 39]) can be used to add aggregation to maintainable VCs such as Merkle trees, this is too slow in practice (see §5.2). This brings us to this paper's main concern: *Can we build an efficient VC that is both maintainable and aggregatable?* In this paper, we answer this positively and present *Hyperproofs*. Similar to Merkle trees, Hyperproofs are $\log n$-sized and determine a tree. This makes updating all proofs very efficient in logarithmic time. However, Hyperproofs are built from *polynomial commitments* [24, 37] rather than hash functions such as SHA-256. This enables a natural aggregation algorithm that is 10× to 100× faster than "SNARKing" multiple Merkle proofs.

In addition to aggregation and maintainability, Hyperproofs have another very useful property: *homomorphism*. Specifically, trees (and digests) for two vectors can be combined into a single tree (and digest) for their sum. This has several applications. First, homomorphism allows us to obtain *unstealability*, a new property which incentivizes proof computation in applications such as stateless cryptocurrencies. In a nutshell, unstealability allows a prover to *cryptographically bind the proofs she computes with her identity, in an irreversible manner*. This way, honest provers can be rewarded for the proofs they compute while malicious provers cannot *steal* other provers' proofs. Second, homomorphism makes updating digests (and Hyperproofs) more convenient than updating Merkle roots (and proofs), which requires having the proof(s) for the changed position(s) in the vector. Third, homomorphism allows authenticating data in a streaming setting [38].

Challenges. In designing Hyperproofs, we surmount three key challenges. First, computing n proofs in Papamanthou-Shi-Tamassia (PST) polynomial commitments [37] takes $O(n^2)$ time and is too slow. Second, aggregation of PST proofs is difficult without generic SNARKs [23, 39], which would be too expensive. Third, making proofs unstealable is not enough: such proofs must remain maintainable, aggregatable and verifiable with respect to the same digest.

Evaluation. In §5.1, we show Hyperproofs are small (1.44 KiB), they verify quickly (17.4 milliseconds) and are fast to maintain (2.6 milliseconds per update). In §5.2, we show Hyperproof aggregation is much faster than Merkle proof aggregation: 10× faster when using Poseidon hashes [22], which likely need more cryptanalysis, and 100× faster when using provably-secure Pedersen hashes. However, our faster aggregation comes at a cost of slower verification for aggregated proofs and a larger 52 KiB aggregate proof size. Nonetheless, when considering both aggregation and verification time, Hyperproofs remain 10× to 100× faster. Lastly, in §5.3, when we benchmark the overhead of agreeing on a block in a stateless cryptocurrency, Hyperproofs are 32× faster than Merkle trees.

Limitations. To commit to a vector of size n, Hyperproofs requires public parameters consisting of $2n - 1$ group elements, which must

Scheme	Aggregatable	Maintainable	Homomorphic	Unstealable	Transparent
Merkle [30]	×†	✓	×	×	✓
Bilinear$_{O(1)}$ [17, 21, 47, 49]	✓	×	✓	×	×
Bilinear$_{\log n}$ [48, 50]	×	✓	✓	×	×
RSA [11, 16, 17, 27]	✓	×	✓	×	×*
Lattice [38, 40]	×	✓	✓	×	✓
Hyperproofs	✓	✓	✓	✓	×

†: Merkle proofs can be aggregated via SNARKs [23], but are too slow (see §5.2).
*: RSA-based VCs from class groups [13] are transparent but much slower.

Table 1: Comparison with other VCs; more details in the appendix in Table 4.

be generated via a *trusted setup*, typically decentralized via multi-party computation protocols [12]. In future work, we hope to have a *transparent setup* by using assumptions in hidden-order groups. We also do not explore the subtleties of fully-integrating unstealable proofs into a statelessly-validated cryptocurrency. Lastly, our macrobenchmarks only measure the computational overhead of VCs that arises on the critical path to a consensus decision. While our results show Hyperproofs lead to 32× faster decisions, we do not claim this is sufficient to make the stateless setting practical.

1.1 Overview of Techniques

Vectors as multilinear extensions (MLEs). We build upon previous work [56, 57] that represents a vector of size $n = 2^\ell$ as a *multilinear extension (MLE)* polynomial. For example, the MLE of $\mathbf{a} = [5, 2, 8, 3]$ is $f(x_2, x_1) = 5(1-x_2)(1-x_1) + 2(1-x_2)x_1 + 8x_2(1-x_1) + 3x_2x_1$. Note that f correctly "selects" the right a_i given the binary expansion of i as input: $f(0,0) = 5$, $f(0,1) = 2$, $f(1,0) = 8$ and $f(1,1) = 3$.

PST commitments to MLEs. To commit to a vector, we compute a Papamanthou-Shi-Tamassia (PST) commitment (see §2.2) to its MLE. For example, the PST commitment to f above is $\mathsf{C} = g_1^{f(s_1,s_2)} \in \mathbb{G}_1$, where $(s_1, s_2) \in \mathbb{Z}_p^2$ are secret points encoded in the *public parameters* of the scheme and g_1 is the generator of $\mathbb{G}_1$. For vectors of size 4, these public parameters consist of $g_1^{s_1}, g_1^{s_2}, g_1^{(1-s_2)(1-s_1)}, g_1^{(1-s_2)s_1}, g_1^{s_2(1-s_1)}, g_1^{s_2s_1}$. Importantly, we show that the selectively-secure variant of PST commitments is actually adaptively-secure when restricted to only proving evaluations on the Boolean hypercube $\{0,1\}^\ell$. This reduces our proof size compared to previous work based on PST [56, 57].

Multilinear trees. To prove that a_i is the ith value in the vector $\mathbf{a} = [a_0, \ldots, a_{n-1}]$, we compute a *PST evaluation proof* for $f(i_\ell, \ldots, i_1) = a_i$ w.r.t. the commitment C, where $(i_\ell, \ldots, i_1)$ is the binary representation of i. Unfortunately, this takes $O(n)$ time *per position*. Thus, computing all n proofs would take $O(n^2)$ time which is prohibitive. We reduce this to $O(n \log n)$ by computing a novel *multilinear tree (MLT)* of proofs using a divide-and-conquer approach. Importantly, our MLT is *maintainable*: updating all proofs after a change to the vector only requires $O(\log n)$ time.

Proof aggregation. A proof for a_i consists of PST commitments $(w_{i,\ell}, \ldots, w_{i,1}) \in \mathbb{G}_1^\ell$, such that the following *pairing equation* holds:

$$e(\mathsf{C}/g_1^{a_i}, g_2) = \prod_{j \in [\ell]} e(w_{i,j}, g_2^{s_j - i_j}), \tag{1}$$

where $e : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$ is a *pairing* and $g_2^{s_j}$'s are additional $O(\ell)$-sized PST public parameters in $\mathbb{G}_2$. To aggregate b proofs, we prove *knowledge* of PST commitments that pass Eq. 1 for each i, resulting in a succinct $O(\log(b\ell))$ aggregated proof size. Our key ingredient is an *inner-product argument (IPA)* by Bünz et al. [15] for proving several pairing equations hold.

Homomorphism and unstealablity. As we mentioned, Hyperproofs are *homomorphic*: exponentiating a PST evaluation proof $(w_{i,\ell}, \ldots, w_{i,1})$ by a constant α yields a proof for position i but in a vector whose values are multiplied by α. We observe that if α is the secret key of a *proof-serving node (PSN)*, this makes the proof unstealable by other nodes who do not have α. Importantly, the proof can still be verified against the digest C, except the verifier must also give the node's corresponding public key g_2^α: $e(\mathsf{C}/g_1^{a_i}, g_2^\alpha) = \prod_{j \in [\ell]} e(w_{i,j}^\alpha, g_2^{s_j - i_j})$. As an optimization, proof-serving nodes can exponentiate the PST public parameters by α before computing proofs. This way, when computing a multilinear tree (MLT) with these parameters, all proofs are implicitly unstealable and the MLT remains maintainable.

1.2 Related work

Below, we relate our VC to previous work and summarize in Table 1.

Merkle trees. A Hyperproof consists of $\log n$ (algebraic) hashes, which can be as small as Merkle hashes if using 256-bit elliptic curves [5]. Hyperproofs are orders of magnitude slower to compute and update, when compared to normal Merkle trees hashed with SHA-256. However, when compared to aggregatable Merkle trees that use SNARK-friendly hash functions (e.g., Poseidon-128 [22]), Hyperproofs are only slightly slower (see §5.3) and make up for it with much faster aggregation, homomorphism and unstealability.

SNARK-based works. Ozdemir et al. [35] explore using SNARKs to prove knowledge of changes that update a vector with digest d into a new vector with digest d'. Lee et al. [28] also use SNARKs to prove correctness of state transitions in replicated state machines, without having to send the state changes. Neither work explores unstealability nor maintaining and aggregating proofs efficiently.

Algebraic VCs. Zhang et al. [56, 57] were the first to build VCs from PST commitments to MLEs. However, their $O(\log n)$-sized proofs are concretely larger and do not support updates. Some VCs have $O(1)$-sized proofs [11, 16, 17, 21, 25, 27, 49], which inherently require $\Theta(n)$ time to update all proofs after a change. Aggregation and verification in these VCs is concretely, and sometimes asymptotically, faster (see Table 4). They also have smaller aggregated proofs. However, these VCs are not efficiently maintainable.

Previous maintainable VCs [38, 40, 48, 50] do not support aggregation; at least not without expensive generic argument systems (e.g., SNARKs). The lattice-based construction from [38, 40] is also homomorphic and additionally transparent, with constant-sized

public parameters. However, it is too slow for practice and non-aggregatable. The *authenticated multipoint evaluation tree (AMT)* construction from [48, 50] can be viewed as the dual to our construction, but from univariate polynomials rather than multivariate. Unfortunately, it is non-aggregatable, its trusted setup requires $O(n^2)$ time and it has larger $O(n \log n)$-sized public parameters.

Recent work [3, 11, 51] enhances VCs into *key-value commitments (KVCs)*, where arbitrary keys (rather than vector positions) are mapped to values. Unfortunately, all of these constructions have constant-sized proofs and are thus not maintainable. Some VCs have transparent setup [11, 16, 27], support incremental aggregation [16], have a "specialiazable" CRS [16] and provide time/space trade-offs when computing proofs [11, 16]. Hyperproofs do not have any of these features. Lastly, no previous VC scheme is unstealable, although some (e.g., aSVCs [49] and AMTs [50]) can be made so using our techniques.

2 PRELIMINARIES

Notation. Let $[0, n) = \{0, 1, \ldots, n-1\}$. An ℓ-bit number i has *binary representation* $\mathbf{i} = (i_\ell, \ldots, i_1)$ if, and only if, $i = \sum_{k=0}^{\ell-1} i_{k+1} 2^k$. Note that i_ℓ is the MSB of i and i_1 is the LSB. We often use $\mathbf{i}$ as i's binary representation and i_k as its kth bit, without explicit definition. Let $r \in_R S$ denote picking an element from S uniformly at random.

Pairings. $(p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, e, g_1, g_2) \leftarrow \mathsf{BilGen}(1^\lambda)$ denotes generating groups $\mathbb{G}_1$, $\mathbb{G}_2$ and $\mathbb{G}_T$ of prime order p, with g_i a generator of $\mathbb{G}_i$, and a *pairing* $e : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$ such that $\forall u \in \mathbb{G}_1, w \in \mathbb{G}_2$ and $a, b \in \mathbb{Z}_p$, $e(u^a, w^b) = e(u, w)^{ab}$. A useful property of $e(\cdot, \cdot)$ is that $e(u, h)e(v, h) = e(uv, h), \forall u, v, h \in \mathbb{G}_1^2 \times \mathbb{G}_2$. In this paper, we assume *Type III bilinear groups* (i.e., without efficiently-computable homomorphisms between $\mathbb{G}_1$ and $\mathbb{G}_2$ or viceversa), which are needed by the *inner-product argument* from §2.4 and are also more efficient in practice. Let $1_\mathbb{G}$ denote the identity in a group $\mathbb{G}$.

Vectors. Bolded, lower-case symbols such as $\mathbf{a} = [a_0, \ldots, a_{n-1}] \in \mathbb{Z}_p^n$ typically denote vectors of field elements. Bolded, upper-case symbols such as $\mathbf{A} = [A_1, \ldots, A_m] \in \mathbb{G}^m$ typically denote vectors of group elements. $|\mathbf{A}|$ denotes the size of the vector $\mathbf{A}$. $\mathbf{A}^x = [A_1^x, \ldots, A_m^x], x \in \mathbb{Z}_p$, $\mathbf{A} \circ \mathbf{B} = [A_1 B_1, A_2 B_2, \ldots, A_m B_m]$, and $\langle \mathbf{A}, \mathbf{B} \rangle = \prod_{j=1}^m e(A_j, B_j)$ denotes a *pairing product*. Let $\mathbf{A}_L = [A_1, \ldots, A_{m/2}]$ and $\mathbf{A}_R = [A_{m/2+1}, \ldots, A_m]$ denote the left and right halves of $\mathbf{A}$. Let $\mathbf{A} || 1_\mathbb{G}$ denote a vector of size $2|\mathbf{A}|$ that "extends" $\mathbf{A}$ to the right with the identity of $\mathbb{G}$. (Similarly, $1_\mathbb{G} || \mathbf{A}$ "extends" $\mathbf{A}$ to the left.)

2.1 Multilinear extension (MLE) of a vector

Let $n = 2^\ell$ and $\mathbf{x} = (x_\ell, \ldots, x_1)$. A vector $\mathbf{a} = [a_0, \ldots, a_{n-1}] \in \mathbb{Z}_p^n$ can be represented as a *multilinear extension* polynomial $f : \mathbb{Z}_p^\ell \to \mathbb{Z}_p$ which maps each position i to a_i:

$$f(\mathbf{i}) = f(i_\ell, \ldots, i_2, i_1) = a_i, \forall i \in [0, n) \tag{2}$$

For example, the MLE of $\mathbf{a} = [5, 2, 8, 3]$ is $f(x_2, x_1)$ defined as:

$$5(1-x_2)(1-x_1) + 2(1-x_2)x_1 + 8x_2(1-x_1) + 3x_2x_1 \tag{3}$$

In general, the *unique* multilinear extension f of $\mathbf{a}$ is [45, Fact 3.5]:

$$f(\mathbf{x}) = f(x_\ell, \ldots, x_1) = \sum_{j=0}^{n-1} a_j \mathcal{S}_{j,\ell}(x_\ell, \ldots, x_1) = \sum_{j=0}^{n-1} a_j \mathcal{S}_{j,\ell}(\mathbf{x}) \tag{4}$$

where $\mathcal{S}_{j,\ell}, j \in [0, 2^\ell)$ are *selector multinomials* defined as:

$$\mathcal{S}_{j,\ell}(\mathbf{x}) = \prod_{k=1}^{\ell} \mathsf{sel}_{j_k}(x_k), \text{s.t. } \mathsf{sel}_{j_k}(x_k) = \begin{cases} x_k, & \text{if } j_k = 1 \\ 1 - x_k, & \text{if } j_k = 0 \end{cases}, \tag{5}$$

with $S_{0,0}(\mathbf{x}) = 1$. In our example from Eq. 3, we have $\ell = 2$ and so: $\mathcal{S}_{0,2}(\mathbf{x}) = (1-x_2)(1-x_1), \mathcal{S}_{1,2}(\mathbf{x}) = (1-x_2)x_1, \mathcal{S}_{2,2}(\mathbf{x}) = x_2(1-x_1)$ and $\mathcal{S}_{3,2}(\mathbf{x}) = x_2x_1$. We often refer to sel_{j_k} as a *selector monomial*. Importantly, note that:

$$\mathcal{S}_{j,\ell}(i_\ell, \ldots, i_1) = \mathcal{S}_{j,\ell}(\mathbf{i}) \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}, \forall i \in [0, 2^\ell) \tag{6}$$

By these properties above, we can see why Eq. 2 holds for any i:

$$f(\mathbf{i}) = \sum_{j=0}^{n-1} a_j \mathcal{S}_{j,\ell}(\mathbf{i}) = a_i \mathcal{S}_{i,\ell}(\mathbf{i}) + \sum_{j=1, j\neq i}^{n-1} a_j \mathcal{S}_{j,\ell}(\mathbf{i}) = a_i \cdot 1 + 0 \tag{7}$$

In other words, an MLE f acts as a "multiplexer", choosing the right a_i based on the input position i, given as $\mathbf{i}$ in binary.

MLE decomposition. An MLE of size $n = 2^\ell$ can be *decomposed* into two MLEs of size $n/2$ [57]. For example, split $\mathbf{a}$ from Eq. 3 into its left and right halves $\mathbf{a}_0 = [5, 2]$ and $\mathbf{a}_1 = [8, 3]$, with MLEs $f_0 = 5(1 - x_1) + 2x_1$ and $f_1 = 8(1 - x_1) + 3x_1$, respectively. Then, observe that the MLE f for $\mathbf{a}$ is a combination of f_0 and f_1: i.e., $f = (1 - x_2)f_0 + x_2 f_1$. In general, the MLE f of any $\mathbf{a}$ decomposes as:

$$f(\mathbf{x}) = (1 - x_\ell) f_0(x_{\ell-1}, \ldots, x_1) + x_\ell f_1(x_{\ell-1}, \ldots, x_1) \tag{8}$$

Note that for $\mathbf{a} = [a_0, a_1]$ of size 2, the MLEs f_0, f_1 are *trivial* (i.e., of size 1) and are simply set to a_0 and a_1, respectively. When decomposing larger MLEs, we use $f_{b_\ell, \ldots, b_k}$ to denote the MLE of the $\mathbf{a}_{b_\ell, \ldots, b_k}$ subvector, which is a vector over a_i's with $i_\ell = b_\ell, i_{\ell-1} = b_{\ell-1}, \ldots, i_k = b_k$ (sorted by i). For example, in a vector $\mathbf{a} = [a_1, \ldots, a_8]$, f_{01} is the MLE of $\mathbf{a}_{01}$, which contains all indices i whose first two bits are 01: i.e., $\mathbf{a}_{01} = [a_2, a_3]$.

2.2 PST commitments to MLEs

Papamanthou, Shi and Tamassia [37] extend Kate-Zaverucha-Goldberg (KZG) univariate polynomial commitments [24] to multivariate ones. We refer to their scheme as PST and restrict its use to multilinear extensions (MLEs), introduced above.

Commitments. PST works over a bilinear group obtained via BilGen. The PST commitment to a multilinear extension f for a vector $\mathbf{a}$ of size $n = 2^\ell$ is a single group element in $\mathbb{G}_1$:

$$\mathsf{pst}(f) = g_1^{f(s_\ell, \ldots, s_1)} = g_1^{\sum_{j=0}^{n-1} a_j \mathcal{S}_{j,\ell}(\mathbf{s})} = \prod_{j=0}^{n-1} \left(g_1^{\mathcal{S}_{j,\ell}(\mathbf{s})} \right)^{a_j} \tag{9}$$

Here, $\mathbf{s} = (s_\ell, \ldots, s_1)$ are *trapdoors* generated via a *trusted setup* that outputs n-sized *public parameters*: $g_1^{\mathcal{S}_{j,\ell}(\mathbf{s})} = g_1^{\mathcal{S}_{j,\ell}(s_\ell, \ldots, s_1)}, \forall j \in [0, 2^\ell)$. Importantly, the setup discards $\mathbf{s}$, since knowledge of it directly breaks PST's security [36]. We stress that $\mathsf{pst}(f)$ can be computed without knowing $\mathbf{s}$, as per Eq. 9. Lastly, PST commitments are *homomorphic*, with $\mathsf{pst}(f + f') = \mathsf{pst}(f)\mathsf{pst}(f')$ for any MLEs f, f'.

Evaluation proofs. Papamanthou, Shi and Tamassia give a way to prove evaluations $f(\mathbf{i})$ against $\mathsf{pst}(f)$ [37], where $\mathbf{i}$ is the binary

$\mathsf{PST.Prove}(f, \ell, \mathbf{i} = (i_\ell, \ldots, i_1)) \to \pi_i$:

1. If $\ell = 0$ (i.e., f is a constant), return $\varnothing$.
2. Otherwise, divide f by $x_\ell - i_\ell$, obtaining quotient $q_\ell(x_{\ell-1}, \ldots, x_1)$ and remainder $r_\ell(x_{\ell-1}, \ldots, x_1)$ such that $f = q_\ell \cdot (x_\ell - i_\ell) + r_\ell$.
3. Return $\left(g_1^{q_\ell(\mathbf{s})}, \mathsf{PST.Prove}(r_\ell, \ell - 1, (i_{\ell-1}, \ldots, i_1))\right)$

Figure 1: $O(n)$-time algorithm for computing a single PST evaluation proof π_i for $f(\mathbf{i})$ w.r.t. an MLE f of size $n = 2^\ell$.

representation of $i \in [0, n)$. Their key observation, which we refer to as the *PST decomposition*, is that:

$$f(\mathbf{i}) = z \Leftrightarrow \exists q_j\text{'s}, f(\mathbf{x}) - z = \sum_{j \in [\ell]} q_j(x_{j-1}, \ldots, x_1) \cdot (x_j - i_j) \quad (10)$$

This yields a *PST evaluation proof* for $f(\mathbf{i}) = z$ consisting of commitments $w_j = g_1^{q_j(\mathbf{s})}$ to these *quotient polynomials* q_j. To compute the q_j's, the prover first divides f by $x_\ell - i_\ell$, obtaining q_ℓ and a remainder r_ℓ. Then, the prover continues recursively on the remainder r_ℓ, which no longer has variable x_ℓ. Specifically, the prover divides r_ℓ by $x_{\ell-1} - i_{\ell-1}$, obtaining $q_{\ell-1}$ and $r_{\ell-1}$. And so on, until he obtains the last quotient q_1 with remainder $r_1 = f(\mathbf{i})$ (see Fig. 1 and [36, Lemma 1]). Overall, this takes $T(n) = O(n) + T(n/2) = O(n)$ time, including the time to commit to the q_j's.

Note that the q_j's are actually MLEs of size $n/2, n/4, \ldots, 1$. As a result, PST's actual public parameters are $g_1^{\mathcal{S}_{j,k}(\mathbf{s})}, \forall k \in [0, \ell], \forall j \in [0, 2^k)$, so as to also be able to commit to these quotient MLEs. Lastly, the parameters form a tree (see Fig. 2) and are thus of size $2n - 1$ $\mathbb{G}_1$ elements.

A verifier who has the commitment $\mathsf{pst}(f)$, the claimed evaluation $(i, f(\mathbf{i}) = z)$ and a logarithmic-sized, publicly-known *verification key* $g_2^{s_j}, \forall j \in [\ell]$ can verify the proof using $\ell + 1$ pairings:

$$e(\mathsf{pst}(f)/g_1^z, g_2) = \prod_{j \in [\ell]} e(w_j, g_2^{s_j - i_j}) \quad (11)$$

Selective security. The check above ensures Eq. 10 holds when $\mathbf{x} = \mathbf{s}$, which is sufficient for security since $\mathbf{s}$ is random and secret. Indeed, Papamanthou et al. prove security under ℓ-SDH (see Assumption A.2), but only in a *selective* sense, where the adversary picks the evaluation point $\mathbf{i} = (i_\ell, \ldots, i_1)$ before the PST public parameters are generated [36, Appendix C.1]. In contrast, we prove adaptive security for any points on the Boolean hypercube (see Appendix D.3). Lastly, note that an adversary who knows $\mathbf{s}$ can forge proofs by solving for the $q_j(\mathbf{s})$'s that satisfy Eq. 10 at $\mathbf{x} = \mathbf{s}$. This is why the trusted setup must discard $\mathbf{s}$.

2.3 Vector Commitments (VCs)

We formalize VCs below, similar to Catalano and Fiore [17].

Definition 2.1 (VC). *A VC scheme is a set of* ppt *algorithms:*

(1) $\mathsf{Gen}(1^\lambda, n) \to \mathsf{pp}$*: Given security parameter* λ *and maximum vector size* n*, outputs randomly-generated public parameters* pp.

(2) $\mathsf{Com}_{\mathsf{pp}}(\boldsymbol{a}) \to \mathsf{C}$*: Outputs digest* C *of* $\boldsymbol{a} = [a_0, \ldots, a_{n-1}] \in \mathbb{Z}_p^n$.

(3) $\mathsf{Open}_{\mathsf{pp}}(i, \boldsymbol{a}) \to \pi_i$*: Outputs a proof* π_i *for position* i *in* $\boldsymbol{a}$.

(4) $\mathsf{OpenAll}_{\mathsf{pp}}(\mathbf{a}) \to (\pi_0, \ldots, \pi_{n-1})$*: Outputs all proofs* π_i *for* $\boldsymbol{a}$.

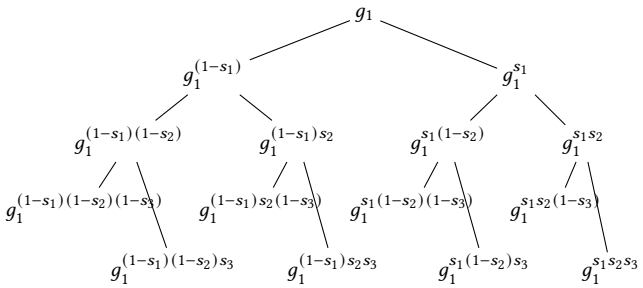


Figure 2: PST (and Hyperproofs) public parameters. The uth path in this tree is actually the update key upk_u from Eq. 17.

(5) $\mathsf{Agg}_{\mathsf{pp}}(I, (a_i, \pi_i)_{i \in I}) \to \pi_I$*: Combines individual proofs* π_i *for values* a_i *into an aggregated proof* π_I.

(6) $\mathsf{Ver}_{\mathsf{pp}}(\mathsf{C}, I, (a_i)_{i \in I}, \pi_I) \to \{0, 1\}$*: Verifies proof* π_I *that each position* $i \in I$ *has value* a_i *against digest* C.

(7) $\mathsf{UpdDig}_{\mathsf{pp}}(u, \delta, \mathsf{C}) \to \mathsf{C}'$*: Updates digest* C *to* C′ *to reflect position* u *changing by* $\delta \in \mathbb{Z}_p$.

(8) $\mathsf{UpdAllProofs}_{\mathsf{pp}}(u, \delta, \pi_0, \ldots, \pi_{n-1}) \to (\pi'_0, \ldots, \pi'_{n-1})$*: Updates* all *proofs* π_i *to* π'_i *to reflect position* u *changing by* $\delta \in \mathbb{Z}_p$.

(9) $\mathsf{UpdProof}_{\mathsf{pp}}(u, \delta, \pi_i) \to \pi'_i$*: Updates proof* π_i *to* π'_i *to reflect position* u *changing by* $\delta \in \mathbb{Z}_p$.

Observations: For simplicity, we give our algorithms oracle access to the public parameters pp of the scheme. This way, each algorithm can easily access the subset of the parameters it needs.

We formalize OpenAll and UpdAllProofs since, in some VCs, these algorithms are faster than n calls to Open and UpdProof, respectively. In this sense, we stress that the UpdAllProofs algorithm can work in sublinear time, since it does not necessarily need to read all input or write all output (e.g., in Merkle trees, UpdAllProofs only reads $\log n$ sibling hashes and overwrites another $\log n$ hashes).

Definition 2.2 (VC Correctness). *A VC is correct, if for all* $\lambda \in \mathbb{N}$ *and* $n = \mathsf{poly}(\lambda)$*, for all* $\mathsf{pp} \leftarrow \mathsf{Gen}(1^\lambda, n)$*, for all vectors* $\mathbf{a} = [a_0, \ldots, a_{n-1}]$*, if* $\mathsf{C} = \mathsf{Com}_{\mathsf{pp}}(\mathbf{a})$ *and* $\pi_i = \mathsf{Open}_{\mathsf{pp}}(i, \mathbf{a}), \forall i \in [0, n)$ *(or from* $\mathsf{OpenAll}_{\mathsf{pp}}(\mathbf{a})$*), then, for any polynomial number of updates* (u, δ) *resulting in a new vector* $\mathbf{a}'$*, if* C′ *and* π'_i*, for all* i*, are the updated digest and proofs obtained via calls to* $\mathsf{UpdDig}_{\mathsf{pp}}$ *and* $\mathsf{UpdProof}_{\mathsf{pp}}$ *(or to* $\mathsf{UpdAllProofs}_{\mathsf{pp}}$*) respectively, then (1)* $\Pr[1 \leftarrow \mathsf{Ver}_{\mathsf{pp}}(\mathsf{C}', \{i\}, a'_i, \pi'_i)] = 1$ *for all* i*; (2)* $\forall I \subseteq [n], \Pr[1 \leftarrow \mathsf{Ver}_{\mathsf{pp}}(\mathsf{C}', I, (a'_i)_{i \in I}, \mathsf{Agg}_{\mathsf{pp}}(I, (a'_i, \pi'_i)_{i \in I}))] = 1$.

Definition 2.3 (VC Soundness). $\forall$ ppt *adversaries* $\mathcal{A}$,

$$\Pr\left[\begin{array}{c} \mathsf{pp} \leftarrow \mathsf{Gen}(1^\lambda, n), \\ (\mathsf{C}, I, J, (a_i)_{i \in I}, (a'_j)_{j \in J}, \pi_I, \pi'_J) \leftarrow \mathcal{A}(1^\lambda, \mathsf{pp}) : \\ 1 \leftarrow \mathsf{Ver}_{\mathsf{pp}}(\mathsf{C}, I, (a_i)_{i \in I}, \pi_I) \wedge \\ 1 \leftarrow \mathsf{Ver}_{\mathsf{pp}}(\mathsf{C}, J, (a'_j)_{j \in J}, \pi'_J) \wedge \\ \exists k \in I \cap J \text{ s.t. } a_k \neq a'_k \end{array}\right] \leq \mathsf{negl}(\lambda)\,.$$

Observation: This soundness definition allows the digest C to be produced adversarially. This is stronger than what is required in our cryptocurrency setting from §4, where the digest is produced correctly from the agreed transactions. Nonetheless, having a stronger definition makes our VC from §3 more widely useful.

2.4 Inner Product Arguments (IPA)

In this subsection, we describe a non-interactive inner-product argument (IPA) by Bünz et al. [15] where a *prover* convinces a *verifier*, who has a *commitment key* $\mathbf{ck}$ and a commitment $\mathbf{C}$, that the prover can open $\mathbf{C}$ to $\mathbf{A} \in \mathbb{G}_1^m, \mathbf{B} \in \mathbb{G}_2^m$ and $Z \in \mathbb{G}_T$ such that $Z = \langle \mathbf{A}, \mathbf{B} \rangle = \prod_{j=1}^{m} e(A_j, B_j)$. More formally, this is an argument for the language:

$$\mathcal{L}_{\mathsf{IPA}}^{m} = \left\{ (\mathbf{ck}, \mathbf{C}) \;\middle|\; \exists \mathbf{A} \in \mathbb{G}_1^m, \mathbf{B} \in \mathbb{G}_2^m, \text{ s.t. } \mathbf{C} = \mathsf{CM}(\mathbf{ck}; \mathbf{A}, \mathbf{B}, \langle \mathbf{A}, \mathbf{B} \rangle) \right\}$$

Commitments to group elements. Above, CM denotes the commitment scheme by Abe et al. [1] for vectors $\mathbf{A}, \mathbf{B} \in \mathbb{G}_1^m \times \mathbb{G}_2^m$ and their pairing product $Z = \langle \mathbf{A}, \mathbf{B} \rangle = \prod_{i=1}^{m} e(A_i, B_i)$. It uses a randomly-generated *commitment key* $\mathbf{ck} = (\mathbf{v}, \mathbf{w}) \in \mathbb{G}_1^m \times \mathbb{G}_2^m$. The commitment $\mathbf{C} \in \mathbb{G}_T^3$ to $\mathbf{A}, \mathbf{B}$ and Z is:

$$\mathbf{C} = \mathsf{CM}(\mathbf{ck}; \mathbf{A}, \mathbf{B}, Z) = (\langle \mathbf{A}, \mathbf{v} \rangle,\ \langle \mathbf{w}, \mathbf{B} \rangle,\ Z) \stackrel{\text{def}}{=} (C_1, C_2, C_3) \quad (12)$$

This commitment scheme is not hiding but is binding under SXDH (see Assumption A.1) [1, 2].

IPA algorithms. For ease of presentation, Fig. 3 describes the *interactive* variant of the IPA, where the prover interacts with the verifier over $\log m$ rounds. This interactive IPA is knowledge-sound assuming Abe et al. commitments are binding [15, Theorem 1]. It is made non-interactive via the Fiat-Shamir transform [19] and proved secure in a new *algebraic commitment model* and in the *random oracle model (ROM)* [15, Appendix D.2]. Our work uses this non-interactive variant, whose algorithms we give below:

(1) $\mathcal{G}_{\mathsf{IPA}}(1^\lambda, m) \to (PK, VK)$: Returns $PK = VK = \langle \mathsf{BilGen}(1^\lambda), \mathbf{ck} = (\mathbf{v} \in_R \mathbb{G}_2^m, \mathbf{w} \in_R \mathbb{G}_1^m) \rangle$.

(2) $\mathcal{P}_{\mathsf{IPA}}(PK, \mathbf{A}, \mathbf{B}) \to \pi$: Runs the Fiat-Shamir transformation of the interactive prover $\mathcal{P}_{\mathsf{IPA}}^{\leftrightarrow}$ from Fig. 3 on input $(\mathbf{ck}; \mathbf{A}, \mathbf{B})$, obtaining a transcript π. Returns the transcript π as the proof.

(3) $\mathcal{V}_{\mathsf{IPA}}(VK, \mathbf{C}, \pi) \to \{0, 1\}$: Runs the Fiat-Shamir transformation of the interactive verifier $\mathcal{V}_{\mathsf{IPA}}^{\leftrightarrow}$ from Fig. 3 on input $(\mathbf{ck}; \mathbf{C}, \pi)$, obtaining a success bit b. Returns b.

Faster verification in IPA. Bünz et al. show that both $\mathcal{P}_{\mathsf{IPA}}$ and $\mathcal{V}_{\mathsf{IPA}}$ take $O(m)$ time and the proof size is $|\pi| = O(\log m)$. They also show how to reduce $\mathcal{V}_{\mathsf{IPA}}$ time by using a "structured" commitment key $\mathbf{ck} = (\mathbf{v}, \mathbf{w})$, similar to a q-SDH common reference string (see Assumption A.2) This allows the verifier to outsource the computations of $\mathbf{v}'$ and $\mathbf{w}'$ to the untrusted prover and reduces verification time from $O(m)$ to $O(\log m)$. However, this comes at the cost of additionally relying on the q-SDH assumption (see Assumption A.2) and the q-ASDBP assumption (see Assumption A.4). Our work implicitly assumes this optimized $\mathcal{V}_{\mathsf{IPA}}$, which we later implement in §5. We refer the reader to [15, Section 5] for the details of this optimization, which is beyond the scope of this paper.

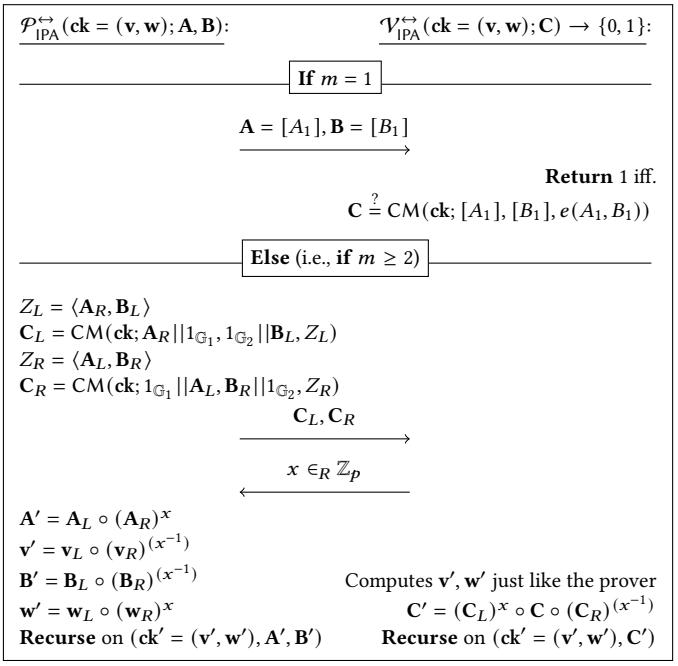


Figure 3: The *interactive* IPA by Bünz *et al.* for $m = 2^k$ (wlog). Its *non-interactive* counterpart is in §2.4, denoted by $\mathcal{P}_{\mathsf{IPA}}$ and $\mathcal{V}_{\mathsf{IPA}}$. The prover convinces the verifier that he knows $(\mathbf{A}, \mathbf{B}) \in \mathbb{G}_1^m \times \mathbb{G}_2^m$ such that $\mathbf{A}, \mathbf{B}, Z$ are committed in $\mathbf{C}$ (under commitment key $\mathbf{ck}$) and that $Z = \langle \mathbf{A}, \mathbf{B} \rangle$. See §2 for IPA-specific notation such as $1_{\mathbb{G}_b}$, $\mathbf{A}_L$, $\mathbf{A} \circ \mathbf{B}$, CM, $\mathbf{A}_R || 1_{\mathbb{G}}$ or $\mathbf{A}_L^x$.

3 HYPERPROOFS

In this section, we intuitively explain how Hyperproofs work, often referring to a *prover* who computes the vector's *digest*, as well as proofs, and to a *verifier* who verifies proofs against this digest. Without loss of generality, our discussion will assume vectors of size exactly $n = 2^\ell$. Hyperproofs represents a vector $\mathbf{a} = [a_0, \ldots, a_{n-1}]$ as a multilinear extension (MLE):

$$f(\mathbf{x}) = \sum_{i=0}^{n-1} a_i \mathcal{S}_{i,\ell}(\mathbf{x}),$$

where $\mathcal{S}_{i,\ell}$ are selector multinomials as per Eq. 5. The *digest* of the vector $\mathbf{a}$ is, a Papamanthou-Shi-Tamassia (PST) commitment to f:

$$\mathsf{pst}(f) = g_1^{f(\mathbf{s})},$$

where $\mathbf{s}$ is the PST trapdoor (see §2.2). Thus, our public parameters are the same as PST's parameters depicted in Fig. 2.

3.1 Multilinear trees (MLTs)

A Hyperproof for position i is just a PST evaluation proof (see §2.2) for $f(\mathbf{i})$. Unfortunately, if one uses the PST.Prove algorithm from Fig. 1 to compute *all* PST evaluation proofs, this takes $O(n^2)$ time. Below, we show how to compute all n proofs faster, in $O(n \log n)$ time, by avoiding unnecessary computations (see Fig. 5).

Denote the proof for $f(\mathbf{i})$ as $\pi_i = (\pi_{i,\ell}, \ldots, \pi_{i,1})$. Next, observe that if we compute all proofs π_i via n calls to:

$$\mathsf{PST.Prove}(f, \ell, (i_\ell, \ldots, i_1)), \forall i \in [n],$$

they actually all have the same first quotient q_ℓ committed in $\pi_{i,\ell}$! This is because all n PST.Prove calls initially divide f by $x_\ell - i_\ell$, which actually yields the same quotient, independent of i_ℓ. To see this, recall the MLE decomposition from Eq. 8 and reorganize it in

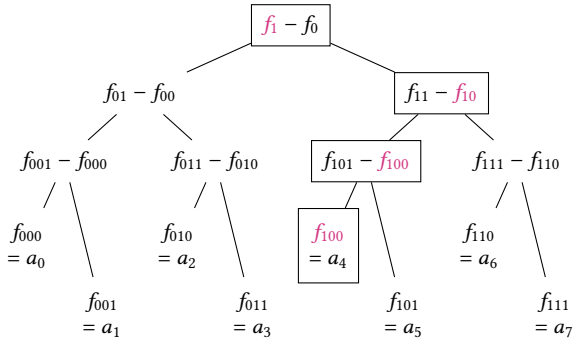


Figure 4: A multilinear tree (MLT) of size 8. Recall from §2 that $f_{b_\ell,\ldots,b_k}$ denotes the MLE of $\mathbf{a}_{b_\ell,\ldots,b_k}$. Each node stores a PST commitment to the depicted MLE: e.g., root stores $\mathsf{pst}(f_1 - f_0)$, not $f_1 - f_0$. The proof for a_i consists of all nodes along a_i's path to the root (e.g., for a_4, the boxed nodes). Sibling leaves $[a_{2j}, a_{2j+1}]$ have the same proof. If, say, a_4 changes, all pink-colored MLEs change, and all boxed commitments must be updated.

two ways as:

$$f = (1 - x_\ell) \cdot f_0 + x_\ell \cdot f_1 \Leftrightarrow$$
$$f = (f_1 - f_0) \cdot (x_\ell - 1) + f_1 \tag{13}$$
$$= (f_1 - f_0) \cdot x_\ell + f_0\,, \tag{14}$$

where f_0 is the MLE for the left half $\mathbf{a}_0$ of $\mathbf{a}$ and f_1 is the MLE for the right half $\mathbf{a}_1$ (recall from §2). Since both divisions yield the same $q_\ell = f_1 - f_0$ quotient, all π_i's share the same $\pi_{i,\ell}$ commitment to q_ℓ! We depict this q_ℓ as the root of a *multilinear tree (MLT)* in Fig. 4.

Next, recall that each one of the n PST.Prove calls recurses on its remainder, which we saw above was either f_0 or f_1. Specifically, the first $n/2$ calls for $i \in [0, n/2)$ (i.e., $i_\ell = 0$) recurse on $\mathsf{PST.Prove}(f_0, \ell - 1, (i_{\ell-1}, \ldots, i_1))$, and the other $n/2$ calls for $i \in [n/2+1, n)$ (i.e., $i_\ell = 1$) recurse on $\mathsf{PST.Prove}(f_1, \ell-1, (i_{\ell-1}, \ldots, i_1))$. But by the same argument above, each group of $n/2$ calls returns the same first quotient commitment. For example, for the first group, we have quotient $f_{01} - f_{00}$:

$$f_0 = (f_{01} - f_{00})(x_{\ell-1} - 1) + f_{01} \tag{15}$$
$$= (f_{01} - f_{00})x_{\ell-1} + f_{00}\,, \tag{16}$$

Similarly, for the second group, the quotient will be $f_{11} - f_{10}$. Both quotients are depicted as the children of the root in Fig. 4. Continuing recursively in this fashion yields our *multilinear tree (MLT)* from Fig. 4. We describe the algorithm for computing it in Fig. 5 and we argue correctness of MLT proofs in Appendix D.1.

3.2 Updates and homomorphism

Updating digests and MLTs. Suppose a_4 changes by δ in our MLT from Fig. 4. Then, by Eq. 4, we know that $\mathbf{a}$'s MLE will change to:

$$f' = f + x_3(1 - x_2)(1 - x_1)\delta = f + \mathcal{S}_{4,3}(\mathbf{x})\delta$$

$\mathsf{MLT.Compute}(f, \ell) \to [t_1, \ldots, t_{2^\ell - 1}]$:

1. If $\ell = 0$ (i.e., f is a constant), return $\emptyset$.
2. Otherwise, $\forall b \in \{0, 1\}$, divide f by $x_\ell - b$, obtaining quotient $f_1 - f_0$ and remainder f_b such that $f = (f_1 - f_0) \cdot (x_\ell - b) + f_b$.
3. Return $\left(g_1^{(f_1 - f_0)(\mathbf{s})}, \mathsf{MLT.Compute}(f_0, \ell - 1), \mathsf{MLT.Compute}(f_1, \ell - 1)\right)$

Figure 5: Computes an MLT in $O(n \log n)$ time consisting of PST evaluation proofs for *all* $f(\mathbf{i})$ w.r.t. an MLE f of $\mathbf{a}$ of size $n = 2^\ell$. In contrast, n naive calls to PST.Prove would take $O(n^2)$. Recall that f_0 and f_1 are MLEs for the left and right halves of $\mathbf{a}$. Returns the tree stored in preorder in an array.

But what about the MLT? The following highlighted MLEs from Fig. 4 will be updated to:

$$f'_{100} = f_{100} + \delta$$
$$f'_{10} = f_{10} + (1 - x_1)\delta$$
$$f'_1 = f_1 + (1 - x_2)(1 - x_1)\delta$$
$$f' = f + x_3(1 - x_2)(1 - x_1)\delta$$

These MLEs changing affect the MLEs along a_4's path. For example, the root MLE $f_1 - f_0$ also changes by the same amount as f_1: i.e., by $+(1-x_2)(1-x_1)\delta$. Furthermore, their corresponding commitments are easy to update via the PST homomorphism. For example, the new root will be $\mathsf{pst}(f_1 - f_0) \cdot g_1^{(1-s_2)(1-s_1)\delta}$. However, note that updating commitments requires knowing $g_1^{(1-s_2)(1-s_1)}$, which is referred to as an *update key*. We delve into this next.

Update keys. Recall that $\mathcal{S}_{u,k}(\mathbf{x})$ is the selector multinomial for position $u \in [0, 2^k)$ in an MLE of size 2^k (see Eq. 5). However, to easily reason about updates, it is useful to define $\mathcal{S}_{u,k}$ even when $u \geq 2^k$ as $\mathcal{S}_{u,k} = \mathcal{S}_{u \bmod 2^k, k}$. As explained above, updating the MLT after a_u changes by δ requires some auxiliary information referred to as an *update key* for position u. This consists of commitments to all selector multinomials for u in MLEs of size $1, 2, \ldots, 2^\ell$:

$$\mathsf{upk}_u = \left\{g_1^{\mathcal{S}_{u,k}(\mathbf{s})} : k \in [0, \ell]\right\} = \left\{\mathsf{upk}_{u,k} : k \in [0, \ell]\right\} \tag{17}$$

Recall that $\mathcal{S}_{u,0}(\mathbf{x}) = 1$, so that $\mathsf{upk}_{u,0} = g_1, \forall u \in [0, 2^\ell)$. Then, the MLT commitments $(w_{u,\ell}, \ldots, w_{u,1})$ along u's path are updated as:

$$w'_{u,k} = w_{u,k} \cdot (\mathsf{upk}_{u,k-1})^\delta = w_{u,k} \cdot (g_1^{\mathcal{S}_{u,k-1}(\mathbf{s})})^\delta, \forall k \in [\ell] \tag{18}$$

Note that this implies that any proof $\pi_i = (w_{i,\ell}, \ldots, w_{i,1})$ can be updated after a change at u: one simply has to identify the "intersection" of u's proof with i's proof and apply the update as above, as if updating a pruned MLT consisting of just π_i. More formally, suppose i and u have the same t most significant bits (i.e., $i_k = u_k, \forall k \in \{\ell, \ell - 1, \ldots, \ell - t + 1\}$). Then, the updated proof π'_i is initially set to π_i and (partially) updated as:

$$w'_{i,k} = w_{i,k} \cdot (\mathsf{upk}_{u,k-1})^\delta, \forall k \in \{\ell, \ldots, \ell - t\}, 1 \leq k \leq \ell \tag{19}$$

The digest updates more simply as:

$$\mathsf{pst}(f') = \mathsf{pst}(f) \cdot g_1^{\mathcal{S}_{u,\ell}(\mathbf{s})} = \mathsf{pst}(f) \cdot \left(\mathsf{upk}_{u,\ell}\right)^\delta \tag{20}$$

Lastly, we note that the update keys actually coincide with our public parameters (see Fig. 2).

$\mathcal{G}_{\mathsf{BATCH}}(1^\lambda, b, \ell) \to (PK, VK)$: Return $\mathcal{G}_{\mathsf{IPA}}(1^\lambda, b \cdot \ell)$

$\mathcal{P}_{\mathsf{BATCH}}(PK, (\mathbf{A}_i, \mathbf{B}_i)_{i\in[b]}) \to \pi$:

1. Let $\mathbf{A} = [\mathbf{A}_1 || \mathbf{A}_2 || \ldots || \mathbf{A}_b]$ and $\mathbf{B} = [\mathbf{B}_1 || \mathbf{B}_2 || \ldots || \mathbf{B}_b]$
2. Let $C_1 = \langle \mathbf{A}, \mathbf{v} \rangle$ (i.e., 1st component of $\mathsf{CM}(\mathbf{ck}; \mathbf{A}, \mathbf{B}', 1_{\mathbb{G}_T})$)
3. Let $r_i = H(C_1, \mathbf{B}, i) \in \mathbb{Z}_p$ for $i = 1, \ldots, b$
4. Let $\mathbf{B}' = [\mathbf{B}_1^{r_1} || \mathbf{B}_2^{r_2} || \ldots || \mathbf{B}_b^{r_b}]$
5. Let $\pi^* = \mathcal{P}_{\mathsf{IPA}}(\mathbf{ck}, \mathbf{A}, \mathbf{B}')$ and return $\pi = (C_1, \pi^*)$.

$\mathcal{V}_{\mathsf{BATCH}}(VK, (\mathbf{B}_i, Z_i)_{i\in[b]}, \pi) \to \{0, 1\}$:

1. Parse the proof $\pi = (C_1, \pi^*)$
2. Let $r_i = H(C_1, \mathbf{B}, i) \in \mathbb{Z}_p$ for $i = 1, \ldots, b$
3. Let $\mathbf{B}' = [\mathbf{B}_1^{r_1} || \mathbf{B}_2^{r_2} || \ldots || \mathbf{B}_b^{r_b}]$ and $Z' = \prod_{i=1}^b Z_i^{r_i}$
4. Let $C_2 = \langle \mathbf{w}, \mathbf{B}' \rangle$ (i.e., 2nd component of $\mathsf{CM}(\mathbf{ck}; \mathbf{A}, \mathbf{B}', 1_{\mathbb{G}_T})$)
5. Let $\mathbf{C} = (C_1, C_2, Z')$ and return $\mathcal{V}_{\mathsf{IPA}}(\mathbf{ck}, \mathbf{C}, \pi^*)$.

Figure 6: Our argument for $\mathcal{L}^{b,\ell}_{\mathsf{BATCH}}$ used to aggregate Hyperproofs. H is a random oracle and $(\mathcal{G}_{\mathsf{IPA}}, \mathcal{P}_{\mathsf{IPA}}, \mathcal{V}_{\mathsf{IPA}})$ is the Bünz et al. IPA from §2.4.

MLTs are homomorphic. Since our multilinear tree stores an MLE commitment at each node, we observe that the MLT itself is *homomorphic*: the MLT for $\mathbf{a}+\mathbf{b}$ can be obtained by "multiplying" $\mathbf{a}$'s MLT with $\mathbf{b}$'s MLT. In other words, every node w in the new MLT is the product of the nodes w in the MLTs for $\mathbf{a}$ and $\mathbf{b}$. Specifically, $\mathsf{pst}(f''_w) = \mathsf{pst}(f_w + f'_w) = \mathsf{pst}(f_w)\mathsf{pst}(f'_w)$, where f_w, f'_w, f''_w denote the MLE stored at node w in the MLT for $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{a}+\mathbf{b}$, respectively. This enables our unstealability construction from §3.4 and has other applications to authenticating data in the *streaming* setting [38].

3.3 Aggregating proofs

Recall that a proof $(w_1, \ldots, w_\ell)$ for a_i in the vector $\mathbf{a}$ of size $n = 2^\ell$ is just an ℓ-sized PST evaluation proof (see §2.2) and verifies as:

$$e(\mathsf{C}/g_1^{a_i}, g_2) = \prod_{k=1}^{\ell} e(w_k, g_2^{s_k - i_k}), \tag{21}$$

where C is the digest and $(g_2^{s_k - i_k})_{k\in[\ell]}$ is position i's public *verification key*.

Warm-up: Compressing proofs. It is useful to first discuss compressing a size-ℓ proof for a_i to size $\log \ell$ via the IPA from §2.4. For this, we let $\mathbf{A} = [w_1 \ldots w_\ell], \mathbf{B} = [g_2^{s_1 - i_1} \ldots g_2^{s_\ell - i_\ell}], Z = e(\mathsf{C}/g_1^{a_i}, g_2)$ and prove that $(Z, \mathbf{B})$ is in the following language:

$$\mathcal{L}^{\ell}_{\mathsf{PROD}} = \left\{ Z \in \mathbb{G}_T, \mathbf{B} \in \mathbb{G}_2^\ell \;\middle|\; \exists \mathbf{A} \in \mathbb{G}_1^\ell, Z = \langle \mathbf{A}, \mathbf{B} \rangle \right\} \tag{22}$$

Next, we can use the IPA from §2.4. Specifically, assume our $\mathcal{L}_{\mathsf{PROD}}$ prover and verifier share a commitment key $\mathbf{ck} = (\mathbf{v}, \mathbf{w})$. First, the prover gives $C_1 = \langle \mathbf{A}, \mathbf{v} \rangle$ to the verifier. Second, the verifier computes $C_2 = \langle \mathbf{w}, \mathbf{B} \rangle$ and lets $C_3 = Z$. Thus, the verifier now has a commitment $\mathbf{C} = (C_1, C_2, C_3)$ to $\mathbf{A}$,$\mathbf{B}$ and Z. Third, the prover simply runs $\mathcal{P}_{\mathsf{IPA}}$ from §2.4 and convinces the verifier that the committed values satisfy $Z = \langle \mathbf{A}, \mathbf{B} \rangle$ and thus that the Hyperproof verifies as per Eq. 21.

Aggregating proofs. Next, we observe that aggregating many proofs $(\pi_1, \ldots, \pi_b)$, each for a position p_i in $\mathbf{a}$, reduces to proving membership in $\mathcal{L}^{\ell}_{\mathsf{PROD}}$ *for each* $(Z_i, \mathbf{B}_i)$, where $Z_i = e(\mathsf{C}/g_1^{a_{p_i}}, g_2)$ and $\mathbf{B}_i$ is position p_i's verification key. But doing this naively would result in a large, $O(b \log \ell)$ aggregated proof size. Instead, we seek a more succinct argument for the following new language:

$$\begin{aligned}\mathcal{L}^{b,\ell}_{\mathsf{BATCH}} &= \left\{ (Z_i \in \mathbb{G}_T, \mathbf{B}_i \in \mathbb{G}_2^\ell)_{i\in[b]} \;\middle|\; ((Z_i, \mathbf{B}_i) \in \mathcal{L}^{\ell}_{\mathsf{PROD}})_{i\in[b]} \right\} \\ &= \left\{ (Z_i \in \mathbb{G}_T, \mathbf{B}_i \in \mathbb{G}_2^\ell)_{i\in[b]} \;\middle|\; (\exists \mathbf{A}_i \in \mathbb{G}_1^\ell, Z_i = \langle \mathbf{A}_i, \mathbf{B}_i \rangle)_{i\in[b]} \right\}\end{aligned} \tag{23}$$

In other words, membership in $\mathcal{L}^{b,\ell}_{\mathsf{BATCH}}$ guarantees that $\forall i \in [b], \exists \mathbf{A}_i$:

$$Z_i = \prod_{j=1}^{\ell} e(A_{i,j}, B_{i,j}), \tag{24}$$

where $A_{i,j}$ is the jth entry of $\mathbf{A}_i$. Note that we cannot use the TIPP argument from [15] to prove membership in $\mathcal{L}_{\mathsf{BATCH}}$, since it can only prove that $\forall i, Z_i = \langle X_i, Y_i \rangle$, where $(X_i, Y_i) \in \mathbb{G}_1 \times \mathbb{G}_2$. Instead, we design a new argument for $\mathcal{L}^{b,\ell}_{\mathsf{BATCH}}$ (see Fig. 6) which uses a random linear combination to combine the ℓ-sized equations from above into a single $b\ell$-sized one:

$$\prod_{i=1}^{b} Z_i^{r_i} = \prod_{i=1}^{b} \left(\prod_{j=1}^{\ell} e(A_{i,j}, B_{i,j}) \right)^{r_i} \tag{25}$$

$\mathsf{Gen}(1^\lambda, n) \to \mathsf{pp}$: Let $(p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, e, g_1, g_2) \leftarrow \mathsf{BilGen}(1^\lambda)$. Let $\mathbf{s} = (s_1, \ldots, s_\ell) \in_R \mathbb{Z}_p^\ell$, where $n = 2^\ell$. Let pp consist of

- $\mathsf{pst}(\mathcal{S}_{j,k}) = g_1^{\mathcal{S}_{j,k}(\mathbf{s})}, \forall k \in [0, \ell], \forall j \in [0, 2^k)$;
- $g_2^{s_k}, \forall k \in [\ell]$;
- $(PK, VK) \leftarrow \mathcal{G}_{\mathsf{BATCH}}(1^\lambda, b, \ell)$.

We refer to $(g_2^{s_k - i_k})_{k\in[\ell]}$ as position i's *verification key.*

$\mathsf{Com}_{\mathsf{pp}}(\mathbf{a}) \to \mathsf{C}$: Let $\mathsf{C} = \mathsf{pst}(f) = g_1^{f(\mathbf{s})} = g_1^{f(s_1, \ldots, s_\ell)}$, where f is $\mathbf{a}$'s MLE.

$\mathsf{OpenAll}_{\mathsf{pp}}(\boldsymbol{a}) \to (\pi_0, \ldots, \pi_{n-1})$: Return the MLT as per §3.1.

$\mathsf{Open}_{\mathsf{pp}}(i, \mathbf{a}) \to \pi_i$: Compute only the ith path in the MLT and return it.

$\mathsf{Agg}_{\mathsf{pp}}(I, \{a_i, \pi_i\}_{i\in I}) \to \pi_I$: Let $m = |I|$ and let $\mathbf{A}_1, \mathbf{A}_2, \ldots, \mathbf{A}_m$ denote proofs $(\pi_i)_{i\in I}$, ordered by i, and $\mathbf{B}_1, \ldots, \mathbf{B}_m$ denote their corresponding verification keys. Return $\mathcal{P}_{\mathsf{BATCH}}(PK, (A_k, B_k)_{k\in[m]})$.

$\mathsf{Ver}_{\mathsf{pp}}(\mathsf{C}, I, \{a_i\}_{i\in I}, \pi_I) \to \{0, 1\}$: If $I = \{i\}$, parse $\pi_I = (w_1, \ldots, w_\ell)$ and ensure that

$$e(\mathsf{C}/g_1^{a_i}, g_2) = \prod_{j=1}^{\ell} e(w_j, g_2^{s_j - i_j}).$$

Otherwise, let $m = |I|$ and $\mathbf{B}_1, \ldots, \mathbf{B}_m$ denote the verification keys for the proofs, ordered by their position i. Let $Z_1, Z_2, \ldots, Z_m$ be all the $e(\mathsf{C}/g_1^{a_i}, g_2)$'s, also ordered by i. Return $\mathcal{V}_{\mathsf{BATCH}}(VK, (B_k, Z_k)_{k\in[m]}, \pi_I)$.

$\mathsf{UpdDig}_{\mathsf{pp}}(u, \delta, \mathsf{C}) \to \mathsf{C}'$: Let $\mathsf{C}' = \mathsf{C} \cdot (g_1^{\mathcal{S}_{u,\ell}(\mathbf{s})})^\delta$.

$\mathsf{UpdAllProofs}_{\mathsf{pp}}(u, \delta, \pi_0, \ldots, \pi_{n-1}) \to (\pi'_0, \ldots, \pi'_{n-1})$: Assume u's MLT path is $(w_1, \ldots, w_\ell)$. Update this path as $w'_k = w_k \cdot (\mathsf{upk}_{u,k-1})^\delta$ (for $k = 1, \ldots, \ell$) as per Eq. 18.

$\mathsf{UpdProof}_{\mathsf{pp}}(u, \delta, \pi_i) \to \pi'_i$: Update via UpdAllProofs as if π_i was a pruned, single-path MLT (see Eq. 19).

Figure 7: Algorithms for Hyperproofs, implicitly parameterized by the max number of proofs b that can be aggregated into a single proof.

It is well known that, if the r_i's are uniformly random, verifying the combined equation above is sufficient (see Lemma C.1). As a result, our argument for $\mathcal{L}^{b,\ell}_{\mathsf{BATCH}}$ uses the IPA from §2.4 on this combined equation in a black-box manner. (This is similar to the previous $\mathcal{L}^{\ell}_{\mathsf{PROD}}$ argument, except it involves larger vectors and randomization.) We give a precise description of its $(\mathcal{G}_{\mathsf{BATCH}}, \mathcal{P}_{\mathsf{BATCH}}, \mathcal{V}_{\mathsf{BATCH}})$ algorithms in Fig. 6, show how it fits in our VC construction in Fig. 7, and prove security in Appendix C.3.

Aggregation time and proof size. It is easy to see from Fig. 6 that the $\mathcal{P}_{\mathsf{BATCH}}$ time (i.e., the time to aggregate b proofs) is $O(b \cdot \ell)$ and the $\mathcal{V}_{\mathsf{BATCH}}$ time (i.e., the time to verify the aggregated proof) is $O(b \cdot \ell)$. Unfortunately, even though our $\mathcal{L}^{b,\ell}_{\mathsf{BATCH}}$ argument uses the IPA with fast, $O(\log(b \cdot \ell))$-time KZG-based verification (see §2.4), the $\mathcal{V}_{\mathsf{BATCH}}$ verifier still needs to do $O(b \cdot \ell)$ work on the $\mathbf{B}_i$ vectors. (Note that this $O(b \cdot \ell)$ verifier work seems inherent for processing the b verification keys.) Lastly, the argument size (i.e., the aggregated proof size) is $O(\log(b \cdot \ell)) = O(\log b + \log \ell)$.

Cross-aggregation. In addition to aggregating proofs w.r.t. the some commitment C, we can also *cross-aggregate* proofs w.r.t. different commitments [21]. Specifically, suppose we have b proofs π_i for positions p_i, each w.r.t. a (potentially-different) digest C_i for a vector with MLE f_i. Then, we can use the same $\mathcal{P}_{\mathsf{BATCH}}$ prover from Fig. 6 to cross-aggregate these proofs. To verify, the verifier now computes the Z_i's from Eq. 24 by using the right commitment and evaluation point: i.e., $Z_i = e(C_i / g_1^{f_i(p_i)}, g_2)$. For simplicity, we do not formalize VCs with cross-aggregation, but our soundness proofs from Appendix D.3 can easily carry over to this setting.

3.4 Unstealable proofs

In this subsection, we show how to incentivize proof computation by allowing provers, who store the vector and maintain proofs, to *watermark* the proofs they compute. Such watermarked proofs are cryptographically-bound to their prover's identity, which means the prover can be monetarily rewarded for having computed them (e.g., in cryptocurrencies). Importantly, this cryptographic binding cannot be undone by adversaries. In other words, "stealing" a proof by replacing its watermark with your own, is no easier than computing the proof from scratch like everyone else. We call such watermarked proofs *unstealable*, formalize and prove their security and make Hyperproofs unstealable. We show unstealability of proofs is useful in the cryptocurrency setting in §4 and we envision other applications could benefit from it.

Strawman: unstealability via digital signatures. Note that a digital signature on the VC proof will not make it unstealable. This is because the signature, which is appended to the VC proof, can be simply removed by the adversary and replaced with his own signature. Instead, we need a different approach that somehow embeds the signature into the proof itself so that it cannot be removed.

Unstealability via exponentiations. We make a proof $\boldsymbol{\pi}_i = (w_{i,\ell}, \ldots, w_{i,1})$ unstealable by exponentiating it with α as:

$$\boldsymbol{\pi}_i^{\alpha} = (w_{i,\ell}^{\alpha}, \ldots, w_{i,1}^{\alpha}) \stackrel{\mathsf{def}}{=} (\hat{w}_{i,\ell}, \ldots, \hat{w}_{i,1}), \tag{26}$$

where α is the prover's *watermarking secret key (WSK)*. The corresponding *watermarking public key (WPK)* is g_2^{α} together with a zero-knowledge proof of knowledge (ZKPoK) of α (e.g., a Schnorr proof [42] as per Appendix B). To verify a proof watermarked with g_2^{α}, one first checks that the ZKPoK of α verifies. Second, one checks the proof as normal as per Eq. 21, but accounts for the WPK g_2^{α}:

$$e(C / g_1^{a_i}, g_2^{\alpha}) \stackrel{?}{=} \prod_{k \in [\ell]} e(\hat{w}_{i,k}, g_2^{s_k - i_k}) \tag{27}$$

The key intuition is that an adversary can no longer remove α from the watermarked proof, since this seems to require exponentiating by α^{-1}, which the adversary does not know. Indeed, in Appendix D.5, we prove that no PPT adversary can steal proofs under ℓ-DHI (see Assumption A.3).

Aggregation-preserving unstealability. One important property of our unstealable proofs is that they remain aggregatable via a call to Agg from Fig. 7. Intuitively, this is because the right-hand side of the watermarked verification from Eq. 27 remains the same as for normal verification in Eq. 21. However, the left-hand side changes. Thus, when verifying an aggregated proof via Ver (see Fig. 7), the verifier has to account for the WPKs when computing the Z_i's. In other words, the verifier needs to have these WPKs. In our application setting from §4, we anticipate the verifier will already have all of the WPKs, instead of receiving them with the aggregated proof.

Homomorphism-preserving unstealability. Our approach to watermarking proofs preserves the PST and MLT homomorphisms. This has a few advantages. First, watermarked proofs can still be updated. Specifically, assuming position u changed by δ, the watermarked proof $\boldsymbol{\pi}_i^{\alpha}$ from Eq. 26 can be updated as before (see Eq. 19) if one uses *watermarked update keys* $(\mathsf{upk}_{u,k})^{\alpha}$. Second, an MLT of watermarked proofs can be computed directly, if the prover uses *watermarked public parameters*. The prover can obtain these in a one-time pre-processing step that exponentiates all parameters from Fig. 2 with the WSK α:

$$\left(g_1^{\mathcal{S}_{u,k}(\mathbf{s})}\right)^{\alpha} = g_1^{\alpha \mathcal{S}_{u,k}(\mathbf{s})} \stackrel{\mathsf{def}}{=} \hat{g}_1^{\mathcal{S}_{u,k}(\mathbf{s})}, \forall k \in [0, \ell], \forall u \in [0, 2^k) \tag{28}$$

Importantly, these are still valid Hyperproofs parameters, except under a new base $\hat{g}_1 = g_1^{\alpha}$. As a result, the prover can directly compute a *watermarked MLT* using these new parameters. This is important, as it allows precomputing watermarked proofs, ensuring that serving such proofs is as efficient as serving normal proofs. Third, a watermarked MLT is efficiently maintainable, just like a normal MLT. This follows from the updatability of watermarked proofs argued above.

New UVC algorithms. We must slightly change our VC API from Definition 2.1 into an *unstealable VC (UVC)* API that accounts for watermarked proofs and watermarking key-pairs. First, we introduce two new algorithms:

1. $\mathsf{WtrmkGen}(1^{\lambda}) \to (\mathsf{wsk}, \mathsf{wpk})$: Generates a random $(\mathsf{wsk}, \mathsf{wpk})$ watermarking key pair.
2. $\mathsf{WtrmkParams}(\mathsf{pp}, \mathsf{wsk}) \to \mathsf{wpp}$: Returns watermarked public parameters wpp, under wsk, as per Eq. 28.

Second, the verification algorithm $\mathsf{Ver}_{\mathsf{pp}}(C, I, (a_i, \mathsf{wpk}_i)_{i \in I}, \pi_I)$ additionally takes as input the watermarking PK wpk_i that each proof π_i is watermarked with. Third, the algorithms for creating and

updating proofs now operate on *watermarked* public parameters. In the interest of brevity, we give the full UVC API, with a new correctness definition, in the appendix in Definition D.1.

UVC soundness. We model UVC soundness similar to VC soundness, except we account for watermarked proofs (see Definition D.3 in the appendix). Informally, we prevent adversaries from creating two inconsistent proofs for the same index k, even if those proofs are watermarked with different, adversarially-generated WPKs.

UVC unstealability. We also need to formally define our new unstealability notion, which we only explained intuitively so far. We do this via a game where an adversary sees a commitment to an unknown vector and up to $n-1$ watermarked proofs for the vector. His goal is to output a new proof watermarked under a WPK he knows. We restrict ourselves to individual proofs only (see Definition 3.1), but generalize to aggregated proofs in the appendix (see Definition D.4).

Definition 3.1 (UVC Unstealability for individual proofs). $\forall$ ppt *adversaries* $\mathcal{A} = (\mathcal{A}_0, \mathcal{A}_1)$,

$$\Pr\left[\begin{array}{c} \mathsf{pp} \leftarrow \mathsf{Gen}(1^\lambda, n), \\ (\mathsf{wsk}, \mathsf{wpk}) \leftarrow \mathsf{WtrmkGen}_{\mathsf{pp}}(1^\lambda), \\ \mathsf{wpp} \leftarrow \mathsf{WtrmkParams}(\mathsf{pp}, \mathsf{wsk}), \\ \mathsf{C} \leftarrow \mathsf{Com}_{\mathsf{pp}}(\boldsymbol{a}), \text{ where } a_i \in_R \mathbb{Z}_p, \\ (I \subsetneq [n], \mathsf{st}) \leftarrow \mathcal{A}_0(1^\lambda, \mathsf{pp}, \mathsf{C}, \mathsf{wpk}), \\ \left(\pi_i \leftarrow \mathsf{Open}_{\mathsf{wpp}}(i, \boldsymbol{a})\right)_{i \in I}, \\ (j \in [n], \pi', \mathsf{wpk}') \leftarrow \mathcal{A}_1(1^\lambda, \mathsf{st}, (a_i, \pi_i)_{i \in I}) : \\ \mathsf{Ver}_{\mathsf{pp}}(\mathsf{C}, \{j\}, (a_j, \mathsf{wpk}'), \pi') = 1 \wedge \mathsf{wpk} \neq \mathsf{wpk}' \end{array}\right] \leq \mathsf{negl}(\lambda)\,.$$

Observation: "Stealing" proofs implicitly assumes $\mathcal{A}$ does not have (and cannot predict) the vector. This is why $\mathbf{a}$ is randomly picked: to discount non-attacks such as $\mathcal{A}$ computing a proof from scratch (via knowledge of the vector) and watermarking it.

3.5 Analysis

Correctness and security. Correctness follows by inspection, but we do argue correctness of the MLT in Appendix D.1. We prove our VC is sound as per Definition 2.3 in Appendix D.3. We prove our unstealable VC is (1) sound as per Definition D.3 in Appendix D.4 and (2) unstealable as per Definition 3.1 in Appendix D.5.

Asymptotics. Gen can compute all $O(n)$ public parameters from Fig. 2 in $O(n)$ time, since it knows the trapdoors $(s_1, \ldots, s_\ell)$. Com runs in time $O(n)$, requiring one exponentiation per vector element. OpenAll takes $T(n) = 2T(n/2)+O(n) = O(n \log n)$ exponentiations, since it must commit to all MLEs in Fig. 4. Open only takes $T(n) = T(n/2)+O(n) = O(n)$ exponentiations, since it must commit to one path of MLEs from Fig. 4. UpdDig takes one exponentiation as per Eq. 20 and UpdProof takes ℓ exponentiations as per Eq. 19. Finally, Agg takes $O(b\ell)$ time, where b is the number of proofs that are aggregated, outputting a proof of size $O(\log b + \log \ell)$, which can be verified by Ver in $O(b\ell)$ time. A detailed asymptotic comparison of our VC with other works can be found in the appendix in Table 4.

4 HYPERPROOFS FOR CRYPTOCURRENCIES

In this section, we discuss how Hyperproofs can be used to speed up validation in *payment-only* stateless cryptocurrencies.

Stateless validation. In account-based cryptocurrencies [53], *validators* such as miners and P2P nodes store a large amount of *state* to validate transactions and blocks in the consensus protocol. This state consists of each user's account balance and can be represented as a vector that maps each user's public key to their balance. Recent work [11, 18, 21, 31, 41, 46, 49] trades off storage of the state with additional bandwidth and computation. This approach, known as *stateless validation*, commits to the state using a vector commitment (VC) scheme and allows validators to store only the digest rather than the full state. Next, transactions and blocks are augmented with proofs for the accessed state, so validators can check validity against the digest, instead of storing the full state.

Challenges. There are several challenges in stateless validation. First, when creating a transaction, the sending user needs to include a proof that they have enough balance. In this sense, users should be able to fetch their proof from *proof-serving nodes (PSNs)* [41, 49], who maintain (a subset of) all proofs. Thus, *PSNs should be able to efficiently update all proofs*, as new blocks are confirmed. Second, *PSNs should be incentivized to maintain proofs*. Third, a miner must now include each transaction's proof in a proposed block, so that other miners can statelessly validate this block. This calls for *proofs to be efficiently aggregatable*, to save block space. Finally, when validating a block, miners must now verify such an aggregated proof. Thus, *aggregated proofs should verify fast.*

Why rely on proof-serving nodes? In theory, each user can maintain their proof locally by keeping up with all confirmed transactions and updating their proof (e.g., as per Eq. 19). However, this overwhelms users with large computation (i.e., updating proofs) and large communication (i.e., downloading new blocks). This is why well-incentivized, efficient proof-serving nodes (PSNs) are important: they eliminate this burden from users by allowing them to fetch their latest proof.

Hyperproofs for stateless validation As described above, in the stateless validation setting, it is important to minimize the time for (1) PSNs to update all proofs to reflect the latest block, (2) miners to propose a new block, with aggregated proofs and (3) other miners to verify this block, including its aggregated proof. In §5.3, we show experimentally that Hyperproofs outperforms other VCs in this task. This is because VCs with $O(1)$-sized proofs [16, 17, 21, 27, 49] require $O(n)$ time to update all proofs, while Hyperproofs only requires $O(\log n)$. Furthermore, when compared to Merkle trees, aggregation is 10× to 100× faster in Hyperproofs (see §5.2).

5 EVALUATION

In this section, we measure the performance of Hyperproofs and explore their applicability for stateless validation. We do not compare to VCs with constant-sized proofs due to their impractical $O(n)$ cost to update all proofs. Instead, we focus on Merkle trees with SNARK-based aggregation. We use the Golang bindings of the `mcl` library [33] to implement Hyperproofs. We use BLS12-381, a pairing-friendly elliptic curve, which offers 128 bits of security. A serialized $\mathbb{G}_1, \mathbb{G}_2$ and $\mathbb{G}_T$ element in `mcl` takes 48, 96, and 576 bytes, respectively. A single exponentiation takes 106 μs in $\mathbb{G}_1$ and 250 μs in $\mathbb{G}_2$. Each experiment ran *single-threaded* on an Intel Core i7-4770 CPU @ 3.40GHz with 8 cores and 32 GiB of RAM. Unless stated

$\ell = \log_2 n$	**22**	**24**	**26**	**28**	**30**
Com (min)	3.15	12.61	50.44	201.79*	807.19*
OpenAll (hrs)	0.644	2.78	12.06*	52.19*	225.80*
UpdDig			47.76 μs		
UpdAllProofs (ms)	1.74	1.96	2.15	2.37	2.58
Indiv. Ver (ms)	8.15	8.22	9.10	10.09	10.93
Agg (s)	105.14	109.49	114.03	118.31	122.62
Aggr. Ver (s)	12.94	14.06	15.11	16.22	17.41
Indiv. proof size (KiB)	1.06	1.15	1.25	1.34	1.44
Aggr. proof size (KiB)			51.6		

Table 2: Single-threaded microbenchmarks for Hyperproofs. Running times with an asterisk symbol (*) are too long and have been interpolated. We measure aggregation of 1024 proofs. OpenAll and Com are only measured once. UpdDig and UpdAllProofs times are averages after a batch of 1024 changes to the vector. All algorithms are parallelizable.

otherwise, we perform 4 runs of each experiment and report their average. Also, vectors in this section are of size $n = 2^\ell$.

5.1 Microbenchmarks

We microbenchmark Hyperproofs in Table 2. All microbenchmarks pick vectors and updates randomly and are *single-threaded*, but trivially parallelizable.

Public parameters. To commit to vectors of size n, Hyperproofs needs a large *proving key* consisting of $2n-1$ $\mathbb{G}_1$ elements depicted in Fig. 2. For $\ell = 28$, this requires around 24 GiB of space (see Table 5). *Verification keys* are all derived from $(g_2^{s_k})_{k\in[\ell]}$. Furthermore, to aggregate b proofs, Abe et al. commitment keys [2] are needed consisting of $\ell \cdot b$ $\mathbb{G}_1$ and $\ell \cdot b$ $\mathbb{G}_2$ elements. For $\ell = 28$ and $b = 1024$, this only adds 3.94 MiB. Watermarking the public parameters as per §3.4 requires $2n-1$ exponentiations in $\mathbb{G}_1$. For $\ell = 28$, this takes 15.87 hours. However, this is a one-time cost.

Committing and computing multilinear trees. We commit to a vector of size n via an $O(n)$-sized multi-exponentiation. For $\ell = 28$, this takes 202 minutes. Computing a multilinear tree (MLT) involves committing to the MLEs in each node via a multi-exponentiation (see Fig. 4). For $\ell = 28$, this takes 52.2 hours (or 1.63 hours with 32 threads). We expect to at least double performance via faster multi-exponentiation algorithms, which `mcl` lacks.

Updating the digest and the multilinear tree. For updating the digest, we measure the time to apply a batch of 1024 updates via a multi-exponentiation, divide this time by 1024 and obtain an average time of 48 μs per update. For the MLT, we also measure the time to apply a batch of 1024 updates. This way, we can use multi-exponentiations when updating nodes in the tree. Dividing the total time by 1024, gives us an average time of 1.74 ($\ell = 22$) to 2.58 milliseconds ($\ell = 30$) per update. Recall from §3.4 that updates will be just as fast for watermarked multilinear trees.

Proof size and verification time. Individual proof size is ℓ $\mathbb{G}_1$ elements and is competitive with Merkle trees (e.g., for $\ell = 30$, 1.44 KiB in MLTs versus 960 bytes in MHTs). Verifying a proof requires $\ell + 1$ pairings, which we optimize into a multi-pairing (i.e., first compute $\ell + 1$ Miller loops and then compute a single final exponentiation). This way, verifying a proof ranges from 8.2 ($\ell = 22$) to 11 milliseconds ($\ell = 30$). If the proof is watermarked, we discount the WPK from the proof size, since the verifier could already have the WPK, depending on the application. Furthermore, this overhead would be acceptable: 224 bytes (see Appendix B). Lastly, verifying the ZKPoK for the WPK requires two $\mathbb{G}_2$ exponentiations, which adds around 500 μs to the proof verification time.

Proof aggregation. Let I be the set of transactions to be aggregated via $\mathsf{Agg}_{\mathsf{pp}}$, which calls $\mathcal{P}_{\mathsf{BATCH}}$ from Fig. 6. In our benchmarks, $b = |I| = 1024$. As shown in Table 2, aggregating 1024 transactions takes between 105 ($\ell = 22$) to 123 seconds ($\ell = 30$). Verifying such an aggregated proof takes between 13 ($\ell = 22$) to 17.5 ($\ell = 30$) seconds. These times are not affected by watermarking. In §5.2, we show our aggregation is 10× to 100× faster than SNARKs.

Aggregated proof size. Our aggregated proof size is 52 KiB for any $\ell = 22, \ldots, 30$. This is an artifact of the IPA proof size depending on the smallest power of two $\geq \log(b \cdot \ell)$, which is the same for all ℓ's above when $b = 1024$. As with individual proofs, watermarking does not affect proof size when the verifier has the WPKs.

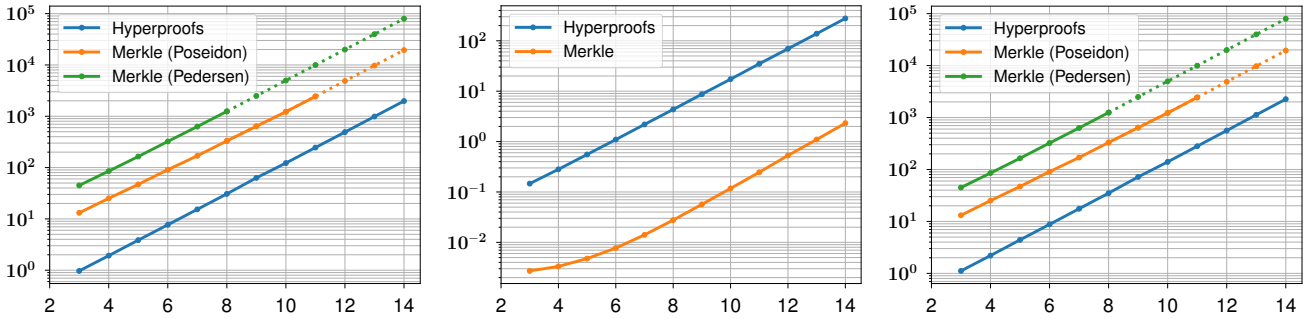
5.2 Comparison with SNARKs

In this subsection, we show that Hyperproof aggregation is anywhere from 10× to 100× faster than Merkle proof aggregation via SNARKs (see Fig. 8), depending on the choice of hash function. This comes at the cost of larger proofs (52 KiB versus 192 bytes) and slower verification. Nonetheless, the end-to-end time to prove-and-verify remains around 10× to 100× faster in Hyperproofs.

Experimental setup. We fix the height of both the Merkle tree and our MLT to $\ell = 30$, and measure performance when aggregating $b \in \{2^2, 2^4, \ldots, 2^{14}\}$ proofs. We compare to an implementation by Ozdemir *et al.* [35] in Rust [34] which uses the state-of-the-art SNARK by Groth [23] to prove *knowledge* of changes to a Merkle tree, updating it from digest d to digest d'. To benchmark proof aggregation, we notice that proof aggregation would involve half of the work done by the Ozdemir *et al.* prover, and directly use their code. This is because proving knowledge of b changes involves first verifying b Merkle proofs for the original values "inside the SNARK" and then updating the Merkle root with the changes, which also involves b Merkle path verifications. For the SNARK verifier, we directly measure its work, which involves a $\mathbb{G}_1$ multi-exponentiation linear in the number of proofs aggregated and 3 pairings.

Choice of Merkle hash function. Choosing a "SNARK-friendly" hash function for the Merkle tree can significantly reduce the prover time. In this sense, we use the recently-proposed Poseidon-128 hash function [22], which only requires 316 R1CS constraints per invocation inside the SNARK, but lacks sufficient cryptanalysis. As a more secure choice, we also use the Pedersen hash function [54] used in Zcash [7], which is collision-resistant under the hardness of discrete log, but requires 2753 constraints per invocation [35].

Proving time. The SNARK prover time is dominated by several multi-exponentiations and Fast Fourier Transforms (FFTs) of size linear in the number of R1CS constraints. For example, when aggregating $b = 2^{10}$ proofs in a Poseidon-hashed Merkle tree of height



(a) Aggregation time (seconds) (b) Verification time (seconds) (c) Aggregation plus verification time (seconds)

Figure 8: SNARK-based Merkle proof aggregation versus Hyperproof aggregation. The x-axis is $\log_2$(# of proofs being aggregated). Dotted lines are extrapolated, due to the SNARK prover running out of memory. We use the 128-bit secure variant of Poseidon.

$\ell = 30$, the number of constraints is 10 million. As a result, SNARK aggregation is very slow, taking 1224 seconds. In contrast, when aggregating b Hyperproofs, also in a height ℓ MLT, our IPA-based prover from Fig. 6 only computes $O(b\ell)$ pairings and $O(b\ell)$ $\mathbb{G}_1, \mathbb{G}_2$ and $\mathbb{G}_T$ exponentiations. This only takes 123 seconds. On average, as shown in Fig. 8(a), aggregating Hyperproofs is 10× faster than aggregating Merkle-Poseidon proofs and 100× faster than Merkle-Pedersen.

Prover memory. The SNARK prover also requires memory at least linear in the number of constraints. As a result, on our machine with 32 GiB of RAM, SNARK aggregation runs out of memory when aggregating $\geq 2^{11}$ proofs with Poseidon hashing (20 million constraints) or $\geq 2^8$ proofs with Pedersen (23 million constraints). Nonetheless, we extrapolate the proving times in Fig. 8. In contrast, our IPA-based aggregation from Fig. 6 has a much lower memory footprint and never runs out of memory.

Verification time. In general, verifying a SNARK proof requires 3 pairings and a $\mathbb{G}_1$ multi-exponentiation of size equal to the number of verifier-provided inputs [23]. In particular, when aggregating b Merkle proofs, this multi-exponentiation will be of size $2b+1$, since the verifier must input the digest and the b leaves $(i, v_i)_{i \in I}$ being verified. We implement verification in Golang using `mcl` [33] and report the times in Fig. 8(b). (We cannot use the Ozdemir et al. code, since the verifier only inputs two digests and checks *knowledge* of b changes to the Merkle tree.) When aggregating $b = 2^{10}$ proofs, it takes 0.11 seconds to verify a SNARK proof and 17.4 seconds to verify an aggregated Hyperproof. While verification is slower in Hyperproofs, when accounting for both the time to prove and verify in Fig. 8(c), Hyperproofs are faster.

SNARKs without trusted setup. Recent SNARKs [14, 44, 55] are *transparent* (i.e., do not need a trusted setup). Even better, these SNARKs often have faster provers than pairing-based SNARKs. However, compared to Hyperproofs, they are still too slow, have larger proof sizes and consume too much memory. For example, aggregating $b = 2^{14}$ Merkle proofs requires 2^{28} R1CS constraints if using Poseidon hashes. The prover time would be around 2.58 hours using Spartan [44, Figure 7] and 1.53 hours using Virgo [55]. This is close to 5× and 3× slower than Hyperproofs, respectively. The proof size would be around 1.83 MiB using Spartan and 350 KiB using Virgo (estimated using the open-source code of [55]). This is around 36× and 7× bigger than Hyperproofs, respectively. The performance is even worse if we use Pedersen hashes. Moreover, these transparent SNARKs are not as memory-efficient as Hyperproofs: Virgo scales to 2^{24} constraints, similar to pairing-based SNARKs (i.e., fails aggregating when $b \geq 2^{11}$ proofs) while Spartan scales to 2^{26} constraints (i.e., fails for $b \geq 2^{13}$). Lastly, other transparent arguments (e.g., STARKs [6], Aurora [8], Hyrax [52], Ligero [4]) have similar drawbacks. We defer to [44, 55] for a detailed discussion on trade-offs.

5.3 Macrobenchmarks

Our *single-threaded* experiments measure the VC-induced overheads of statelessly reaching consensus on a new block (see §4), which consist of: (1) aggregating proofs during block proposal, (2) verifying an aggregated proof during block validation and (3) proof-serving nodes (PSNs) updating all proofs after a new block, so that the next proposed block can use these proofs. In Table 3, we show Hyperproofs are 32× faster than Poseidon-hashed Merkle trees when proposing and validating blocks and remain competitive in terms of proof maintenance cost. Interestingly, stateless validation involves a more complex SNARK, which worsens the 10× slowdown of Merkle trees from §5.2. We assume MLTs and Merkle trees of height $\ell = 30$ and blocks of 1024 transactions. We do not compare to VCs with $O(1)$-sized proofs, due to their large proof maintenance cost (i.e., 2^ℓ $\mathbb{G}_1$ exponentiations, or 31.7 hours).

Limitations: Our macrobenchmarks do not account for all the subtleties that would arise in a full prototype, such as communication overheads, or miners needing to update the proofs in the current block they are working on due to another competing block. They also do not account for the overhead of signature verification, which is not affected by the chosen VC scheme. Instead, they focus on the three key operations whose overheads should be minimized: block proposal, block validation and proof maintenance. Lastly, while we show Hyperproofs are faster than other VCs for stateless validation, we do not claim they make the stateless setting practical.

Block transitions with Hyperproofs versus Merkle trees. In a stateless cryptocurrency, the ith block stores the digest d_i of all users' balances at that point in time. When block $i+1$ arrives, it

Scheme	Hyperproofs	Merkle tree with Poseidon-128 + SNARK
Block proposal	2.23 min	80.78 min
Block validation	17.51 sec	0.18 sec
Proof maintenance	5.14 sec	4.7 sec

Table 3: Single-threaded, stateless cryptocurrency macrobenchmarks that measure the time to prepare a block for proposal, to validate a proposed block and to update all proofs after a new block is seen. Trees have height $\ell = 30$ and blocks have 1024 transactions. A Poseidon-128 hash takes 113 μs using the `go-iden3-crypto` library [20].

must prove that its new digest d_{i+1} correctly reflects the updated balances, after applying its transactions. With Hyperproofs, the block includes an aggregated proof for the balance of each user sending money. This way, a validator can ensure that a block is spending valid coins and then can compute d_{i+1} from d_i via UpdDig, subtracting coins from each sending user's account and adding coins to each receiving user.

With SNARK-based Merkle trees, it is not possible to update the digest d_{i+1} given the old digest d_i, the SNARK aggregation proof, and the changes in balances: the Merkle proofs for all the changed leaves are also needed as auxiliary information. But including these Merkle proofs in the block would defeat the point of aggregating them via SNARKs! Therefore, the SNARK circuit must be extended to also verify the transition between d_i and d_{i+1}. Specifically, the circuit additionally proves that d_{i+1} is obtained by applying the changes in the block to d_i. A block of b transactions involves $2b$ changes to the Merkle tree, and each change requires two Merkle path verifications inside the circuit. Therefore, the circuit involves $4 \cdot b$ Merkle path verifications (4× more expensive than the aggregation circuit from §5.2).

Block overhead. As described above, stateless cryptocurrency blocks additionally store the digest of the state and an aggregated proof for all transactions. Both Merkle trees and Hyperproofs have similar digest sizes (i.e., 32 bytes versus 48 bytes). However, aggregated Hyperproofs are 52 KiB, whereas SNARK-aggregated Merkle proofs are only 192 bytes. Nonetheless, relative to the size of the full block, Hyperproof overhead is modest and only decreases with larger blocks. Furthermore, we foresee optimizing the IPA from Fig. 3 to reduce the proof size. Lastly, using unstealability to incentivize proof-serving nodes (which Merkle trees do not support) adds 224 bytes of WPK overhead per PSN involved in the block. As an alternative, if the set of PSNs is fixed or grows slowly, then WPKs can be stored as part of the public parameters of the system.

Transaction overhead. Transactions propagating through the P2P network in a stateless cryptocurrency need to include proofs. With Hyperproofs, this only requires a 1.44 KiB proof for the sender's balance. With Poseidon-hashed Merkle trees, this requires two 960 byte proofs, or 1.875 KiB, one for the sender and one for the receiver. This is because, to update the Merkle root, the miner also needs the receiver's Merkle proof as auxiliary information, whereas in Hyperproofs the digest can be updated homomorphically.

Block proposal. With Hyperproofs, a miner proposing a block with 1024 transactions has to (1) verify 1024 individual Hyperproofs, (2) aggregate these proofs, (3) and update the digest. With Merkle trees, this remains the same, except steps (2) and (3) are done in the SNARK. Table 3 shows block proposal is 36× faster in Hyperproofs than in SNARKs due to faster aggregation/digest updates.

Block validation. To validate an incoming block, a miner has to verify its aggregated proof and check its commitment was computed correctly via UpdDig. In Table 3, we see that SNARKs are much faster to verify than b aggregated Hyperproofs (97×), which require $O(b\ell)$ $\mathbb{G}_1$, $\mathbb{G}_2$ and $\mathbb{G}_T$ exponentiations to verify. While SNARKs also incur $O(b)$ cost, this only involves a fast $\mathbb{G}_1$ multi-exponentiation. Nonetheless, when considering the time to propose *and* validate a block, Hyperproofs remains 32× faster.

Proof maintenance. Recall that having updated proofs ready to be served is important in stateless cryptocurrencies, since users need to fetch and include their proofs when sending a transaction to a miner. Fortunately, a PSN can update all proofs in $O(\ell)$ time in both Hypeproofs and Merkle trees. Table 3 gives the concrete *batch* update time after 1024 transactions (or 2048 changes to the tree). Batch-updating Merkle trees is slightly faster than applying each update sequentially, because each node in the Merkle tree need only be updated once, by recomputing a Poseidon-128 hash, which takes 113μs. In contrast, when batch-updating MLTs, each node still needs to be updated several times to account for all the leaves that changed underneath it, as per Eq. 19. While we optimize this using a multi-exponentiation, MLTs will be slightly slower.

6 CONCLUSION

We presented Hyperproofs, a new VC that supports efficiently *maintaining* and *aggregating* proofs. We showed Hyperproofs aggregation is 10× to 100× faster than SNARK-based Merkle trees, depending on choice of hash function, and are only slightly slower when maintaining proofs. Although aggregated Hyperproofs are slower to verify, they are overall faster when used to statelessly propose and validate blocks in cryptocurrencies due to their faster aggregation. Hyperproofs is also the first *unstealable VC*, allowing proof-serving nodes to efficiently precompute proofs watermarked with their identity so they can be rewarded for their work. Importantly, these watermarked proofs remain updatable and aggregatable.

Future work. It would be interesting to apply our aggregation and unstealability techniques to *Verkle trees* [26, 29], which are q-ary rather than binary Merkle trees. This would also help extend Hypeproofs into a key-value commitment (KVC) scheme that maps arbitrary keys to values. Building Hyperproofs from assumptions in hidden-order groups would eliminate the large public parameters and, potentially, the trusted setup. Using more malleable inner-product arguments would allow us to update aggregated proofs too. Lastly, optimizing the arguments from Figs. 3 and 6 for our Hyperproof setting could speed up aggregation and verification times as well as reduce proof size.

Open source implementation. Our code will be available at:

https://github.com/hyperproofs/hyperproofs

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A ASSUMPTIONS

Our work builds upon the inner product argument (IPA) by Bünz et al. (see §2.4), which in turn relies on Abe et al. commitments to group elements [1], which are secure under the *Symmetric-eXternal Diffie-Hellman (SXDH)* assumption defined below.

Assumption A.1 (SXDH). *Let* $(p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, e, g_1, g_2) \leftarrow \mathsf{BilGen}(1^\lambda)$. *The Decisional Diffie-Hellman (DDH) problem in* $\mathbb{G}$ *is to decide whether* $c = ab$, *given* (g, g^a, g^b, g^c) *where* $g \in_R \mathbb{G}$. *The SXDH assumption is that DDH holds in both* $\mathbb{G}_1$ *and* $\mathbb{G}_2$.

We prove Hyperproofs satisfy soundness, as per Definition 2.3, under *q-Strong Diffie-Hellman (q-SDH)* assumption, defined below.

Assumption A.2 (q-SDH [10]). *For any* ppt *adversary* $\mathcal{A}$,

$$\Pr\left[\begin{array}{c}(p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, e, g_1, g_2) \leftarrow \mathsf{BilGen}(1^\lambda), s \in_R \mathbb{Z}_p^*, \\ \mathsf{pp} = ((p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, e, g_1, g_2), g_2^s, g_1^s, \ldots, g_1^{s^q}) : \\ (a, g_1^{\frac{1}{s+a}}) \leftarrow \mathcal{A}(1^\lambda, \mathsf{pp})\end{array}\right] \leq \mathsf{negl}(\lambda)$$

In addition to VC soundness, we prove Hyperproofs have unstealability, as per Definition 3.1, under the *q-Diffie-Hellman Inversion (q-DHI)* assumption [9, 32], defined below.

Assumption A.3 (q-DHI). *For any* ppt *adversary* $\mathcal{A}$,

$$\Pr\left[\begin{array}{c}(p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, e, g_1, g_2) \leftarrow \mathsf{BilGen}(1^\lambda), s \in_R \mathbb{Z}_p^*, \\ \mathsf{pp} = ((p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, e, g_1, g_2), g_2^s, g_1^s, \ldots, g_1^{s^q}) : \\ g_1^{1/s} \leftarrow \mathcal{A}(1^\lambda, \mathsf{pp})\end{array}\right] \leq \mathsf{negl}(\lambda)$$

The faster verifier for the Bünz et al. IPA from §2.4 relies on a modified Abe et al. commitment scheme which uses "structured" commitment keys. This modified commitment scheme is binding under the *q-Auxiliary Structured Double Pairing (q-ASDBP)* assumption in $\mathbb{G}_1$ and $\mathbb{G}_2$ introduced in [15]. First, we present this assumption in $\mathbb{G}_2$ below.

Assumption A.4 (q-ASDBP$_{\mathbb{G}_2}$). *For any* ppt *adversary* $\mathcal{A}$,

$$\Pr\left[\begin{array}{c}(p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, e, g_1, g_2) \leftarrow \mathsf{BilGen}(1^\lambda), \beta \in_R \mathbb{Z}_p^*, \\ \mathsf{pp} = ((p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, e, g_1, g_2), g_1^\beta, \left(g_2^{\beta^{2i}}\right)_{i \in [1,q)} \\ (A_0, \ldots, A_{q-1}) \leftarrow \mathcal{A}(1^\lambda, \mathsf{pp}) : \\ (A_0, \ldots, A_{q-1}) \neq \mathbf{1}_{\mathbb{G}_1} \wedge 1_{\mathbb{G}_T} \neq \prod_{i=1}^{q-1} e(A_i, g_2^{\beta^{2i}})\end{array}\right] \leq \mathsf{negl}(\lambda)$$

Second, the $\mathbb{G}_1$ variant of q-ASDBP is defined similarly by swapping $\mathbb{G}_2$ with $\mathbb{G}_1$. Third, the q-ASDBP assumption holds in the generic group model [15].

B ZERO-KNOWLEDGE PROOFS OF KNOWLEDGE

Recall from §3.4 that a watermarking public key (WPK) g_2^α must come with a zero-knowledge proof of knowledge (ZKPoK) of α. For this, we rely on Schnorr ZKPoKs of α [42], defined as $\mathsf{zkpok}_\alpha = (z, y) \in \mathbb{G}_2 \times \mathbb{Z}_p$, where:

$$y = g_2^c, \text{ where } c \in_R \mathbb{Z}_p \tag{29}$$

$$z = c + H(g_2, g_2^\alpha, y)\alpha \tag{30}$$

To verify zkpok_α, one checks if:

$$g_2^z \stackrel{?}{=} y(g_2^\alpha)^{H(g_2, g_2^\alpha, y)} = g_2^c g_2^{H(g_2, g_2^\alpha, y)\alpha} = g_2^{c + H(g_2, g_2^\alpha, y)\alpha}, \tag{31}$$

where $H(\cdot)$ is modeled as a random oracle.

C INNER PRODUCT ARGUMENT (IPA)

C.1 Non-interactive arguments of knowledge

Let $\mathcal{L}$ be an NP relation such that $\mathbf{x} \in \mathcal{L}$ if, and only if, there exists a witness $\mathbf{w}$ such that $\mathcal{L}(\mathbf{x}; \mathbf{w}) = 1$. A non-interactive argument of knowledge for $\mathcal{L}$ (e.g., SNARKs [39]) allows a *verifier* to efficiently verify that $\mathbf{x} \in \mathcal{L}$, without using $\mathbf{w}$, but via a (small) proof provided by an untrusted *prover*. A non-interactive argument of knowledge consists of three ppt algorithms, $(\mathcal{G}, \mathcal{P}, \mathcal{V})$:

(1) $(PK, VK) \leftarrow \mathcal{G}(1^\lambda, \mathcal{L})$: Generates the proving and verification key for the program $\mathcal{L}$.
(2) $\pi \leftarrow \mathcal{P}(PK, \mathbf{x}; \mathbf{w})$: Generates a proof π to prove that there exists $\mathbf{w}$ such that $\mathcal{L}(\mathbf{x}; \mathbf{w}) = 1$.
(3) $\{0, 1\} \leftarrow \mathcal{V}(VK, \pi, \mathbf{x})$: Checks if the proof π is valid for $\mathbf{x}$ using the verification key VK.

The main property of arguments of knowledge is *knowledge soundness*, formalized in Definition C.1. The intuition is that, if the verifier accepts a proof for $\mathbf{x}$, then the prover must "know" a witness $\mathbf{w}$ for $\mathbf{x}$ and therefore $\mathbf{x} \in \mathcal{L}$. Knowledge is modeled by an extractor $\mathcal{E}$ that can output such a $\mathbf{w}$ by inspecting the prover's tape.

Definition C.1 (Knowledge soundness). *We say that an argument of knowledge* $(\mathcal{G}, \mathcal{P}, \mathcal{V})$ *for NP relation* $\mathcal{L}$ *has* knowledge soundness *if, for any* ppt $\mathcal{A}$, *there is a* ppt *extractor* $\mathcal{E}$ *such that:*

$$\Pr\left[\begin{array}{c|c}1 \leftarrow \mathcal{V}(VK, \pi, \mathbf{x}) & \begin{array}{c}(PK, VK) \leftarrow \mathcal{G}(1^\lambda, \mathcal{R}), \\ (\pi, \mathbf{x}) \leftarrow \mathcal{A}(PK, VK), \\ \mathbf{w} \leftarrow \mathcal{E}(PK, VK, \pi, \mathbf{x}), \\ \mathcal{L}(\mathbf{x}; \mathbf{w}) \neq 1\end{array}\end{array}\right] \leq \mathsf{negl}(\lambda)$$

C.2 Random linear combinations lemma

Lemma C.1. *Let* $Z_i \in \mathbb{G}_T$, $\mathbf{A}_i \in \mathbb{G}_1^m$ *and* $\mathbf{B}_i \in \mathbb{G}_2^m$ *for* $i = 1, \ldots, N$. *Assume each* r_i *is chosen uniformly at random from* $\mathbb{Z}_p$. *Then, with probability at least* $1 - 1/p$, *all Eq.* 24 *are satisfied iff. Eq.* 25 *is satisfied.*

Proof sketch. Clearly if Eq. 24 are satisfied then Eq. 25 is also satisfied. For the other direction, by the Schwartz-Zippel lemma [43, 58], if at least one equation from Eq. 24 does not hold, Eq. 25 holds at randomly selected values r_i with probability $1/p$, completing the proof. □

C.3 Security proof for $\mathcal{L}^{b,\ell}_{\mathsf{BATCH}}$ argument

Theorem C.1. $(\mathcal{G}_{\mathsf{BATCH}}, \mathcal{P}_{\mathsf{BATCH}}, \mathcal{V}_{\mathsf{BATCH}})$ *from Fig. 6 is a non-interactive argument of knowledge for* $\mathcal{L}^{b,\ell}_{\mathsf{BATCH}}$ *from Eq.* 23 *that has knowledge soundness according to Definition C.1 under the same assumptions as the non-interactive IPA from §2.4 (i.e., algebraic commitment model [15], the random oracle model,* $(2b\ell)$*-SDH,* $(b\ell)$*-ASDBP).*

Proof sketch. This follows from Lemma C.1 and the knowledge soundness of the Bünz et al. IPA, which is used in a black box fashion. □

D HYPERPROOFS

D.1 MLT correctness

We argue below why our multilinear tree from §3.1 yields correct proofs as per Definition 2.2. Formally, the quotients $q_j(x_j, \ldots, x_1)$,

Table 4: Comparison with other VCs, which are not simultaneously aggregatable and maintainable. Proof sizes are in terms of group elements. n is the size of the vector, π_i is a proof for position i and π_I is an aggregated proof for k positions. Times are in terms of group exponentiations and field operations. (In RSA-based VCs [11, 16, 17, 27], we count $O(\ell)$ group operations as an exponentiation, where ℓ is the size of VC elements.) All schemes* support UpdDig and UpdProof (see Definition 2.1). However, CF-CDH and Pointproofs have $O(n)$-sized update keys, which can be too large for some applications.

Scheme	$\lvert\pi_i\rvert$	$\lvert\pi_I\rvert$	OpenAll time	Agg time	UpdAllProofs time	Trans-parent?	Homo-morphic?	Gen time	$\lvert$pp$\rvert$
AMT [48]	$\log n$	×	$n \log n$	×	$\log n$	×	✓	n^2	$n \log n$
aSVC [49]	1	1	$n \log n$	$k \log^2 k$	n	×	✓	$n \log n$	n
BBF [11]	1	1	$n \log^2 n$	$k \log n$	$n \log n$	×†	×	1	1
CF-CDH [17, 21, 27]	1	1	n^2	k	n	×	✓	n^2	n^2
CF-RSA [16, 17, 27]	1	1	$n \log n$	$k \log^2 k$	n	×†	✓	1	1
CFG+RSA [16]	1	1	$n \log^2 n$	$k \log k \log n$	n	×†	×	1	1
Lattice [38, 40]	$\log n$	×	n	×	$\log n$	✓	✓	1	$\log n$
Merkle	$\log n$	×	n	×	$\log n$	✓	×	1	1
Merkle SNARK	$\log n$	1	n	$k \log n \log (k \log n)$	$\log n$	×	×	1	1
Pointproofs [21]	1	1	$n \log n$	k	n	×	✓	n	n
Hyperproofs	$\log n$	$\log(k \log n)$	$n \log n$	$k \log n$	$\log n$	×	✓	n	n

†: BBF, CF-RSA and CFG+RSA avoid the trusted setup if instantiated using class groups of imaginary quadratic order, which are known to be slower than RSA groups.

*: Merkle trees, BBF VCs and CFG+RSA require *dynamic* update hints, rather than *static* update keys, for digest and proof updates. Also, only the *weakly-binding* variant of CFG+RSA supports digest updates.

$\forall j \in [\ell]$ for a proof π_i in our MLT from Fig. 4 are computed as:

$$q_j(x_{j-1}, \ldots, x_1) = f_{i_\ell,\ldots,i_{j+1},1} - f_{i_\ell,\ldots,i_{j+1},0} \tag{32}$$

One can prove that that these quotients satisfy the PST decomposition from Eq. 10, and thus yield a correct proof, for any i via induction. Here, we just show this intuitively. Begin with the first term in the PST decomposition sum from Eq. 10, which is $q_\ell(\mathbf{x})(x_\ell - i_\ell)$. By Eq. 32, this term is equal to:

$$\begin{aligned} q_\ell(\mathbf{x})(x_\ell - i_\ell) &= (f_1(\mathbf{x}) - f_0(\mathbf{x}))\,(x_\ell - i_\ell) \\ &= x_\ell f_1(\mathbf{x}) - i_\ell f_1(\mathbf{x}) - x_\ell f_0(\mathbf{x}) + i_\ell f_0(\mathbf{x}) \\ &= (1 - x_\ell) f_0(\mathbf{x}) + x_\ell f_1(\mathbf{x}) - (1 - i_\ell) f_0(\mathbf{x}) - i_\ell f_1(\mathbf{x}) \\ &= f(\mathbf{x}) - f_{i_\ell}(\mathbf{x}) \end{aligned}$$

The next term in the sum from Eq. 10 would be $q_{\ell-1}(\mathbf{x})(x_{\ell-1} - i_{\ell-1})$ which, by similar reasoning, equals $f_{i_\ell}(\mathbf{x}) - f_{i_\ell,i_{\ell-1}}(\mathbf{x})$. Adding these two terms up, the $f_{i_\ell}(\mathbf{x})$'s cancel out, leaving us with $f(\mathbf{x}) - f_{i_\ell,i_{\ell-1}}(\mathbf{x})$. Continuing with the other terms, everything cancels out except for $f(\mathbf{x}) - f_{i_\ell,\ldots,i_1}(\mathbf{x}) = f(\mathbf{x}) - f(\mathbf{i})$, as per Eq. 10. Therefore, the quotients defined in our MLT from Fig. 4 are correct.

D.2 UVC definitions

In §3.4, we glanced over how to change our VC API from Definition 2.1 to account for unstealability. In particular, we introduced a new notion of unstealability (see Definition 3.1), but we did not account for aggregated proofs. Also, we did not give a correctness definition. Below, we give our final unstealble VC (UVC) definitions, which account for aggregation. We also give the full API for an UVC, which we only sketched in §3.4. Changes from §2.3 are highlighted in blue.

Definition D.1 (UVC). *An* unstealable VC (UVC) *scheme consists of the following algorithms:*

(1) $\mathsf{Gen}(1^\lambda, n) \to \mathsf{pp}$*: Given security parameter* λ *and vector length* n*, it outputs public parameters* pp.

(2) $\mathsf{WtrmkGen}(1^\lambda) \to (\mathsf{wsk}, \mathsf{wpk})$*: Outputs a randomly-generated watermarking secret key (WSK)* wsk *and its corresponding watermarking public key (WPK)* wpk.

(3) $\mathsf{WtrmkParams}(\mathsf{pp}, \mathsf{wsk}) \to \mathsf{wpp}$*: Returns watermarked public parameters that can be used to directly compute watermarked proofs under* wsk.

(4) $\mathsf{Com}_{\mathsf{pp}}(\boldsymbol{a}) \to \mathsf{C}$*: Outputs the digest* C *of* $\boldsymbol{a} = (a_0, \ldots, a_{n-1}) \in \mathbb{Z}_p^n$.

(5) $\mathsf{Open}_{\mathsf{wpp}}(i, \boldsymbol{a}) \to \pi_i$*: Outputs a proof* π_i *for index* i *in* $\boldsymbol{a}$*, watermarked with the WSK from* wpp.

(6) $\mathsf{OpenAll}_{\mathsf{wpp}}(\mathbf{a}) \to (\pi_0, \ldots, \pi_{n-1})$*: Outputs all proofs* π_i *for* $\boldsymbol{a}$*, watermarked with the WSK from* wpp.

(7) $\mathsf{Agg}_{\mathsf{pp}}(I, (a_i, \pi_i)_{i \in I}) \to \pi_I$*: Combines individual proofs* π_i *for values* a_i *into an aggregated proof* π_I.

(8) $\mathsf{Ver}_{\mathsf{pp}}(\mathsf{C}, I, (a_i, \mathsf{wpk}_i)_{i \in I}, \pi_I) \to \{0, 1\}$*: Verifies proof* π_I *that each index* $i \in I$ *has value* a_i *against digest* C*. Additionally checks that the proof for* a_i *was watermarked using* wpk_i.

(9) $\mathsf{UpdDig}_{\mathsf{pp}}(u, \delta, \mathsf{C}) \to \mathsf{C}'$*: Updates digest* C *to* C′ *to reflect index* u *changing by* $\delta \in \mathbb{Z}_p$.

(10) $\mathsf{UpdAllProofs}_{\mathsf{wpp}}(u, \delta, \pi_0, \ldots, \pi_{n-1}) \to (\pi'_0, \ldots, \pi'_{n-1})$*: Updates* all *watermarked proofs* π_i *to* π'_i *after changing index* u *by* δ.

(11) $\mathsf{UpdProof}_{\mathsf{wpp}}(u, \delta, \pi_i) \to \pi'_i$*: Updates watermarked proof* π_i *to* π'_i *after changing index* u *by* δ.

Definition D.2 (UVC Correctness). *An unstealable VC (UVC) is correct, if for all $\lambda \in \mathbb{N}$ and $n = \mathsf{poly}(\lambda)$, for all $\mathsf{pp} \leftarrow \mathsf{Gen}(1^\lambda, n)$, for all vectors $\mathbf{a} = [a_0, \ldots, a_{n-1}]$, if $\mathsf{C} = \mathsf{Com}_{\mathsf{pp}}(\mathbf{a})$ and $\pi_i = \mathsf{Open}_{\mathsf{wpp}_i}(i, \mathbf{a}), \forall i \in [0, n)$, where $\mathsf{wpp}_i = \mathsf{WtrmkParams}(\mathsf{pp}, \mathsf{wsk}_i)$ and $(\mathsf{wsk}_i, \mathsf{wpk}_i) = \mathsf{WtrmkGen}(1^\lambda)$ (or from any combination of $\mathsf{OpenAll}_{\mathsf{wpp}_i}(\mathbf{a})$ calls, with different wpp_i's), then, for any polynomial number of updates (u, δ) outputting a new vector $\mathbf{a}'$, if C' and π'_i, for all i, are the updated digest and proofs via calls to $\mathsf{UpdDig}_{\mathsf{pp}}$ and $\mathsf{UpdProof}_{\mathsf{wpp}_i}$ (or $\mathsf{UpdAllProofs}_{\mathsf{wpp}_i}$) respectively, then:*

(1) $\Pr[1 \leftarrow \mathsf{Ver}_{\mathsf{pp}}(\mathsf{C}', \{i\}, (a'_i, \mathsf{wpk}_i), \pi'_i)] = 1$ *for all* i

(2) $\Pr[1 \leftarrow \mathsf{Ver}_{\mathsf{pp}}(\mathsf{C}', I, (a'_i, \mathsf{wpk}_i)_{i\in I}, \mathsf{Agg}_{\mathsf{pp}}(I, (a'_i, \pi'_i)_{i\in I}))] = 1, \forall I \subseteq [n]$.

Definition D.3 (UVC Soundness). $\forall$ ppt *adversaries $\mathcal{A}$, the following probability*

$$\Pr\left[\begin{array}{c} \mathsf{pp} \leftarrow \mathsf{Gen}(1^\lambda, n), \\ (\mathsf{C}, I, J, (a_i, \mathsf{wpk}_i)_{i\in I}, (a'_j, \mathsf{wpk}_j)_{j\in J}, \pi_I, \pi'_J) \leftarrow \mathcal{A}(1^\lambda, \mathsf{pp}) : \\ 1 \leftarrow \mathsf{Ver}_{\mathsf{pp}}(\mathsf{C}, I, (a_i, \mathsf{wpk}_i)_{i\in I}, \pi_I) \wedge \\ 1 \leftarrow \mathsf{Ver}_{\mathsf{pp}}(\mathsf{C}, J, (a'_j, \mathsf{wpk}_j)_{j\in J}, \pi'_J) \wedge \\ \exists k \in I \cap J \text{ s.t. } a_k \neq a'_k \end{array}\right]$$

is negligible in λ.

Notation: Let $\mathsf{Open}_{\mathsf{wpp}}(I, \mathbf{a})$ return an aggregated proof π_I for all $(a_i)_{i\in I}$, by first obtaining individual proofs $\pi_i = \mathsf{Open}_{\mathsf{wpp}}(i, \mathbf{a})$ and then aggregating them via $\pi_I = \mathsf{Agg}_{\mathsf{pp}}(I, (a_i, \pi_i)_{i\in I})$.

Definition D.4 (UVC Unstealability). $\forall$ ppt *adversaries $\mathcal{A} = (\mathcal{A}_0, \mathcal{A}_1)$, the following probability*

$$\Pr\left[\begin{array}{c} \mathsf{pp} \leftarrow \mathsf{Gen}(1^\lambda, n), \\ (\mathsf{wsk}, \mathsf{wpk}) \leftarrow \mathsf{WtrmkGen}_{\mathsf{pp}}(1^\lambda), \\ \mathsf{wpp} \leftarrow \mathsf{WtrmkParams}(\mathsf{pp}, \mathsf{wsk}), \\ \mathsf{C} \leftarrow \mathsf{Com}_{\mathsf{pp}}(\boldsymbol{a}), \text{ where } a_i \in_R \mathbb{Z}_p, \\ (I_1, \ldots, I_m, \mathsf{st}) \leftarrow \mathcal{A}_0(1^\lambda, \mathsf{pp}, \mathsf{C}, \mathsf{wpk}) \text{ s.t.} \\ I_1 \cup \ldots \cup I_k \subsetneq [n], \\ \left(\pi_{I_k} \leftarrow \mathsf{Open}_{\mathsf{wpp}}(I_k, \boldsymbol{a})\right)_{k\in[m]}, \\ (J \subsetneq [n], \pi_J, (\mathsf{wpk}_j)_{j\in J}) \leftarrow \mathcal{A}_1(\mathsf{st}, (a_{I_k})_{k\in[m]}, (\pi_{I_k})_{k\in[m]}) : \\ \mathsf{Ver}_{\mathsf{pp}}(\mathsf{C}, J, (a_j, \mathsf{wpk}_j)_{j\in J}, \pi_J) = 1 \wedge \exists j \in J, \mathsf{wpk} \neq \mathsf{wpk}_j \end{array}\right]$$

is negligible in λ.

Observation: In Appendix D.5, we prove the weaker version of this definition (i.e., Definition 3.1) holds under ℓ-DHI. However, our proof can be easily generalized to the aggregated setting.

D.3 VC soundness proof

Below, we prove our Hyperproofs construction from §3 is sound as per Definition 2.3. We first show soundness holds for individual (non-aggregated) proofs, and then show how soundness of aggregated proofs follows from that and the knownledge-soundness of the $\mathcal{L}_{\mathsf{BATCH}}$ argument.

D.3.1 Soundness for individual Hyperproofs.

Theorem D.1. *Our* individual log *n-sized (non-aggregated) proofs from §3.1 are sound as per Definition 2.3 under q-SDH (see Assumption A.2).*

Proof. Suppose there exists an adversary $\mathcal{A}$ that breaks Definition 2.3. We show how to build another adversary $\mathcal{B}$ that breaks the ℓ-SDH assumption (see Assumption A.2). We first assume $\mathcal{A}$ returns individual (non-aggregated) proofs and then generalize to $\mathcal{A}$ returning aggregated proofs.

$\mathcal{B}$ is given ℓ-SDH public parameters $\mathsf{pp} = ((p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, e, g_1, g_2), g_2^s, g_1^s, \ldots, g_1^{s^\ell})$, and must (somehow) break ℓ-SDH by outputting $(a, g_1^{\frac{1}{s-a}})$ for some $a \neq s$. For this, $\mathcal{B}$ will leverage $\mathcal{A}$ into helping him.

First, $\mathcal{B}$ "guesses" the index i on which $\mathcal{A}$ will forge, which he can do with probability $1/\mathsf{poly}(\lambda)$, where λ is our security parameter. Second, $\mathcal{B}$ "tweaks" the ℓ-SDH public parameters into the Hyperproofs public parameters from Fig. 2, which he then calls $\mathcal{A}$ with. Specifically, $\mathcal{B}$ sets $s_k - i_k = r_k(s - i_1), \forall k \in [\ell]$, where $r_1 = 1$, the rest of the r_k's are random, and $i_\ell, \ldots, i_1$ is the binary representation of i. Importantly, note that $\mathcal{B}$ can do this without knowledge of s, since $\mathcal{B}$ can compute any product $g_1^{\prod_{i\in S} s_i}, S \in 2^{\{1,2,\ldots,\ell\}}$ from the $g_1^{s^i}$'s. Similarly, $\mathcal{B}$ can compute any $g_2^{s_k}$ from g_2^s. Third, $\mathcal{B}$ calls $\mathcal{A}$ with the "tweaked" public parameters as input and obtains a commitment C and two *inconsistent* proofs $\pi = (w_k)_{k\in[\ell]}$, $\pi' = (w'_k)_{k\in[\ell]}$ for position i having values both v and v'. (If $\mathcal{A}$ outputs proofs for a different index $i' \neq i$, $\mathcal{B}$ retries.)

Since both proofs verify, the following equations hold, where $i_\ell, \ldots, i_1$ is the binary expansion of the index i:

$$e(\mathsf{C}/g_1^v, g_2) = \prod_{k\in[\ell]} e(w_k, g_2^{s_k - i_k}) \tag{33}$$

$$e(\mathsf{C}/g_1^{v'}, g_2) = \prod_{k\in[\ell]} e(w'_k, g_2^{s_k - i_k}) \tag{34}$$

Next, dividing the top equation by the bottom one and substitute $s_k - i_k = r_k(s - i_1), \forall k \in [\ell]$:

$$e(g_1^{v'}/g_1^v, g_2) = \prod_{k\in[\ell]} e(w_k/w'_k, g_2^{s_k - i_k}) \Leftrightarrow \tag{35}$$

$$e(g_1^{v'-v}, g_2) = \prod_{k\in[\ell]} e(w_k/w'_k, g_2^{r_k(s-i_1)}) \Leftrightarrow \tag{36}$$

$$e(g_1, g_2)^{v'-v} = \left(\prod_{k\in[\ell]} e(w_k/w'_k, g_2^{r_k})\right)^{s-i_1} \Leftrightarrow \tag{37}$$

$$e(g_1, g_2)^{\frac{1}{s-i_1}} = \left(\prod_{k\in[\ell]} e(w_k/w'_k, g_2^{r_k})\right)^{\frac{1}{v'-v}} \Leftrightarrow \tag{38}$$

$$e(g_1^{\frac{1}{s-i_1}}, g_2) = \prod_{k\in[\ell]} e\left(\left(w_k/w'_k\right)^{\frac{r_k}{v'-v}}, g_2\right) \Leftrightarrow \tag{39}$$

$$e(g_1^{\frac{1}{s-i_1}}, g_2) = e\left(\prod_{k\in[\ell]} \left(w_k/w'_k\right)^{\frac{r_k}{v'-v}}, g_2\right) \Leftrightarrow \tag{40}$$

Thus, $g_1^{\frac{1}{s-i_1}} = \prod_{k\in[\ell]} \left(w_k/w'_k\right)^{\frac{r_k}{v'-v}}$ and $\mathcal{B}$ can output $(i_1, g_1^{\frac{1}{s-i_1}})$ and break ℓ-SDH. □

D.3.2 Soundness for aggregated Hyperproofs.

Theorem D.2. *Our* aggregated *proofs from §3.3 are sound as per Definition 2.3 under the knowledge-soundness of the* $\mathcal{L}_{\text{BATCH}}$ *argument (see Theorem C.1).*

Proof. Suppose there exists $\mathcal{A}$ that outputs $(\mathsf{C}, I, J, (a_i)_{i\in I}, (a'_j)_{j\in J}, \pi_I, \pi'_J)$ such that $1 \leftarrow \mathsf{Ver}_{\mathsf{pp}}(\mathsf{C}, I, (a_i)_{i\in I}, \pi_I)$ and $1 \leftarrow \mathsf{Ver}_{\mathsf{pp}}(\mathsf{C}, J, (a'_j)_{j\in J}, \pi'_J)$, while $\exists t \in I \cap J$ s.t. $a_t \neq a'_t$. By Theorem C.1, there exist extractors that extract the individual $\log n$-sized proofs. Let $\boldsymbol{\pi}_i = (w_{i,1}, \ldots, w_{i,\ell})$ denote the extracted proofs for all $i \in I$ and $\boldsymbol{\pi}'_j = (w'_{j,1}, \ldots, w'_{k,\ell})$ denote the ones for all $j \in J$. Recall that for $t \in I \cap J$, $\boldsymbol{\pi}_t$ verifies for a_t while $\boldsymbol{\pi}'_t$ verifies for a different a'_t. Thus, we have:

$$e(\mathsf{C}/g_1^{a_t}, g_2) = \prod_{k=1}^{\ell} e(w_{t,k}, g_2^{s_k - t_k}) \tag{41}$$

$$e(\mathsf{C}/g_1^{a'_t}, g_2) = \prod_{k=1}^{\ell} e(w'_{t,k}, g_2^{s_k - t_k}) \tag{42}$$

By the same argument from Appendix D.3, this breaks q-SDH. □

D.4 UVC soundness proof

Theorem D.3. *The unstealable variant of Hyperproofs from §3.4 is sound as per (the updated) Definition D.3.*

Proof sketch. We assume proofs are not aggregated, but the proof proceeds similar to Appendix D.3.2 when proofs are aggregated. The proof proceeds the same as in Appendix D.3, except the reduction $\mathcal{B}$ that calls the adversary $\mathcal{A}$ needs to know the WSKs α, β corresponding to wpk and wpk' respectively in order to output an ℓ-SDH solution $(i_1, g_1^{\frac{1}{s-i_1}})$. Indeed, since the WPKs come with a ZKPoK, $\mathcal{B}$ can extract α, β with non-negligible probability. Next, let $\pi = (w_1, \ldots, w_\ell)$ and $\pi' = (w'_1, \ldots, w'_\ell)$ be the two inconsistent proofs. Similar to the proof in Appendix D.3, $\mathcal{B}$ knows that:

$$e(\mathsf{C}/g_1^{a_i}, g_2^{\alpha}) = \prod_{k=1}^{\ell} e(w_{i,k}, g_2^{s_k - i_k}) \tag{43}$$

$$e(\mathsf{C}/g_1^{a'_i}, g_2^{\beta}) = \prod_{k=1}^{\ell} e(w'_{i,k}, g_2^{s_k - i_k}) \tag{44}$$

Dividing the top by the bottom equation yields:

$$e(g_1^{a'_i - a_i}, g_2) = \prod_{k\in[\ell]} e(w_k^{1/\alpha}/w_k'^{1/\beta}, g_2^{s_k - i_k}) \tag{45}$$

$$= \left(\prod_{k\in[\ell]} e(w_k^{1/\alpha}/w_k'^{1/\beta}, g_2^{r_k})\right)^{s-i_1} \tag{46}$$

Thus, $\mathcal{B}$ can output the ℓ-SDH solution $(i_1, g_1^{\frac{1}{s-i_1}})$ where $g_1^{\frac{1}{s-i_1}} = \prod_{k\in[\ell]} \left(w_k^{1/\alpha}/w_k'^{1/\beta}\right)^{\frac{r_k}{a'_i - a_i}}$. □

D.5 UVC unstealability proof

Theorem D.4. *Hyperproofs are unstealable as per Definition 3.1 under ℓ-DHI.*

Proof. If an adversary $\mathcal{A} = (\mathcal{A}_0, \mathcal{A}_1)$ exists that breaks Definition 3.1, we show how to build a reduction $\mathcal{B}$ that breaks ℓ-DHI Assumption A.3. Specifically, $\mathcal{B}$ will get $(h_2^{\alpha}, (h_1^{\alpha^k})_{k\in[0,\ell]})$ as input and must output $h_1^{1/\alpha}$.

First, $\mathcal{B}$ randomizes the ℓ-DHI instance into $(g_2^{\alpha}, (g_1^{\alpha^k})_{k\in[0,\ell]})$ where $g_2 = h_2^{\gamma}$ and $g_1 = h_1^{\gamma}$ for $\gamma \in_R \mathbb{Z}_p$. Note that a $g_1^{1/\alpha}$ break on this instance can be immediately turned into a $h_1^{1/\alpha}$ break on the original instance, since γ is known. Second, $\mathcal{B}$ must generate random public parameters for Hyperproofs. For this, $\mathcal{B}$ guesses the index j that $\mathcal{A}_1$ outputs a re-watermarked proof π' for and sets $s_k - j_k = \alpha r_k$ for $r_k \in_R \mathbb{Z}_p$. (Recall that $\mathcal{B}$ can do this without knowing α, similar to Appendix D.3) $\mathcal{B}$ also programs the random oracle $H(\cdot)$ for the Schnorr proof system and simulates a ZKPoK of α w.r.t. g_2^{α} denoted by zkpok_{α} and sets $\mathsf{wpk} = (g_2^{\alpha}, \mathsf{zkpok}_{\alpha})$. Specifically, $\mathcal{B}$ picks $(z, c) \in_R \mathbb{Z}_p^2$, sets $y = g_2^z/(g_2^{\alpha})^c$ and programs $H(g_2, g_2^{\alpha}, y)$ to return c. Third, $\mathcal{B}$ picks $(u, r, a_j) \in_R \mathbb{Z}_p$ such that $r - a_j \neq 0$ and sets the commitment $\mathsf{C} = g_1^{\alpha u + r}$. Finally, $\mathcal{B}$ calls $\mathcal{A}_0(1^{\lambda}, \mathsf{pp}, \mathsf{C}, \mathsf{wpk})$

This way, if $\mathcal{A}_1$ outputs a watermarked proof $\pi' = (w'_1, \ldots, w'_\ell)$, then $\mathcal{B}$ can break ℓ-DHI as follows:

$$e(\mathsf{C}/g_1^{a_j}, g_2^{\beta}) = \prod_{k\in[\ell]} e((w'_k), g_2^{s_k - j_k}) \Leftrightarrow \tag{47}$$

$$\mathsf{C}/g_1^{a_j} = \prod_{k\in[\ell]} (w'_k)^{\frac{s_k - j_k}{\beta}} \Leftrightarrow \tag{48}$$

$$g_1^{\alpha u + r}/g_1^{a_j} = \prod_{k\in[\ell]} (w'_k)^{\frac{\alpha r_k}{\beta}} \Leftrightarrow \tag{49}$$

$$g_1^{u} g_1^{\frac{r - a_j}{\alpha}} = \prod_{k\in[\ell]} (w'_k)^{\frac{r_k}{\beta}} \Leftrightarrow \tag{50}$$

$$g_1^{\frac{1}{\alpha}} = \left(g_1^{-u} \prod_{k\in[\ell]} (w'_k)^{\frac{r_k}{\beta}}\right)^{\frac{1}{r - a_j}} \tag{51}$$

But before this can happen, $\mathcal{A}_0$ might ask $\mathcal{B}$ for proofs $\pi_i = (w_k)_{k\in[\ell]}$ watermarked with α for arbitrary $i \in I$. $\mathcal{B}$ will simulate such a proof π_i, without knowing α as follows. First, $\mathcal{B}$ computes random z_k's in $\mathbb{Z}_p$ such that $\sum_{k\in[\ell]} z_k r_k = u$. Specifically, $\mathcal{B}$ picks random z_k's for all $k \geq 2$ and sets $z_1 = \frac{u - \sum_{k\in[2,\ell]} z_k r_k}{r_1}$. (Recall that u and the r_k's were fixed before giving $\mathcal{A}_0$ the commitment C and pp.). Second, $\mathcal{B}$ computes $a_i = r - \sum_{k\in[\ell]} z_k \delta_{i,k}$, where $\delta_{i,k} = i_k - j_k, \forall k \in [\ell]$. (Note that a_i is uniform random both when $j = i$ and when $j \neq i$.) Finally, $\mathcal{B}$ sets $w_k = g_1^{\alpha z_k}, \forall k \in [\ell]$. Note that this proof for a_i verifies against C and WSK α since:

$$e(\mathsf{C}/{g_1}^{a_i}, g_2^{\alpha}) = \prod_{k\in[\ell]} e(w_k, g_2^{s_k - i_k}) = \prod_{k\in[\ell]} e(w_k^{\frac{s_k - i_k}{\alpha}}, g_2) \Leftrightarrow$$

$$g_1^{\alpha u + r - a_i} = \prod_{k\in[\ell]} g_1^{z_k(s_k - j_k + \delta_{i,k})} = \prod_{k\in[\ell]} g_1^{z_k(\alpha r_k + \delta_{i,k})}$$

$$= g_1^{\alpha \sum_{k\in[\ell]} z_k r_k + \sum_{k\in[\ell]} z_k \delta_{i,k}} = g_1^{\alpha u + r - a_i}$$

Thus, $\mathcal{B}$ can simulate proofs to $\mathcal{A}_1$ and turn his outputted proof π' into an q-DHI break. This completes our proof. □

D.6 Public parameter size

Table 5: The size of the public parameters from Fig. 2 for various values of $\ell = \log_2 n$. Recall that the *verification key* consists of all selector monomial commitments $g_2^{s_k}, \forall k \in [\ell]$, while the *proving key* consists of all selector multinomial commitents $g_1^{\mathcal{S}_{j,k}(\mathbf{s})}, \forall k \in [0, \ell], j \in [0, 2^k)$ (see Fig. 2).

$\ell = \log_2 n$	Verification key	Proving key
22	2.11 KiB	384 MiB
24	2.3 KiB	1.5 GiB
26	2.49 KiB	6 GiB
28	2.68 KiB	24 GiB
30	2.88 KiB	96 GiB