FIN: Practical Signature-Free Asynchronous Common Subset in Constant Time

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ABSTRACT

Asynchronous common subset (ACS) is a powerful paradigm enabling applications such as Byzantine fault-tolerance (BFT) and multi-party computation (MPC). The most efficient ACS framework in the information-theoretic setting is due to Ben-Or, Kelmer, and Rabin (BKR, 1994). The BKR ACS protocol has been both theoretically and practically impactful. However, the BKR protocol has an $O(\log n)$ running time (where n is the number of replicas) due to the usage of n parallel asynchronous binary agreement (ABA) instances, impacting both performance and scalability. Indeed, for a network of $16\sim64$ replicas, the parallel ABA phase occupies about $95\%\sim97\%$ of the total runtime in BKR. A long-standing open problem is whether we can build an ACS framework with O(1) time while not increasing the message or communication complexity of the BKR protocol.

In this paper, we resolve the open problem, presenting the first constant-time ACS protocol with $O(n^3)$ messages in the informationtheoretic and signature-free settings. Moreover, as a key ingredient of our new ACS framework and an interesting primitive in its own right, we provide the first information-theoretic multivalued validated Byzantine agreement (MVBA) protocol with O(1) time and $O(n^3)$ messages. Both results can improve—asymptotically and concretely-various applications using ACS and MVBA in the information-theoretic, quantum-safe, or signature-free settings. As an example, we implement FIN, a BFT protocol instantiated using our framework. Via a 121-server deployment on Amazon EC2, we show FIN is significantly more efficient than PACE (CCS 2022), the state-of-the-art asynchronous BFT protocol of the same type. In particular, FIN reduces the overhead of the ABA phase to as low as 1.23% of the total runtime, and FIN achieves up to 3.41x the throughput of PACE. We also show that FIN outperforms other BFT protocols with the standard liveness property such as Dumbo and Speeding Dumbo.

1 INTRODUCTION

Overview. This paper is about resolving a long-standing open problem in fault-tolerant distributed computing and multi-party computation (MPC). We present the first practical O(1)-time and $O(n^3)$ -message asynchronous common subset (ACS) protocol [6, 8], while prior constructions have either $O(\log n)$ time and $O(n^3)$ messages or have O(1) time and $O(n^4)$ messages.

History. The concept of asynchronous common subset (ACS) is due to Ben-Or, Canetti, and Goldreich (BCG) in the context of

asynchronous MPC—under a different name called *agreement on a core set* [6]. BCG proposed an ACS construction using two core building blocks in fault-tolerant distributed computing—reliable broadcast (RBC) and asynchronous binary agreement (ABA). Soon later, Ben-Or, Kelmer, and Rabin (BKR) presented a refined and practical ACS construction using *n* RBC and *n* ABA instances [8]. Meanwhile, BKR renamed "agreement on a core set" as "agreement on a common subset."

Information-theoretic and signature-free settings. The ACS notion has been historically associated with the information-theoretic setting: as emphasized by Cachin et al. [11, Section 4], "the primitive of agreement on a core set (which) is used in the information-theoretic model." Indeed, ACS constructions typically rely on *ideal* building primitives in distributed computing such as RBC, ABA, and common coins [10, 15].

An equivalent setting is the *signature-free setting* focusing on protocols assuming the existence of common coins.¹ In particular, the line of work for signature-free ABA—which is at the core of practical ACS constructions—begins with the seminal work by Rabin [50] and is followed by [23, 42, 45, 46, 54].

The unique benefits of information-theoretic/signature-free ACS. The information-theoretic ACS paradigm (BCG and BKR) has been enormously impactful, empowering numerous applications as well as implementations, such as MPC [5, 8, 9, 14, 18–21, 40], Byzantine fault tolerence (BFT) [2, 9, 29, 39, 43, 56, 57], distributed key generation [25, 27, 37], and proactive secret sharing [34, 35, 53]. For instance, we illustrate of benefits of information-theoretic ACS using MPC applications:

• MPC. A more efficient information-theoretic ACS directly improves a large number of asynchronous MPC protocols with fairness and guaranteed output delivery that use information-theoretic ACS (BCG and BKR) [5, 8, 9, 14, 18–21, 40]. In this approach, one just replaces RBC with asynchronous complete secret sharing [8].² The only MPC system with fairness and guaranteed output delivery falls into this category [40].

Even just being used for the BFT purpose, information-theoretic ACS has unique benefits and features, when compared to the other two main BFT approaches—multivalued validated Byzantine agreement (MVBA) based BFT[11] and direct acyclic graph (DAG) based BFT [36]:

• Quantum safety. Information-theoretic ACS protocols and DAGbased protocols can achieve the desirable quantum safety property (but no quantum liveness) as defined in [36], where the safety

1

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 $^{^{1}}$ Here we directly borrow the term from the line of work [45–47]; the setting may also be called cryptography-free.

²In fact, the original BKR framework was presented as an asynchronous MPC framework using asynchronous complete secret sharing and ABA.

protocol	messages	time	# (R)ABA
BKR variants [29, 39, 43]	$O(n^3)$	$O(\log n)$	n
PACE [54]	$O(n^3)$	$O(\log n)$	n
FIN (§7; this paper)	$O(n^3)$	O(1)	O(1)

Table 1: Comparison among ACS implementations relying on common coins.

of the system is always achieved even in the presence of a quantum adversary. In contrast, this is not the case for MVBA-based approach.

- Weaker cryptographic assumptions. Information-theoretic ACS and DAG-based BFT can be realized using the standard Computational Diffie-Hellman (CDH) assumption; others would need to use pairing assumptions to save communication costs.
- Liveness. Unlike information-theoretic ACS protocols that achieve standard liveness, some other BFT protocols (including DAGbased ones) either require unbounded memory for liveness (DAGRider [36], Dumbo-NG [30]) or achieve weak liveness (Bullshark [31], Tusk [24]).

The open problem. Almost all ACS implementations use the BKR ACS construction, which runs n parallel ABA instances, thereby achieving $O(n^3)$ messages and $O(\log n)$ time. The ABA phase involving n parallel ABA instances is the well-known performance and scalability bottleneck reported in various BFT implementations [29, 33, 54]. For example, the ABA phase in a network with $16\sim64$ replicas occupies about $95\%\sim97\%$ of the total runtime in BKR [33]. Recently, Zhang and Duan proposed a refined PACE ACS framework [54] that offers up to 6x peak throughput of BEAT [29] (that improved upon HoneyBadgerBFT [43]). In PACE ACS, the n ABA instances are made fully parallelizable to gain in improved performance, but the running time remains $O(\log n)$ —remaining the critical bottleneck.

A long-standing open problem, ever since the 1990s, in ACS—or generally in fault-tolerant distributed computing and cryptography—is:

Can we build ACS—in the information-theoretic setting or signature-free setting—with $O(n^3)$ messages and O(1) time?

Note that the only known such ACS construction terminating in constant time is due to Ben-Or and El-Yaniv [7]. The construction uses n^2 RBC instances with $O(n^4)$ messages; earlier works such as HoneyBadger [43, Footnote 4] and Dumbo [33, Footnote 5] explained why [7] has lower throughput than BKR and that the communication of [7] bottlenecks the system.

Our results and our approach. In this work, we resolve the open problem, showing an ACS protocol with $O(n^3)$ messages and O(1) time. At a high level, we begin with a new multivalued validated Byzantine agreement (MVBA) protocol [11] in the information-theoretic setting with $O(n^3)$ and O(1) time. Then we reduce the ACS problem to MVBA and RBC.

Recall that the notion of MVBA is different from the conventional multivalued Byzantine agreement in that MVBA assumes the existence of a global predicate, and replicas only decide values satisfying the global predicate. To build an MVBA protocol towards our goal, we use RBC, common coins, and reproposable ABA (RABA)—a notion due to Zhang and Duan [54]. RABA, like ABA, can be readily

built from common coins and authenticated channels and terminate in expected constant time. Our MVBA uses n parallel RBC instances and an expected constant number of RABA instances. Our MVBA protocol is also the first information-theoretically secure and signature-free MVBA protocol—by directly assuming common coins—with $O(n^3)$ messages and O(1) time.

We also show that one can use a weaker RBC primitive to realize a computation and bandwidth more efficient MVBA protocol while still achieving $O(n^3)$ messages and O(1) time.

Our transformation from MVBA to ACS is simple and efficient, consisting of *n* parallel RBC instances and a single MVBA instance. Both our MVBA and ACS constructions are efficient, inheriting the fast path in RABA, enabling rapid termination.

Our contributions. We make the following contributions:

- We present the first information-theoretically secure and signature-free ACS protocol with O(1) time and $O(n^3)$ messages, assuming ideal building blocks only. In contrast to prior constructions, our ACS protocol requires only an expected constant number of binary agreement instances. Our protocol directly improves various ACS-enabled applications such as asynchronous multi-party computation [8, 14, 21, 40].
- As a core ingredient of our ACS construction and a primitive that is interesting in its own right, we present the first signature-free multivalued validated Byzantine agreement protocol with O(1) time and O(n³) messages while existing such MVBA protocols have O(log n) time and O(n³) messages. Moreover, our MVBA protocols lead to instantiations having lower communication than existing ones. We also show an efficient and tailored MVBA construction, optimizing both communication cost and computational efficiency.
- To demonstrate the efficiency of our ACS protocol, we instantiate it with a practical BFT system called FIN (see Table 1 for a comparison). We implement FIN and PACE, the state-of-theart ACS protocol of the same kind. Additionally, we also assess Dumbo [33] and Speeding Dumbo [32], two asynchronous and computationally-secure BFT protocols with O(1) time and the standard liveness guarantee. Via a 121-instance deployment on Amazon EC2, we show that for all metrics, FIN consistently and drastically outperforms PACE when n > 16 and all the above protocols when n > 22. Compared to PACE, FIN significantly reduces the overhead of the ABA phase to only 1.23%-5.22% of the total runtime, in contrast to the 15.10%-83.66% overhead in PACE. Moreover, the performance difference between FIN and the other three protocols becomes significantly higher as n increases; for instance, when n = 121 (tolerating 40 failures), the peak throughput of FIN is 3.41x that of PACE, 4.15x that of Speeding Dumbo, and 8.79x that of Dumbo.

2 RELATED WORK

Interactive consistency and vector consensus. The ACS problem can be viewed as an asynchronous version of the *interactive consistency* problem defined for synchronous systems by Pease, Shostak, and Lamport [49]. Replicas in asynchronous interactive consistency reach an agreement on a vector with the values proposed by *all* correct replicas.

In contrast, the ACS abstraction (also called asynchronous interactive consistency by Ben-Or and El-Yaniv [7]) naturally requires that the output of each correct replica contains n-f values such that at least n-2f elements are proposed by correct replicas. Namely, ACS requires only that the majority of the values were proposed by correct replicas. Indeed, it is impossible to guarantee that the vector has the values of all correct replicas in asynchronous settings.

ACS is also called vector consensus in some literature [22, 28, 44, 48]. Note that ACS is different from set agreement [17] that provides only an approximation of agreement.

ACS constructions (in information-theoretic and signaturefree settings). BKR ACS reduces asynchronous BFT to reliable broadcast (RBC) and asynchronous binary agreement (ABA) [8]. In BKR ACS, all replicas run an RBC phase to reliably broadcast their proposals. Then they run an ABA phase with n parallel instances. The *i*-th ABA instance agrees on if the proposal of p_i has been delivered in the RBC phase. Upon RBC delivery of a proposal from p_i , the replica proposes 1 to the *j*-th ABA instance. If a correct replica p_i decides 1 for the *i*-th ABA instance, the proposal from p_i is delivered. BKR ACS requires if a replica has not received some proposals during the RBC phase, the replica abstains from proposing 0 until n-f ABA instances terminate with 1. In PACE ACS, ABA instances are replaced using RABA instances, and thus all RABA instances can be run in a fully parallelizable manner [54]. Both BKR and PACE ACS approaches require running n (R)ABA instances terminating in expected constant rounds, leading to expected $O(\log n)$ time in total.

The ACS construction by Ben-Or and El-Yaniv [7] terminates in expected constant time and uses n^2 RBC instances with $O(n^4)$ messages, which is prohibitively expensive.

Another line of ACS constructions reduces the ACS problem to RBC and multivalued Byzantine agreement (MBA) [22, 44]. The construction requires running O(f) sequential MBA instances, resulting in O(n) running time.

Our ACS approach fundamentally differs from existing ones, reducing the ACS problem to information-theoretically secure MVBA and then to RBC with a constant number of RABA instances.

DispersedLedger. DispersedLedger [51] provides two general techniques (allowing committing additional transactions from prior epochs; using asynchronous verifiable information dispersal) to improve the performance of the BKR framework. The techniques equally work for PACE and FIN. Therefore, the DispersedLedger techniques can be used to improve FIN, and FIN can be used in DispersedLedger as its consensus engine for higher performance.

Separating message transmission from consensus. Tusk [24], Bullshark [31], and Dumbo-NG [30] are BFT protocols that separate data transmission from consensus for higher throughput. FIN can use the technique to improve performance.

As mentioned in the introduction, Tusk, Bullshark, and Dumbo-NG do not achieve standard liveness. We comment the fact that they do not have standard liveness is not due to the framework that separates the transmission from consensus.

RBC. We use RBC (abbreviated as BRB in some works) [10] to build our MVBA and ACS construction. For both our MVBA and ACS, RBC dominates their communication. One could use any RBC

constructions to instantiate them. In this paper, we discuss constructions using CT RBC [13] (using hashes), EFBRB [3] (information-theoretically secure), and CCBRB [3] (using hashes and online error correction coding [6]).

For our ACS implementation, we use CT RBC [13]. Internally in our MVBA implementation, we show that one can use a tailored, weaker RBC construction (using collision-resistant hashes only) to build a concretely more efficient protocol.

RABA. The notion of reproposable ABA (RABA) is due to Zhang and Duan [54]. RABA was originally proposed to solve a BKR bottleneck to allow all ABA instances to run in parallel and remove the two-subphase bottleneck. Later, such a primitive was used to develop a quantum secure and adaptively secure asynchronous BFT protocol without trusted setup [56], and to build an asynchronous distributed key generation protocol without random oracles or PKI [55].

Zhang and Duan have argued that RABA could be useful as a general and "first-class distributed computing primitive" [54]. Our results bolster this point of view.

However, the way we use RABA in this paper is fundamentally different from all these works. Indeed, existing protocols that use RABA in the BKR framework need to run *n* parallel RABA instances; in contrast, this paper only needs to run an expected constant number of RABA instances.

MVBA. The notion of MVBA was introduced by Cachin, Kursawe, Petzold, and Shoup [11].

In the computational model (assuming—in addition to common coins—cryptographic tools such as threshold signatures), Abraham, Malkhi, and Spiegelman proposed an MVBA protocol [1] with $O(Ln^2 + \kappa n^2)$ communication, optimal word complexity, and the quality property, where L is the length of the input from each replica. Lu et al. [41] reduced the communication from $O(Ln^2 + \kappa n^2)$ to $O(Ln + \kappa n^2)$ by additionally using constant-size vector commitments [16].

In the information-theoretic and signature-free setting, the asynchronous distributed key generation protocol by Das et al. [27] implies an MVBA, as demonstrated in a recent work [25]. The MVBA protocols [25, 27] are information-theoretically secure (assuming information-theoretically secure common coins or Rabin dealer [50]), terminating in $O(\log n)$ time. In contrast, the MVBA proposed in this paper has the same message complexity as those in [25, 27], but terminates in O(1) time. Also, our MVBA protocol can lead to instantiations with lower communication than those from [25, 27].

Our MVBA protocol may also be used—possibly in a non-trivial manner—in certain (but not all) protocols in [34, 35, 53] without increasing the communication or making stronger assumptions.

From atomic broadcast to ACS. There are asynchronous atomic broadcast protocols without using ACS or MVBA (e.g., [36]). It is, however, unclear how to *efficiently* transform an atomic broadcast protocol to ACS, which has direct and practical applications beyond just BFT state machine replication (e.g., multi-party computation, asynchronous distributed key generation).

3 SYSTEM MODEL AND PROBLEM STATEMENT

We consider a system with n replicas, $\{p_1, \dots, p_n\}$, where f out of them may fail arbitrarily (Byzantine failures). A non-Byzantine replica is called a correct replica. All protocols we consider assume that $f \leq \lfloor \frac{n-1}{3} \rfloor$, which is optimal. A (Byzantine) *quorum* is a set of $\lceil \frac{n+f+1}{2} \rceil$ replicas. Without loss of generality, this paper may assume n=3f+1 and a quorum size of 2f+1. We assume the existence of point-to-point authenticated channels between each pair of replicas. We consider asynchronous networks making no timing assumptions on message processing or transmission delays.

We consider both adaptive corruption and static corruption. In adaptive corruption, the adversary can choose its set of corrupted replicas at any moment during the execution of the protocol, based on the information it has accumulated thus far. In contrast, in the static adversary model, the adversary is restricted to choosing its set of corrupted replicas at the beginning of the protocol. If all building blocks satisfy adaptive security, then our protocol achieves adaptive security. The ACS protocol implemented in this paper achieves static security, just as in prior protocols [29, 40, 43, 54]; but it can be made adaptively secure if using an adaptively secure common coin protocol [4, 38].

Each protocol instance is associated with a unique tag id. We may omit the identifiers in the pseudocode when no ambiguity arises. We may use subscripts to denote the instance identifier; for instance, RBC $_i$ denotes the reliable broadcast instance tagged with a unique identifier i and initiated by replica p_i .

We present in Appendix A the acronyms we use in this paper.

Asynchronous Common Subset (ACS). In ACS [6, 8], each replica holds an input, and correct replicas reach an agreement on a set of values. An ACS protocol is specified by *acs-propose* and *acs-decide* events. The value *acs-proposed* by a replica is called an input to the ACS protocol, and the value *acs-decided* by a replica is called an output of the protocol. ACS should satisfy the following properties:

- Validity: If a correct replica acs-decides a set v, then |v| ≥ n f and v contains values acs-proposed by at least n - 2f correct replicas.
- Agreement: If a correct replica acs-decides v, then every correct replicas acs-decides v.
- Termination: If all correct replicas acs-propose, then all correct replicas acs-decide.

In this paper, we use the conventional validity notion. Note that as argued in [54], for the validity property, the size of \mathbf{v} (denoted $|\mathbf{v}|$) can be relaxed such that $|\mathbf{v}| \geq f+1$; namely, in many applications, it suffices to ensure a weaker validity notion by requiring that \mathbf{v} contains values from at least one correct replica.

Multivalued validated Byzantine agreement (MVBA). MVBA allows each replica that has an input to agree on a value that satisfies a predicate *Q* known by all replicas [11]. An MVBA protocol satisfies the following properties:

- External validity: Every correct replica that terminates $mvbadecides\ v$ such that Q(v) holds.
- Agreement: If a correct replica mvba-decides v, then any correct replica that terminates mvba-decides v.

- **Integrity**: If all replicas follow the protocol, and if a correct replica mvba-decides v such that Q(v) holds, then some replica mvba-proposed v such that Q(v) holds.
- Termination: If all correct replicas are activated and all messages sent among correct replicas have been delivered, then all correct replicas mvba-decide.

The quality property was introduced by Abraham, Malkhi, and Spiegelman to bound the probability that the decided value was proposed by a correct replica [1]:

Quality: The probability of mvba-deciding a value that was proposed by a correct replica is at least 1/2.

In this paper, we first present an MVBA protocol without the quality property and then show how to modify it to achieve the quality property additionally. For our ACS construction, we only need an MVBA protocol without the quality property though.

The way we present our MVBA protocol follows that of [25]. In particular, the MVBA formalization in [25] requires that the predicate additionally uses some variable depending on the state of each node, a property needed in [25] and our MVBA protocols.

Byzantine fault tolerance (BFT). In a BFT protocol, clients *submit* transactions (requests) and replicas *deliver* them. The client obtains a final response to the submitted transaction from the replica responses. In a BFT system with n replicas, it tolerates $f \leq \lfloor \frac{n-1}{3} \rfloor$ Byzantine failures. The correctness of a BFT protocol is specified as follows:

- Safety: If a correct replica delivers a transaction tx before delivering tx', then no correct replica delivers a transaction tx' without first delivering tx.
- **Liveness**: If a transaction *tx* is *submitted* to all correct replicas, then all correct replicas eventually *deliver tx*.

4 BUILDING BLOCKS

We review the building blocks for our systems. To help understand RABA [54], we first review the notion of ABA. If v is a binary value, we use \bar{v} to denote 1-v.

Asynchronous binary Byzantine agreement (ABA). An ABA abstraction is specified by *aba-propose* and *aba-decide*. Each replica proposes a binary value (aka a vote) and correct replicas will decide on some value. ABA should satisfy the following properties:

- Validity: If all correct replicas aba-propose v, then any correct replica that terminates aba-decides v.
- Agreement: If a correct replica aba-decides v, then any correct replica that terminates aba-decides v.
- Termination: Every correct replica eventually aba-decides some value.
- Integrity: No correct replica aba-decides twice.

Reproposable Asynchronous Binary Agreement (RABA). RABA is a new primitive introduced by Zhang and Duan [54]. In contrast to conventional ABA protocols, where replicas can vote once only, RABA allows replicas to change their votes and vote twice. A RABA protocol is specified by *raba-propose*, *raba-repropose*, and *raba-decide*. For our purpose, RABA is "biased towards 1." A correct replica that proposed 0 is allowed to change its mind and repropose 1. A replica that proposed 1 is not allowed to repropose 0. If a

replica reproposes 1, it does so at most once. RABA (biased towards 1) satisfies the following properties:

- Validity: If all correct replicas raba-propose v and never raba-repropose \bar{v} , then any correct replica that terminates raba-decides v.
- Unanimous termination: If all correct replicas raba-propose v and never raba-repropose \bar{v} , then all correct replicas eventually terminate.
- **Agreement**: If a correct replica *raba-decides v*, then any correct replica that terminates *raba-decides v*.
- **Biased validity**: If f + 1 correct replicas raba-propose 1, then any correct replica that terminates raba-decides 1.
- **Biased termination**: Let Q be the set of correct replicas. Let Q_1 be the set of correct replicas that raba-propose 1 and never raba-repropose 0. Let Q_2 be correct replicas that raba-propose 0 and later raba-repropose 1. If $Q_2 \neq \emptyset$ and $Q = Q_1 \cup Q_2$, then each correct replica eventually terminates.
- Integrity: No correct replica raba-decides twice.

We explain some differences between ABA and RABA. Validity in RABA is slightly different from that for ABA, as we need to modify it to accommodate the RABA syntax. Integrity in RABA is used to ensure that RABA decides only once (even though we have an additional *raba-repropose* event).

Unanimous termination and biased termination are defined to ensure RABA termination in two different scenarios.

Biased validity in RABA requires that if f+1 correct replicas, not all correct replicas, propose 1, then a correct replica that terminates decides 1. We emphasize that the biased validity property was initially defined to build the PACE ACS protocol [54] such that sufficient transactions are delivered (the ACS validity property); however, in this paper, biased validity is essentially to ensure constant-time termination for our ACS construction.

For our implementation, we use the Pisa RABA protocol due to Zhang and Duan [54] assuming common coins and authenticated channels

Byzantine reliable broadcast (RBC). The RBC abstraction allows a sender p_s to reliably broadcast a message to the replicas. An RBC protocol is specified by two events *r-broadcast* and *r-deliver* such that the following properties hold:

- Validity: If a correct replica p_s *r-broadcasts* a message m, then p_s eventually *r-delivers* m.
- **Agreement**: If some correct replica *r-delivers* a message *m*, then every correct replica eventually *r-delivers m*.
- **Integrity**: For any message m, every correct replica r-delivers m at most once. Moreover, if a replica r-delivers a message m with sender p_s , then m was previously broadcast by replica p_s .

For our ACS implementation, we use CT RBC due to Cachin and Tessaro [13] that uses hash functions (with output length κ) and has a communication of $O(n|m| + \kappa n^2 \log n)$.

Common coins. Following prior works [8, 23, 45, 46, 54], we assume our protocols are supplied by a common coin, an object that is introduced by Rabin [50], which delivers the same sequence of random coins to replicas. We use the common coin protocol for the underlying *random leader election* protocol (denoted by Election()) and use it in the underlying RABA protocol.

For our implementation, we use the threshold PRF scheme by Cachin, Kursawe, and Shoup [12].

5 OUR MVBA APPROACH

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MVBA
  Input: value v_i such that a global predicate Q(v_i) holds
  Output: value v_k (proposed by p_k)
  Initialization: r \leftarrow 0
01 upon event mvba-propose(v_i)
02 r-broadcast v<sub>i</sub> for RBC<sub>i</sub>
                                                             {⊳ RBC phase}
     (every replica verifies whether Q(v_i) holds upon receiving v_i
before participating in RBC_i }
     wait for n - f RBC instances to complete \{ \triangleright \text{ Iteration phase} \}
     repeat
05
        k \leftarrow \mathsf{Election}()
06
        if some value is r-delivered in RBC_k
07
           raba-propose 1 for RABA<sub>r</sub>
08
09
           raba-propose 0 for RABA<sub>r</sub>
10
           if later some value is r-delivered in RBC_k
11
             raba-repropose 1 for RABA<sub>r</sub>
12
        if RABA_r outputs 1
13
           wait for RBC<sub>k</sub> to r-deliver value v_k
14
           terminate the protocol and mvba-decide(v_k)
15
```

Figure 1: Our MVBA protocol with a predicate Q. Code for p_i .

5.1 Overview

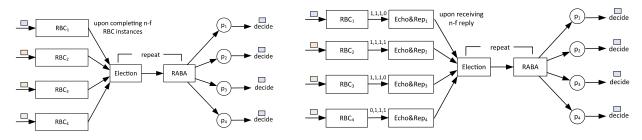
We present our signature-free MVBA protocol with O(1) time and $O(n^3)$ messages. In particular, we reduce MVBA to reliable broadcast, random leader election (via common coins), and reproposable asynchronous binary agreement.

At a high level, our MVBA protocol works as follows. First, each replica p_i runs RBC_i to disseminate its proposal, resulting in n parallel RBC instances. We aim to have replicas agree on the value *r-delivered* by *exactly one* of the *n* RBC instances. A crucial observation is that if n - f correct replicas complete n - f RBC instances, then at least f + 1 correct replicas have r-delivered some values for at least f + 1 RBC instances. If we know this set I of the f + 1 RBC instances, then we are almost done. In particular, all we need to do is to pick any of the RBC instances in I, say, RBC $_k$, and correspondingly, replicas run a RABA instance by proposing 1 once they r-deliver some value for the RBC $_k$ instance. To see why this intuitively works, first note that the agreement property in RBC and the biased termination property in RABA together guarantee termination, and meanwhile, the biased validity property ensures that even if only f + 1 correct replicas, not all correct replicas, propose 1, correct replicas that terminate will decide 1. Observing $|I| \ge f + 1$, if we take a random guess among all 3f + 1 replicas, then with a probability of at least 1/3, we can hit a good $k \in I$. Thus, our MVBA protocol will terminate in expected constant time. Throughout the process, we use ideal building blocks-RBC, RABA, and common coins-no cryptographic primitives such as threshold signatures.

We comment that external validity can be trivially enforced, as long as the predicate is publicly verifiable. Namely, we have treated

protocol	communication (threshold PRF)	time	assumption
CKPS [11]	$O(Ln^2 + \kappa n^2 + n^3)$	O(1)	threshold sig
AMS [1]	$O(Ln^2 + \kappa n^2)$	O(1)	threshold sig
Dumbo-MVBA [41]	$O(Ln + \kappa n^2)$	O(1)	threshold sig; vc
DYX+ MVBA [27]	$O(Ln^2 + \kappa n^3 \log n)$	$O(\log n)$	hash
DXKR MVBA [25]	$O(Ln^2 + \kappa n^3 \log n)$	$O(\log n)$	hash
Our MVBA (Sec. 5.2) + CT RBC [13]	$O(Ln^2 + \kappa n^3 \log n)$	O(1)	hash
Our MVBA (Sec. 5.2) + EFBRB [3]	$O(Ln^2 + \kappa n^2 + n^3 \log n)$	O(1)	none
Our MVBA (Sec. 5.2)+ CCBRB [3]	$O(Ln^2 + \kappa n^3)$	O(1)	hash
Our tailored MVBA (Sec. 5.4)	$O(Ln^2 + \kappa n^3)$	O(1)	hash

Table 2: Comparison of the MVBA protocols using common coins. Here we examine the communication cost of protocols by using threshold PRF [12] to generate common coins. L is the length of the input from each replica and κ is a security parameter. The "assumption" column means the additional assumption besides common coins and authenticated channels. "vc" stands for a constant-size vector commitment which typically requires trusted setup and pairing assumptions. Our protocols lead to the first O(1)-time MVBA protocols without threshold signatures. In addition, instantiating our MVBA (Sec. 5.2) with EFBRB [3] leads to an MVBA protocol achieving lower communication than any instantiations from [25, 27], as our MVBA uses only O(1) RABA and O(1) common coin instances— $O(\kappa n^2)$ bits using threshold PRF.



(a) MVBA without the quality property.

(b) MVBA with the quality property.

Figure 2: Our MVBA protocols.

it as a general predicate. Note, however, that if removing external validity, our MVBA protocol does not directly lead to a multivalued Byzantine agreement (MBA) protocol (e.g., [47]); this is because it does not satisfy the validity property in MBA.³

In this section, we first build MVBA without the quality property, satisfying all the security properties defined in CKPS [11]. We will show this MVBA protocol suffices to build our ACS protocol with O(1) time and $O(n^3)$ messages. Then we show that by including two additional communication rounds (using a variant of HotStuff technique [1, 52]), we can build MVBA with the quality property. Last, we demonstrate a highly efficient MVBA protocol that optimizes concrete communication cost and computational efficiency. Such an MVBA protocol benefits from a core observation that we may not necessarily need a fully-fledged RBC to construct MVBA.

5.2 Our MVBA Protocol

We present the pseudocode of our MVBA protocol in Figure 1. As illustrated in Figure 2a, the protocol has two phases: an RBC phase

with n parallel RBC instances; an iteration phase with only one RABA instance for each iteration.

RBC phase (lines 01-02). In the RBC phase, each replica p_i holds an input v_i that it proposes for the MVBA protocol such that $Q(v_i)$ holds. Upon the event $mvba-propose(v_i)$, replica p_i r-broadcasts v_i for an RBC instance RBC $_i$. Upon receiving value v_i r-broadcast by p_i in RBC $_i$, every replica waits until $Q(v_i)$ holds before participating in RBC $_i$.

Note that there are up to n parallel RBC instances, and every correct replica p_i verifies the predicate for all RBC instances running. As usual, we require that the predicate Q is verifiable across all correct replicas. Meanwhile, in certain applications, we may also require that the predicate Q depends on the internal state st of a particular replica. Namely, it is possible that Q(v, st) for some v and some replica fails to hold at the beginning, but Q(v, st) will hold at some point, all depending on st of the replica.

Iteration phase (lines 03-15). Each replica waits until it *r*-delivers n-f RBC instances before it enters the iteration phase. In each iteration, replicas iterate the Election() function and a RABA instance until a RABA instance outputs 1.

 $^{^3{\}rm Validity}$ in MBA requires that if all correct replicas propose 1, then all replicas that terminate decide 1.

Concretely, in each iteration r, replicas query the Election() function and obtain a random k such that $1 \le k \le n$ (line 05). At lines 06-07, if a replica has previously r-delivered some value in RBC $_k$, it raba-proposes 1 for the RABA instance denoted RABA $_r$ in iteration r. Otherwise, at lines 08-09, the replica raba-proposes 0. If a replica originally raba-proposes 0 but later r-delivers some value in RBC $_k$, p_i raba-reproposes 1 (at lines 10-11), ensuring protocol termination.

After each replica provides some input to RABA_r, it waits for the output of RABA_r. If p_i raba-decides 0, the replica continues to the next iteration r+1 (line 15). If p_i raba-decides 1, it waits for the output of RBC_k denoted as v_k (line 13). Then it mvba-decides v_k and terminates the protocol (line 14).

Complexity and instantiations. To understand the time complexity of our MVBA protocol, we first observe that at the end of the RBC phase, for at least f+1 instances, at least f+1 correct replicas have r-delivered some values (see Lemma 5.4 in this section). Second, the biased validity property of RABA guarantees that for any of these f+1 instances, replicas will raba-decide 1. As the Election() function selects any of the f+1 instances with at least a probability of 1/3, the protocol terminates in expected O(1) time.

Unlike prior signature-free MVBA constructions [25, 27] that terminate in expected $O(\log n)$ time (summarized in Table 2), our MVBA protocols have expected O(1) time.

Both the message complexity and the communication complexity of our MVBA are bounded by the n parallel RBC instances. As each RBC instance has $O(n^2)$ messages, our MVBA construction has $O(n^3)$ messages.

Table 2 summarizes the communication cost of our protocols assuming a threshold PRF [12]. For instance, if we use EFBRB [3] (an information-theoretically secure RBC), we obtain an MVBA protocol assuming common coins only. If we use CCBRB [3], we obtain a protocol using hashes and common coins and having lower communication than instantiations from [25, 27]. In particular, our MVBA protocol with EFBRB results in a protocol with $O(Ln^2 + \kappa n^2 + n^3 \log n)$ communication (the term $O(\kappa n^2)$ is due to the cost of common coins in O(1) RABA and Election() instances); in contrast, [25, 27] with EFBRB lead to MVBA protocols with $O(Ln^2 + \kappa n^3 + n^3 \log n)$ communication.

We comment that if the underlying RBC and common coin protocols are instantiated using information-theoretically secure protocols, then our MVBA protocols are also information-theoretically secure.

Proof of our MVBA. We show that the MVBA protocol presented in Sec. 5.2 achieves external validity, agreement, integrity, and termination.

Theorem 5.1 (External validity). Every correct replica that terminates mvba-decides v such that Q(v) holds.

PROOF. If any correct replica *mvba-decides v*, it has *raba-decided* 1. Hence, according to the validity property of RABA, at least one correct replica has *raba-proposed* 1 or *raba-reproposed* 1. If the correct replica *raba-proposes* 1 or *raba-reproposes* 1, it has *r-delivered*

v in the corresponding RBC instance. In the RBC instance, according to our specification of MVBA, every replica verifies whether Q(v) holds upon receiving any value from another replica in the RBC phase. If a correct replica r-delivered v in an RBC instance, then at least one correct replica has previously received v and verified that Q(v) holds. Hence, if one correct replica mvba-decides v, then at least one correct replica has verified that Q(v) holds. As Q(v) is verifiable across all correct replicas, every correct replica that mvba-decides v must have that Q(v) holds.

THEOREM 5.2 (AGREEMENT). If a correct replica mvba-decides v, then any correct replica that terminates mvba-decides v.

PROOF. Assume a correct replica p_i mvba-decides v. Then p_i must have raba-decided 1 in some RABA $_r$ for iteration r > 0, and for any iteration $\hat{r} < r$, it holds that RABA $_r$ outputs 0.

Now we assume another correct p_j mvba-decides v'. We now prove by contradiction that v' = v. We distinguish two cases for p_j : p_j mvba-decides in iteration r' = r; p_j mvba-decides in iteration $r' \neq r$.

Case 1: p_j mvba-decides v' in round r. In this case, we assume that p_j obtains k' from the Election() function and $RBC_{k'}$ outputs v'. As Election() outputs a *common* coin for the same input r, it must hold that k=k'. Hence, if $v\neq v'$, the agreement property of RBC would be violated.

Case 2: If r' > r, then according to our protocol, p_j raba-decides 1 in RABA $_{r'}$ and raba-decides 0 for any lower iteration r', including r. This violates the agreement property of RABA $_r$. Similarly, the argument holds for the case r' < r.

Theorem 5.3 (Integrity). If all replicas follow the protocol, and if a correct replica mvba-decides v such that Q(v) holds, then some replica mvba-proposed v such that Q(v) holds.

PROOF. If a correct replica p_i mvba-decides v, it raba-decides 1 in some RABA $_r$ and r-delivers v in RBC $_k$ where k is the corresponding common coin. According to the integrity property of RBC, v was previously broadcast by replica p_k .

Lemma 5.4. If all correct replicas enter the iteration phase, then for at least f + 1 RBC instances, at least f + 1 correct replicas have r-delivered some values.

PROOF. Instead of *directly* bounding the number of correct replicas that have *r-delivered* some values, we bound the number of instances where fewer than f+1 correct replicas have *r-delivered* some values. First, we observe that all correct replicas *r-deliver* some values for (2f+1)(2f+1) RBC instances in total. As there are at most (3f+1)(2f+1) instances for correct replicas, the total number of instances where correct replicas do not *r-deliver* some values are upper bounded by $(3f+1)(2f+1)-(2f+1)(2f+1)=2f^2+f$. Hence, the number of RBC instances where fewer than f+1 correct replicas have *r-delivered* some values is bounded by $\frac{2f^2+f}{f+1}<\frac{2f^2+2f}{f+1}=2f$. That is, the number of RBC instances where at least f+1 correct replicas have *r-delivered* some values is at least f+1.

We comment that PACE observed a similar claim that works in a context with a different goal.

 $^{^4\}mathrm{Note}$ that if we assume the Election() function is a random permutation instead of a random function, then we can also use RABA_k to uniquely and unambiguously denote the RABA instance.

Theorem 5.5 (Termination). If all correct replicas are activated and all messages sent among correct replicas have been delivered, then all correct replicas mvba-decide.

PROOF. If all correct replicas start the protocol, then according to the validity property of RBC, every correct replica completes at least n-f RBC instances. During the iteration phase, we first show that any iteration r will complete and then show that eventually, some RABA $_r$ will output 1. For each iteration r, we assume that k is returned by the Election() function in iteration r.

We first show that every iteration r will complete. For each iteration r, we distinguish three cases: 1) all correct replicas have r-delivered some value in RBC_k ; 2) at least one correct replica has r-delivered some value in RBC_k , and at least one correct replica has not r-delivered any value in RBC_k ; 3) none of the correct replicas have r-delivered any value in RBC_k .

Case 1: Due to the unanimous termination property, it holds that $RABA_r$ terminates.

Case 2: If at least one correct replica has r-delivered some value in RBC_k , then from the agreement property of RBC , any correct replica eventually r-delivers some value. According to our protocol, any correct replica that provides 0 as RABA input (in which case it has not r-delivered any value in RBC_k when the iteration begins) will eventually raba-repropose 1. Thus, the biased termination condition of RABA will eventually be satisfied. Hence, RABA_r will terminate, and iteration r will eventually complete.

Case 3: If none of the correct replicas r-deliver any value in RBC_k , iteration r will complete due to the unanimous termination property of RABA. Otherwise, if at least one correct replica later r-delivers some value in RBC_k , then according to case 2, iteration r will complete (due to the biased termination of RABA).

We now prove that eventually, in some iteration r, RABA $_r$ outputs 1, so the protocol will terminate. From Lemma 5.4, for at least f+1 RBC instances, at least f+1 correct replicas have r-delivered some value after they enter the iteration phase. Let I be the set of the f+1 RBC instances. Due to the biased validity property of RABA, RABA $_r$ outputs 1. As Election() outputs a uniformly random coin for each iteration, we have that with probability $\frac{f+1}{3f+1} \approx \frac{1}{3}$, it holds that $k \in I$

After RABA_r outputs 1, every correct replica waits for the output of RBC_k . If RABA_r outputs 1, at least one correct replica has $\mathit{raba-proposed}\ 1$ or $\mathit{raba-reproposed}\ 1$. (Otherwise, the unanimous termination property of RABA would be violated.) Therefore, at least one correct replica has $\mathit{r-delivered}\$ some value in RBC_k . From the agreement property of RBC, every correct replica eventually $\mathit{r-delivers}\$ some value in RBC_k and then $\mathit{mvba-decides}\$.

THEOREM 5.6. Our MVBA protocol has expected O(1) running time

PROOF. From Lemma 5.4, for at least f+1 RBC instances, at least f+1 correct replicas have r-delivered some value after they enter the iteration phase. Due to the biased validity property of RABA, RABA $_r$ outputs 1. Since Election() outputs a uniformly distributed random coin for each iteration, it holds that with probability $\frac{f+1}{3f+1} \approx$

 $\frac{1}{3}$, we have $k \in I$. Therefore, the protocol has expected O(1) running time.

5.3 MVBA with the Quality Property

```
MVBA with the quality property

Initialization: W_i \leftarrow [0]^n

replace line 03 in Figure 1 using the following lines:

01 upon r-delivering v_j for RBC_j

02 W_i[j] \leftarrow 1

03 wait for n-f RBC instances to complete

04 send (ЕСНО, W_i) to all replicas

05 upon (ЕСНО, W_j) from p_j such that there are n-f 1's in W_j

06 if for any W_j[l] = 1, RBC_l outputs some value

07 send (REP, i) to p_j

08 wait for n-f (REP) messages
```

Figure 3: Our MVBA protocol with the quality property. The code for p_i .

We show that by adding two additional communication steps, we can build MVBA with the quality property. As illustrated in Figure 2b, we introduce an echo and reply procedure between the RBC phase and the iteration phase. In the pseudocode, all we need to do is to replace line 03 in Figure 1 using the lines of code in Figure 3. Each replica p_i now additionally maintains one vector W_i to track the set of completed RBC instances. After p_i completes RBC_j , it sets $W_i[j]$ as 1 (lines 01-02). When p_i completes n-fRBC instances, it broadcasts the W_i vector to all replicas (03-04). It then expects to receive n - f (Rep) messages from the replicas, representing that n - f replicas have also r-delivered some values for the same n - f RBC instances. To achieve this goal, for each replica p_i , upon receiving W_i from p_i , p_i first checks whether the vector contains n - f 1's. Then for each $W_i[l] = 1$, p_i waits until some value is r-delivered in RBC₁ (lines 06-07). After that, p_i sends a (Rep) message to p_i (line 08). Upon receiving n - f (Rep) messages, p_i enters the iteration phase.

The protocol achieves quality, mainly because upon receiving a vector W_j , each replica verifies whether it has r-delivered some value in each RBC_l instance for $W_j[l]=1$. Hence, if a correct replica p_j receives n-f (Rep) messages, at least n-f replicas must have r-delivered the same values in the same set of n-f RBC instances due to the agreement property of RBC. Hence, with probability $\frac{2f+1}{3f+1} \approx \frac{2}{3}$, replicas mvba-decide a value from a correct replica. The protocol thus achieves quality.

The way of achieving quality can be viewed as using the technique from [1, 52] and using the agreement property in RBC. We show the correctness of the MVBA in this subsection in Appendix C.

5.4 Tailored MVBA from Weak RBC

While we can use EFBRB and CCBRB for low communication cost in our MVBA protocol, both of them rely on online error correcting

(OEC) code and may suffer some degraded performance during failures (due to the "trial-and-error" OEC pattern). Additionally, EFBRB has significantly more steps than the classic RBC protocols [10, 13].

In this section, we provide a more practical MVBA construction that achieves $O(Ln^2 + \kappa n^3)$ communication. While its communication is the same as that of using CCBRB, our protocol in this subsection outperforms the CCBRB instantiation (in Sec. 5.2) in terms of both concrete communication cost and computational efficiency. First, the construction in this subsection does not use erasure coding or online error correcting code. Hence, the hidden constant in the bulk data term Ln^2 is 1 (namely $1Ln^2$) instead of 3 (at least $3Ln^2$ or more if using erasure coding or error correcting code). Namely, for a large L, the communication cost of this construction is about 1/3 of that of CCBRB-based MVBA. Second, as the construction in this subsection uses hashes only and does not use online error correction, it is computationally more efficient in both gracious and uncivil executions.

We show our tailored MVBA construction in Figure 4. The protocol relies on a weak RBC primitive, which we call WRBC. The workflow of WRBC (lines 21-37) is similar to the 3-phase RBC (e.g., Bracha's broadcast [10]), but we only use hashes in the second and the third phases. As a result, when a replica successfully terminates WRBC, all correct replicas will eventually obtain the hash of the WRBC input message; when needed (later), all correct replicas can retrieve the message based on the hash. (WRBC appears implicitly and partly used in, e.g., [26], and we claim no novelty about WRBC itself.)

Concretely, at lines 21-37, the sender p_j in each WRBC $_j$ first broadcasts its input v_j in (send) messages. Upon receiving the value v_j from p_j , each replica p_i verifies whether the predicate $Q(v_j)$ holds. If so, p_i updates $T_i[j]$ as v_j and then broadcasts a (Echo, $Hash(v_j)$) message (lines 25-26). Otherwise, p_i stores the (Send) message and processes it until $Q(v_j)$ holds (lines 28-29). Upon receiving n-f (Echo, h) messages with the same hash value h, each replica broadcasts a (Ready, h) message (lines 30-31). If a replica p_i receives f+1 (Ready, h) messages but has not sent a (Ready) message, p_i also broadcasts a (Ready, h) message (lines 32-33). Upon receiving 2f+1 (Ready, h) messages, p_i wr-delivers h (line 37). If some h is wr-delivered but the hash of $T_i[j]$ is not h, p_j sets $T_i[j]$ as \bot (lines 35-36).

We now describe our tailored MVBA protocol. There are two major changes on top of our MVBA protocol in Sec. 5.2. First, in the RBC phase, we use WRBC instead of the standard RBC, where for each WRBC $_j$, the sender p_j wr-broadcasts a value v_j and correct replicas wr-deliver $h_j = Hash(v_j)$. Second, as each WRBC instance outputs the hash of the value instead of the original value broadcast by the sender, we need to retrieve the value after replicas reach an agreement in the iteration phase. In particular, in some iteration r where k is the output of the Election() function, after RABA $_r$ outputs 1, each replica first waits for WRBC $_k$ to output some value h_k and then starts the retrieval (lines 13-18). If p_i has some value in $T_i[k]$, it broadcasts a (Value, $T_i[k]$) to all replicas. If p_i does not hold a $T_i[k]$ value, it waits to receive a (Value, v_k) such that $Hash(v_k) = h_k$ and then sets $T_i[k]$ as v_k . After $Hash(T_i[k]) = h_k$, p_i terminates the protocol and mvba-decides $T_i[k]$ (line 19).

```
A Practical MVBA Construction
  Input: Value v_i such that a global predicate Q(v_i) holds
  Output: Value v_k proposed by p_k
  Initialization: r \leftarrow 0, T_i \leftarrow [\bot]^r
01 upon event mvba-propose(v_i)
                                                         {⊳ WRBC phase}
     wr-broadcast(v_i) for WRBC<sub>i</sub>
     wait for n - f WRBC instances to complete \{ \triangleright \text{ Iteration phase } \}
05
        k \leftarrow Election()
06
        if some value is wr-delivered in WRBC_k-
07
           raba-propose 1 for RABA<sub>r</sub>
08
09
          raba-propose 0 for RABA<sub>r</sub>
10
          if later some value is r-delivered in RBC_k
             raba-repropose 1 for RABA<sub>r</sub>
11
12
        if RABA_r outputs 1
13
          wait for WRBC_k to wr-deliver value h_k
14
          if T_i[k] \neq \bot
15
             broadcast (Value, T_i[k])
16
17
             wait for (Value, v_k) such that Hash(v_k) = h_k
18
             T_i[k] \leftarrow v_k
19
          terminate the protocol and mvba-decide(T_i[k])
20
        r \leftarrow r + 1
21 upon event wr-broadcast(v_i) for instance WRBC<sub>i</sub>
     {replica p_i broadcasts (Send, v_i)}
     upon receiving (Send, v_i) from p_i
24
        if Q(v_i) holds
25
          T_i[j] \leftarrow v
26
          broadcast (Есно, Hash(v_i))
28
29
          store the message until Q(v_i) holds
     upon receiving n - f matching (Есно, h)
        broadcast (READY, h)
     upon receiving f + 1 matching (Ready, h) and (Ready) message
    not been sent yet
        broadcast (READY, h)
     upon receiving n - f matching (Ready, h)
35
       if Hash(T_i[j]) \neq h
36
           T_i[j] \leftarrow \bot
        wr-deliver h
```

Figure 4: A Practical MVBA protocol. Code is for p_i .

The communication bottleneck of this protocol is the WRBC phase, as the communication cost for other steps (including the retrieval step) is $O(\kappa n^2)$. For each WRBC instance, the sender broadcasts a message (length L), and replicas exchange hashes of values in the second phase and the third phase, so each instance has $O(n^2)$ messages and $O(Ln + \kappa n^2)$ communication. As there are n parallel WRBC instances, our practical MVBA construction has $O(Ln^2 + \kappa n^3)$ communication. The correctness of our protocol is similar to that of our MVBA in Sec. 5.2. In particular, each WRBC protocol WRBC j can guarantee that if a correct replica wr-delivers

some value h, every correct replica eventually wr-delivers h. Furthermore, at least f+1 correct replicas must set their $T_i[j]$ as value v such that Hash(v)=h. Hence, the value v can be retrieved by any correct replicas. We prove the correctness of this tailored MVBA construction in Appendix D.

6 OUR ACS APPROACH

6.1 Overview

We now present our ACS protocol with O(1) time and $O(n^3)$ messages. At the core of our ACS protocol is the reduction of ACS to our MVBA construction with a specific predicate. In particular, we use nparallel RBC instances for replicas to disseminate their acs-proposed values. Then each replica mvba-proposes a vector of n - f bits, representing the n - f completed RBC instances. Crucially, we define the global predicate of MVBA as the following: given a proposal with n - f bits, each replica considers the proposal valid only if it has completed the same n - f RBC instances. (As we commented earlier, the predicate depends on the state of a particular replica: it is possible that the predicate does not hold at the beginning but will hold at some point.) In this way, we can guarantee that the output of ACS consists of at least n - f acs-proposed values, satisfying the validity property of ACS. As our MVBA component completes in expected O(1) time, our ACS protocol also terminates in expected constant time.

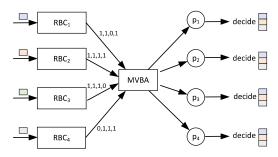


Figure 5: Our ACS protocol.

6.2 The ACS Protocol

We describe our ACS protocol in Figure 5 and the pseudocode in Figure 6. Our ACS protocol has two phases: an RBC phase and an MVBA phase.

RBC phase (lines 02-04). Each replica p_i holds an input v_i . Upon the event $acs-propose(v_i)$, p_i r-broadcasts v_i . Upon completing an RBC instance RBC j, p_i sets W[j] as 1 (lines 03-04), where W is a global map used to track the status of the RBC instances.

MVBA phase (lines 05-12). Replica p_i enters the MVBA phase after completing n - f RBC instances. Each replica p_i sets W_i as W_i and uses W_i as input for MVBA with a predicate $Q(W_j, W)$ (lines 06-07). Note that the 1's in W for each p_i continue to grow as more RBC instances complete. The value mvba-proposed by each p_i is a snapshot of W_i with at least n - f RBC instances completed.

We define a global predicate $Q(W_j, W)$ for each value W_j mvba-proposed by p_j as follows. First, each W_j is a n-bit vector. Second, W_j consists of at least n-f 1's, representing the n-f RBC instances

```
ACS
  Input: value v_i
  Output: n - f values v, among which at least n - 2f are proposed
            by correct replicas
  Initialization: r \leftarrow 0, W \leftarrow [0]^n, \mathbf{v} \leftarrow \bot
  Let Q be the following predicate for MVBA:
     Given value W_i mvba-proposed by some replica p_i, Q(W_i, W) \equiv
     (|W_j| = n \text{ and for at least } n - f l \text{ such that } 1 \le l \le n, W_j[l] = 1
     and W_i \subseteq W)
01 upon event acs-propose(v_i)
     r-broadcast vi for RBCi
                                                            {⊳ RBC phase}
     upon r-delivering v_j for RBC_j
04
        W[j] \leftarrow 1
05
     wait for n - f RBC instances to complete
                                                         {⊳ MVBA phase}
06
     W_i \leftarrow W
     mvba-propose(W_i) with predicate Q(W_i, W)
     upon event mvba-decide(W_k)
09
        for every W_k[j] = 1
            wait for RBC _i to r-deliver value v_i
10
11
            \mathbf{v} \leftarrow \mathbf{v} \cup v_i
12
        terminate the protocol and acs-decide(v)
```

Figure 6: The ACS protocol. Code is for p_i .

that replica p_j has completed. Third, $W_j \subseteq W$. Namely, for each $W_j[l] = 1$, replica p_i must wait until it has r-delivered some value in RBC $_l$. Hence, in MVBA, every replica may need to wait until the global predicate is satisfied for each mvba-proposed message. As discussed previously in Sec. 5, the predicate depends on the internal state W of each replica p_i ; it is possible that $Q(W_j, W)$ fails to hold when p_i receives the mvba-proposed value by p_j , but $Q(W_j, W)$ will hold at some point as p_i r-delivers more RBC instances.

After p_i provides some input to MVBA, it waits for the output of MVBA (line 08). According to the integrity property of MVBA, the output of MVBA W_k must be proposed by some replica such that $Q(W_k, W)$ is satisfied. At lines 09-11, for each $W_k[j] = 1$, p_i waits for the output of RBC $_j$, v_j . Then v_j is added to a set v. After all such RBC instances are complete, p_i acs-decides v (line 12).

Complexity and discussion. Our ACS protocol terminates in expected constant time, as the underlying MVBA protocol runs in expected O(1) time. Moreover, our ACS protocol clearly has $O(n^3)$ messages. The communication cost of our ACS protocol depends on the underlying RBC protocol, as the input length to MVBA is only n. For instance, our ACS protocol with CT RBC has $O(Ln^2 + \kappa n^3 \log n)$ communication.

In contrast to BKR ACS and the state-of-the-art PACE ACS that terminate in $O(\log n)$ time and need n (R)ABA instances, our ACS protocol has O(1) time and uses only O(1) RABA instances.

Note that as the input of MVBA indicates that at least n-2f acs-proposed values from correct replicas will eventually be acs-decided, we do not need the quality property for the underlying MVBA protocol.

6.3 Proof of our ACS Protocol

Theorem 6.1 (Validity). If a correct replica acs-decides a set \mathbf{v} , then $|\mathbf{v}| \ge n-f$ and \mathbf{v} contains values acs-proposed by at least n-2f correct replicas.

PROOF. According to the protocol, every correct replica p_i first $mvba\text{-}decide(W_k)$ and then obtains \mathbf{v} . The predicate of $Q(W_k, W)$ specifies that there are at least n-f 1's in the W_j vector. Then p_i waits for each RBC $_j$ to output some v_j for each $W_k[j]=1$ and includes v_j in \mathbf{v} . As there are n-f values in \mathbf{v} , corresponding to n-f RBC instances, it holds that $|\mathbf{v}| \geq n-f$. Since there are at most f faulty replicas, at least n-2f values are acs-proposed by correct replicas.

Theorem 6.2 (Agreement). If a correct replica acs-decides \mathbf{v} , then every correct replicas outputs \mathbf{v} .

PROOF. If a correct replica p_i acs-decides \mathbf{v} , it first mvba-decides W_k . Then for each $W_k[j] = 1$, v_j is r-delivered by RBC $_j$ and v_j is included in \mathbf{v} . We assume that another correct replica p_j acs-decides $\mathbf{v}' \neq \mathbf{v}$ and then prove the theorem by contradiction.

If p_j acs-decides \mathbf{v}' , it mvba-decides W_k' . According to the agreement property of MVBA, it must hold that $W_k = W_k'$. Then for each $W_k'[j] = 1$, p_j obtains the output of RBC j v_j' and includes v_j' in \mathbf{v}' . If for any j, $v_j \in \mathbf{v}$ and $v_j' \in \mathbf{v}'$ and $v_j \neq v_j'$, the agreement property of RBC is violated. Hence, we have $\mathbf{v} = \mathbf{v}'$.

THEOREM 6.3 (TERMINATION). If all correct replicas acs-propose, then all correct replicas acs-decide.

PROOF. If all correct replicas *acs-propose*, every correct replica p_i starts RBC_i. Due to the validity property of RBC, every correct replica *r-delivers* in RBC_i. Therefore, every correct replica eventually completes n-f RBC instances. Then each correct replica p_i *mvba-proposes* W_i . For each $W_i[j] = 1$, we know that p_i *r-delivers* some value in RBC_j. From the agreement property of RBC, every correct replica eventually *r-delivers* some value in RBC_j. Thus, the predicate $Q(W_i, W)$ eventually holds at every correct replica. Namely, for each $W_i[l] = 1$, each correct replica p_j eventually *r-delivers* some value in RBC_l, so $Q(W_i, W)$ eventually holds at p_j .

According to the termination property of MVBA, every correct replica eventually mvba-decides some value W_k . Furthermore, from the integrity property and external validity property of MVBA, $Q(W_k, W)$ eventually holds, so for each $W_k[j] = 1$, at least one correct replica r-delivers some value in RBC $_j$. Due to the agreement property of RBC, every correct replica eventually r-delivers some value v_j for each RBC $_j$. Thus, every correct replica includes each v_j in its output.

7 A PRACTICAL ACS INSTANTIATION

We use CT RBC as the underlying RBC protocol in ACS. We use our tailored MVBA protocol in Sec. 5.4 as our MVBA protocol. Our tailored MVBA protocol internally uses the hash-based WRBC protocol in Sec. 5.4 and Pisa RABA protocol [54]. We call the resulting instantiation FIN.

8 IMPLEMENTATION AND EVALUATION

We implemented FIN in Golang. In the same library, we implemented PACE [54], the most efficient ACS construction of the same type. Our implementation involves around 9,000 LOC for the two protocols and about 1,000 LOC for evaluation. Additionally, we also assess Dumbo [33]⁵ and Speeding Dumbo (denoted as sDumbo) [32]⁶. Note Dumbo and sDumbo rely on threshold signatures and use stronger pairing assumptions. We do not compare FIN with BFT protocols with unbounded memory (e.g., Dumbo-NG [30]) or weaker liveness properties (e.g., Bullshark [31] and Tusk [24]).⁷

In our implementation, we use gRPC as the communication library. We use HMAC to realize the authenticated channel and use SHA256 as the underlying hash function. We implement threshold PRF [12] to realize common coins for RABA and the random leader election protocol. We use a Golang-based erasure coding library⁸ to implement CT RBC.

We evaluate the performance of our protocols on Amazon EC2 using up to 121 virtual machines (VMs). We use *m5.xlarge* instances for our evaluation. The m5.xlarge instance has four virtual CPUs and 16GB memory. We deploy our protocols in the WAN setting, where replicas are evenly distributed across different regions: us-west-2 (Oregon, US), us-east-2 (Ohio, US), ap-southeast-1 (Singapore), and eu-west-1 (Ireland).

We conduct the experiments under different network sizes and batch sizes. We use f to denote the network size; in each experiment, we use n=3f+1 replicas in total. We use b to denote the batch size, where each replica proposes b transactions in each epoch (i.e., one ACS instance). For each experiment, we run five epochs and report the average performance (for both throughput and latency). The default transaction size is 250 bytes. We also additionally evaluate the performance using a transaction size of 100 bytes and report the performance in Appendix B.

We summarize our main evaluation results as follows.

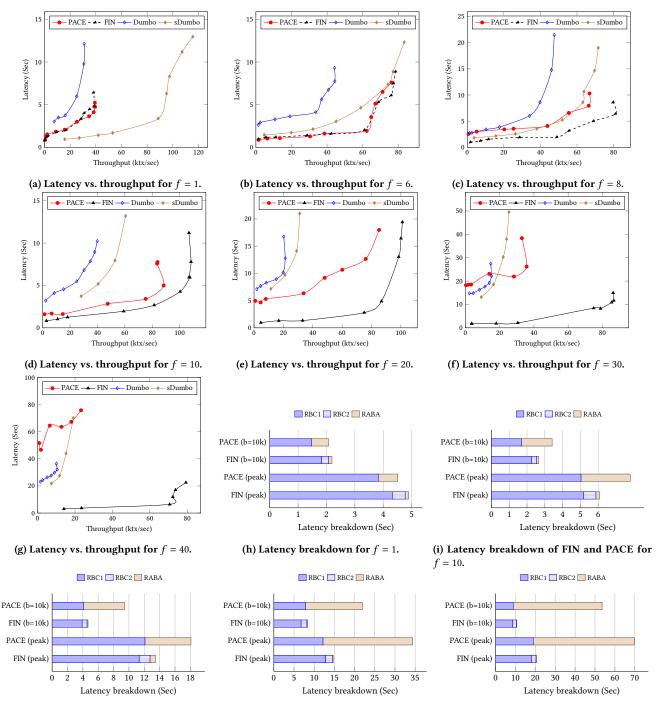
- We evaluate latency vs. throughput and the peak throughput varying f from 1 to 40. We demonstrate that for *both metrics*, FIN consistently and drastically outperforms PACE when n > 16 and all the protocols we evaluated (PACE, Dumbo, sDumbo) when n > 22. The performance difference between FIN and the other protocols drastically increases as n grows. For instance, when n = 121 and f = 40, the peak throughput of FIN is 3.41x that of PACE, 4.15x that of sDumbo, and 8.79x that of Dumbo.
- We also assess the latency breakdown to analyze the improvement of FIN over PACE. The experiments we carefully designed explain well why FIN outperforms PACE. In particular, we show that the RABA phase for FIN (with constant RABA instances) occupies only 1.23%-5.22% of the overall latency, in sharp contrast to PACE, where its RABA phase (with *n* parallel RABA instances) occupies 15.10%-83.66% of the total runtime.
- We evaluate the performance of the protocols under different failure scenarios. Our results show that FIN is highly robust against various failures.

⁵https://github.com/yylluu/dumbo

⁶https://github.com/xygdys/Consensus

⁷Dumbo-NG, Tusk, and Bullshark are BFT protocols that separate data transmission from consensus for higher throughput, and FIN can also use the technique to improve the performance.

⁸https://github.com/klauspost/reedsolomon



(j) Latency breakdown of FIN and PACE for (k) Latency breakdown of FIN and PACE for (l) Latency breakdown of FIN and PACE for f=20. f=40.

Figure 7: Latency vs. throughput and latency breakdown.

• We additionally evaluate the performance with a smaller transaction size of 100 bytes, where we find that FIN outperforms PACE in a more significant manner. For instance, when f = 20,

the throughput of FIN with 100-byte transactions is 2.14x that of PACE.

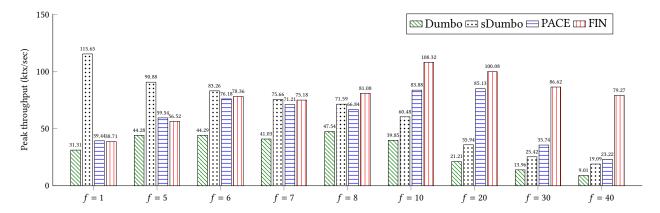


Figure 8: Peak throughput of all the four protocols as f grows.

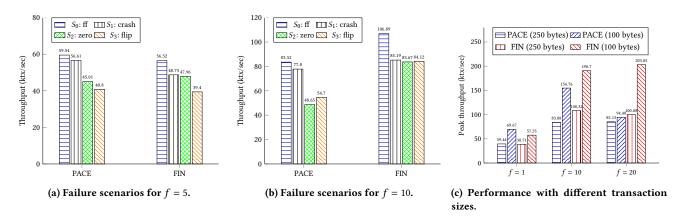


Figure 9: Evaluation results of FIN and PACE under failures and using different transaction sizes.

Latency vs. throughput. We report latency vs. throughput and peak throughput as f increases. We first show the latency vs. throughput of FIN, PACE, Dumbo, and sDumbo for f=1,6,8,10,20,30,40 in Figure 7a-7g. Each point represents one experiment for a particular batch size. In each experiment, we begin with a small batch size and assess its latency and throughput for the batch size. We repeat the process until the throughput does not increase but the latency keeps growing (i.e., when the throughput reaches its peak—peak throughput).

For f=1, the performance of FIN is lower than sDumbo, but only slightly lower than PACE; it is consistently higher than Dumbo. The reason why PACE is slightly more efficient than FIN is that FIN involves two phases of RBC (one for ACS and one inside MVBA), while the parallel RABA instances in PACE do not bottleneck the performance for such small f's. Meanwhile, the reason why FIN achieves lower performance than sDumbo is that sDumbo optimizes the failure-free case where the expensive erasure-coded recovery phase is not triggered; both PACE and FIN can use the same technique for higher failure-free performance. The drawback of using the technique is that it will cause significant performance degradation under failures and attacks.

When f = 6, FIN outpaces sDumbo for all points except the ones where they attain their peaks; FIN starts to experience higher

performance than PACE from this point on. When $f \geq 7$, FIN consistently outperforms all the other protocols we assessed in terms of latency vs. throughput.

The performance gain for FIN over PACE is clearly due to the constant-time termination and the constant number of RABA instances

Peak throughput. We report the peak throughput of all the four protocols we evaluate in Figure 8 for f=1,5,6,7,8,10,20,30,40. As shown in Figure 8, FIN outperforms Dumbo for all cases. When $f \leq 5$, FIN achieves a lower peak throughput than PACE and sDumbo. When f=6, FIN starts to outperform PACE, but the peak performance of FIN is slightly lower than sDumbo. For $f \geq 8$, FIN consistently outperforms all the other three protocols in terms of peak throughput. The performance difference between FIN and the other protocols becomes increasingly significant as f increases. For instance, when f=30, the peak throughput of FIN is 2.42x that of PACE, 3.40x that of sDumbo, and 6.20x that of Dumbo. When f=40, the peak throughput of FIN is 3.41x that of PACE, 4.15x that of sDumbo, and 8.79x that of Dumbo.

Note that there are minor "inconsistencies" between the latency vs. throughput evaluation and the peak throughput evaluation. For example, when f=6, FIN outperforms sDumbo in terms of

latency vs. throughput for almost all points (experiments) but has slightly lower peak throughput. We comment that these minor inconsistencies are completely normal, and users should choose their protocols according to their priorities.

We also observe that as f increases, the peak throughput of protocols of PACE and FIN first increases and decreases. Indeed, as f increases, the number of transactions delivered for both protocols increases, but when f further increases, the network bandwidth consumption dominates the performance. In our experiments, FIN achieves the highest peak throughput when f=10 and PACE achieves its highest peak throughput when f=20. In contrast, the performance of Dumbo and sDumbo degrades as f increases.

Latency breakdown. To help understand why FIN outperforms PACE, we report the latency breakdown for the experiments. In FIN, there are three phases: the RBC phase for ACS (denoted as RBC1), the RBC phase inside MVBA (denoted as RBC2), and the iteration phase with a random leader election and one RABA instance at a time (denoted as RABA). In contrast, PACE has an RBC phase (the same as that in FIN, denoted as RBC1) and a RABA phase with nparallel RABA instances (denoted as RABA). Here the latency of the RBC phase is measured from the beginning of the first RBC instance to the completion of the (n - f)-th RBC instance. Moreover, the latency of the RABA phase for PACE is measured from the beginning of the first RABA instance to the time ACS completes. Finally, the latency of the RABA phase for FIN is measured from the beginning of the iteration phase to the time when ACS completes. The latency breakdown experiments can explain why FIN outperforms PACE, help identify the bottleneck of the two protocols, and assist in understanding the scalability results.

Figure 7h-7l report the latency breakdown for FIN and PACE. We test two settings: a fixed one with $b=10,\!000$, and the smallest batch size where both protocols achieve their peak throughput. We first observe that the RBC1 phases in the two protocols share almost the same latency. This is not surprising, as the RBC1 phase is the only phase that carries bulk data for both protocols, and we use CT RBC for both of them. Additionally, the latency percentage for the RBC2 phase in FIN is comparatively very low. This is because the RBC2 phase does not have any bulk data, and its input size is small (n bits). Hence, the RBC2 phase in FIN does not incur much overhead to the protocol.

In all cases, the RABA latency in PACE is much higher than that for FIN. For f=1 to 40, the latency of the RABA phase in PACE is 15.10%-83.66% of the overall latency. In contrast, the RABA phase in FIN occupies only 1.23%-5.22% of the total runtime.

Moreover, the latency percentage of the RABA phase within the overall PACE latency becomes increasingly larger as f increases, but the RABA latency percentage in FIN remains steady despite an increasing f. Indeed, when f=40, the RABA phase occupies 83.66% of the overall consensus latency for PACE. In contrast, the latency of the RABA phase in FIN is only 1.23% of the overall latency. In fact, even if we consider the latency caused by RBC2 and RABA (i.e., MVBA) in FIN, it only occupies 11.45% of the overall latency. This observation explains well why the performance difference between FIN and PACE becomes increasingly larger as f increases. Indeed, FIN only needs an expected constant number of RABA instances.

Performance under failures. To assess the robustness of FIN, we report the performance of FIN and PACE under various failure scenarios. Following prior works [54, 56], we consider the following scenarios, where the f faulty replicas are evenly distributed in the EC2 regions we use.

- S₀: (failure-free) In this scenario, all replicas are correct.
- S1: (crash) In this scenario, we let f replicas crash by not participating in the protocols.
- S₂: (Byzantine; keep voting 0) In this scenario, we fail f replicas and let them keep voting for 0 in each step of RABA.
- S3: (Byzantine; flipping the RABA input) In this scenario, we fail f replicas and ask them to always vote for a flipped value in RABA.

We fix b to 30,000 and present the throughput for f=5 in Figure 9a and f=10 in Figure 9b. We choose these two settings because they are the settings where the two protocols share similar performance. First, for the crash failure scenario, the performance of both protocols degrades only slightly. For both Byzantine scenarios, the percentage for the performance degradation of FIN is lower than that of PACE. The performance of FIN in Byzantine scenarios degrades by 15.1%-30.2% compared to the failure-free scenario. In contrast, the performance of PACE degrades by 24.39%-41.74%. Notably, for f=10, FIN under all failure scenarios outperforms PACE in its failure-free scenario.

Performance with different transaction sizes. We additionally evaluate the performance of FIN and PACE with a smaller transaction size of 100 bytes as shown in Figure 9c. In this setting, FIN outperforms PACE more significantly. For instance, when f=20, the throughput of FIN with 100-byte transactions is 2.14x that of PACE, while for 250-byte transactions, the throughput of FIN is only 17.56% higher. We also find that the throughput of FIN with 100-byte transactions is roughly 2x that with 250-byte transactions.

9 CONCLUSION

We present the first signature-free ACS protocol with O(1) time and $O(n^3)$ messages, resolving a long-standing open problem in fault-tolerant distributed computing and cryptography. As a core ingredient in our ACS construction and a primitive of independent interests, we present the first signature-free MVBA protocols with O(1) time and $O(n^3)$ messages. In contrast, existing signature-free MVBA protocols have $O(\log n)$ time and $O(n^3)$ messages. From the practical side, we implement a practical ACS protocol called FIN. We demonstrate that FIN significantly outperforms the state-of-theart BFT protocol of the same kind—PACE and outperforms other BFT protocols with standard liveness guarantees.

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A ACRONYMS

We explain the acronyms used in this work in Table 3.

BFT	Byzantine fault tolerance
ACS	asynchronous common subset
MPC	multi-party computation
MVBA	multivalued validated Byzantine agreement
DAG	direct acyclic graph
RBC	reliable broadcast
WRBC	weak reliable broadcast
ABA	asynchronous binary agreement
RABA	reproposable asynchronous binary agreement

Table 3: Acronyms

B ADDITIONAL EVALUATION RESULTS

In Figure 10, we present the throughput vs. latency results for the transaction size of 100 bytes for f=1,10,20. While the trend in this setting is similar to that with the transaction size of 250 bytes, FIN outperforms PACE in a more drastic way. Despite the case for f=1 where the peak throughput of FIN is 17.4% lower than that of PACE, FIN achieves 1.23x and 2.14x the peak throughput of PACE, for f=10 and f=20, respectively.

C PROOF OF OUR MVBA PROTOCOL WITH QUALITY

We show that the MVBA protocol presented in Sec. 5.3 additionally achieves the quality property. All the other properties except termination follow from our MVBA without the quality property. Therefore, we prove quality and termination in this section.

Lemma C.1. If a correct replica enters the iteration phase, then for at least 2f + 1 RBC instances, at least f + 1 correct replicas have r-delivered some values.

PROOF. If a correct replica p_i enters the iteration phase, it receives n-f (Rep) message. Prior to that, p_i has broadcast (Echo, W_i) where W_i consists of at least n-f 1's, i.e., p_i has completed n-f RBC instances. Each replica p_j replies with a (Rep) message only if for any $W_i[l] = 1$, p_j has also r-delivered some value in RBC $_l$. Hence, for any $W_i[l] = 1$, at least f + 1 correct replicas have r-delivered some value.

THEOREM C.2 (QUALITY). The probability of mvba-deciding a value that was proposed by a correct replica is at least 1/2.

PROOF. According to Lemma C.1, for at least 2f+1 RBC instances, at least f+1 correct replicas have r-delivered some value. From the biased validity property of RABA, if any of the 2f+1 RBC instances is selected by the Election() function, RABA will output 1. So every correct replica then mvba-decides. Therefore, the

probability that the decided value was proposed by an adversary is bounded by $\frac{f+1}{3f+1}$. As the probability of deciding a value proposed by a faulty replica is at most $\sum_{k=1}^{\infty} (1/3)^k = 1/2$, the probability of deciding a value that was proposed by a correct replica is at least 1/2.

Theorem C.3 (Termination). If all correct replicas are activated and all messages sent among correct replicas have been delivered, then all correct replicas mvba-decide.

PROOF. If all correct replicas are activated, each correct replica starts one RBC instance. According to the validity property of RBC, at least n-f RBC instances started by the n-f correct replicas will eventually complete. Then, each correct replica p_i sends a (Echo, W_i) message, every replica replies only if it has r-delivered some value in RBC $_l$ for any $W_i[l] = 1$. According to the agreement property of RBC, we know that for the message sent by p_i , every correct replica eventually replies with a (REP) message. Accordingly, every correct replica eventually enters the iteration phase.

During the iteration phase, we first prove that every iteration r completes and then show that eventually some RABA $_r$ outputs 1. For each iteration r, k is returned by the Election() function.

The proof that every iteration r completes is similar to that for the protocol without quality. We include one additional echo-and-reply procedure, where every replica sends its W_i to all replicas and proceeds to the next phase if it receives n - f replies. According to the agreement property of RBC, we know that every correct replicas eventually r-delivers some values in the same set of RBC instances in W_i . Thus, every correct replica completes each epoch.

We now prove that eventually, in some iteration r, RABA $_r$ outputs 1, so the protocol terminates. From Lemma C.1, for at least 2f+1 RBC instances, at least f+1 correct replicas have r-delivered some value after they enter the iteration phase. Let I be the set of the 2f+1 RBC instances. Due to the biased validity property of RABA, RABA $_r$ outputs 1. So with probability $\frac{2f+1}{3f+1} \approx \frac{2}{3}$, it holds that $k \in I$.

After RABA $_r$ outputs 1, every correct replica waits for the output of RBC $_k$. Note that if RABA $_r$ outputs 1, at least one correct replica has raba-proposeed 1 or raba-reproposed 1. This is due to the unanimous termination property of RABA. Therefore, at least one correct replica has r-delivered some value in RBC $_k$. Due to the agreement property of RBC, every correct replica eventually r-delivers some value in RBC $_k$ and then mvba-decides.

D PROOF OF OUR TAILORED MVBA

In this section, we prove the correctness of our tailored MVBA protocol. We first show a few lemmas about WRBC and then show the correctness of our tailored MVBA construction. As external validity is the same as the MVBA protocol presented in Sec. 5.2, we focus on agreement, integrity, and termination in this section.

LEMMA D.1. If a correct replica wr-delivers h and another correct replica wr-delivers h', then h = h'.

PROOF. If a correct replica p_i wr-delivers h, it receives n-f (Ready, h). If another correct replica p_j wr-delivers h', it receives n-f (Ready, h'). Therefore, at least one correct replica has sent

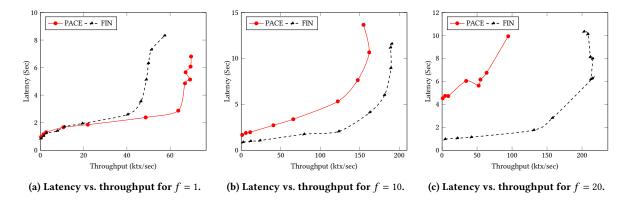


Figure 10: Evaluation results for transaction size of 100 bytes.

both (Ready, h) and (Ready, h'), a contradiction to the fact that every correct replica only sends a (Ready) message once.

Lemma D.2. If a correct replica wr-broadcasts a value v, every correct replica eventually wr-delivers h such that h = Hash(v).

PROOF. For any WRBC instance, if the sender is correct, it is straightforward to see that every correct replica receives the same (Send, v) message, broadcasts a (Echo, m)essage, and receives n - f (Echo, Hash(v)) messages. Then every correct replica eventually sends (Ready, Hash(v)) and will never receive f + 1 (Ready) messages with a value different from Hash(v). Hence, every correct replica eventually wr-delivers h = Hash(v).

LEMMA D.3. For any WRBC instance, if a correct replica wr-delivers some value h, any correct replica eventually wr-delivers some value.

PROOF. If a correct replica wr-delivers h, it receives n-f (Ready, h) messages, among which are least f+1 are sent by correct replicas. Thus, any correct replica that receives f+1 (Ready, h) messages but has not sent any (Ready) message will also send a (Ready, h). Therefore, every correct replica eventually receives n-f (Ready, h) messages and wr-delivers.

Lemma D.4. If a correct replica wr-delivers some value h in $WRBC_j$, at least f+1 correct replica receives (SEND, v) such that Hash(v).

PROOF. If a correct replica wr-delivers h, it receives n-f (Ready, h), among which are least f+1 are sent by correct replicas. Any of the correct replicas receive n-f (Echo, h), among which at least f+1 are sent by correct replicas. The correct replicas must have received (Send, v) from p_i such that Hash(v) = h.

For completeness, below, we provide self-contained proof for agreement, integrity, and termination. Note that the proof is very similar to that for our MVBA protocol in Sec. 5.2.

Theorem D.5 (Agreement). If a correct replica mvba-decides v, then any correct replica that terminates mvba-decides v.

PROOF. If a correct replica p_i mvba-decides v, we assume it has raba-decided 1 in RABA_r for some iteration r>0 and for any iteration $\hat{r}< r$, $\mathsf{RABA}_{\hat{r}}$ outputs 0. Furthermore, if k is returned by the $\mathsf{Election}()$ function in iteration r, WRBC_k outputs h and

 p_i receives some v (from p_k or from a (Value) message) such that H(v) = h.

We assume that another correct replica p_j mvba-decides $v' \neq v$ and prove the theorem by contradiction. We consider two cases: p_j mvba-decides in iteration r; p_j mvba-decides in iteration $r' \neq r$. Case 1: If p_j mvba-decides v' in round r, it obtains k' from Election() function and WRBC $_{k'}$ outputs h'. Then p_j either receives value v' from $p_{k'}$ such that Hash(v') = h' or receives v' from another replica in a (Value) message. As Election() outputs a common coin,

it must hold that k = k'. Thus, if $Hash(v) \neq h'$, p_i wr-delivers

 $h' \neq h$, a violation of Lemma D.1. Furthermore, if $v \neq v'$, the

collision resistance property of the hash function is violated.

Case 2: Without loss of generality, we assume r' > r. According to our protocol, p_j raba-decides 1 in RABA $_{r'}$ and raba-decides 0 for any iteration lower than r', including r. This would violate the agreement property of RABA.

Theorem D.6 (Integrity). If all replicas follow the protocol, and if a correct replica mvba-decides such that Q(v) holds, then some replica mvba-proposed such that Q(v) holds.

Proof. If a correct replica p_i mvba-decides v, it raba-decides 1 in some $RABA_r$ and wr-delivers h = H(v) in $WRBC_k$ where k is the output of the Election() function. According to our protocol, at least n-f replicas have sent (Ready, h) messages, among which at least one is sent by a correct replica. The correct replica has received n-f (Echo, h) messages, where at least f+1 message are sent by correct replicas. The correct replicas must have received a (Send, v) message from p_i such that Hash(v) = h.

Lemma D.7. If all correct replica enter the iteration phase, then for at least f + 1 WRBC instances, at least f + 1 correct replicas have wr-delivered some values.

PROOF. Correctness of the lemma is the same as that in Lemma 5.4, except that RBC is now WRBC. $\hfill\Box$

Theorem D.8 (Termination). If all correct replicas are activated and all messages sent among correct replicas have been delivered, then all correct replicas mvba-decide.

PROOF. If all correct replicas start the protocol, each correct replica starts one WRBC instance. According to Lemma D.2, any

correct replica completes a WRBC instance started by a correct replica. Thus, every correct replica completes at least n-f WRBC instances.

During the iteration phase, we consider each iteration r where k is the corresponding output of the Election() function. We first show that every iteration r completes and then eventually some RABA $_r$ outputs 1.

We first show that every iteration r completes. For each iteration r, there are three cases: 1) all correct replicas have wr-delivered some value in WRBC $_k$; 2) at least one correct replica has wr-delivered some value in WRBC $_k$ and at least one correct replica has not wr-delivered any value in WRBC $_k$; 3) none of the correct replicas have r-delivered any value in WRBC $_k$.

Case 1: According to the unanimous termination property, $RABA_r$ terminates.

Case 2: If at least one correct replica has wr-delivered some value in WRBC $_k$, then according to Lemma D.3, any correct replica eventually wr-delivers some value. Note that any correct replica that provides 0 as the RABA input (it has not wr-delivered any value in WRBC $_k$ when the iteration begins) will eventually raba-repropose 1. Thus, the biased termination condition of RABA is satisfied. RABA $_r$ will terminate and iteration r will complete.

Case 3: If none of the correct replicas wr-deliver any value in WRBC $_k$, iteration r completes due to the unanimous termination property of RABA. Otherwise, if at least one correct replica later wr-delivers some value in WRBC $_k$, then according to the case 2, iteration r completes due to the biased termination property of RABA

We now prove that eventually, in some iteration r, RABA $_r$ outputs 1 so the protocol terminates. From Lemma D.7, for at least f+1 WRBC instances, at least f+1 correct replicas have wr-delivered some value after they enter the iteration phase. Let I be the f+1 WRBC instances. According to the biased validity property of RABA, RABA $_r$ outputs 1. So with probability $\frac{f+1}{3f+1} \approx \frac{1}{3}$, we have $k \in I$

After RABA $_r$ outputs 1, every correct replica waits for the output of WRBC $_k$. We also know that if RABA $_r$ outputs 1, then at least one correct replica has raba-proposed 1 or raba-reproposed 1. Otherwise, the unanimous termination of RABA is violated. Therefore, at least one correct replica has wr-delivered some value h in WRBC $_k$. From Lemma D.3 and Lemma D.1, every correct replica eventually r-delivers h in WRBC $_k$. From Lemma D.4, at least f+1 correct replicas receive (Send, v) from v0 and set their v1 as v2 such that v3 the v4 such that v5 the v6 in Wrba-decides.